THE

BRYANT AND STRATTON

BUSINESS ARITHMETIC.

A New Work, with Practical Problems and Valuable Tables of Reference.

DESIGNED FOR

BUSINESS MEN, COMMERCIAL, AGRICULTURAL AND SCIENTIFIC COLLEGES, NORMAL AND HIGH SCHOOLS, ACADEMIES AND UNIVERSITIES.

BY

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PREFACE.

1. *Design.*—The design of this work is to afford such information to the student, accountant and business man, as will give a clear understanding of the science of numbers and of the art of commercial and business computations. It comprises the philosophy of numbers and their relations, and the complete analysis of operations needed for advanced classes of students, as well as concise rules and formulas, and extensive information of principles, facts and tables, required by practical business men in all departments. By omitting the more philosophical and abstract portions of Part First and selecting somewhat from Part Second, the book may also be used with classes that have but a very limited knowledge of numbers. The pupil is instructed how to reason correctly and then is required to *think for himself*, and in some cases to construct rules as well as to apply those given.

2. *Plan.*—The book is divided into three parts: the scientific, practical, and tabular, fully developed in a clear and logical manner.

PART FIRST comprises the *Science of Numbers.* Special attention is invited to the arrangement of subjects, the complete definitions, the illustrations of principles, analyses of relations and processes, methods of testing and contracting operations, the statistical character of many of the problems, the introduction of subjects by mental problems, the explanation of difficult and peculiar problems, the constant application of known principles to new subjects instead of the old method of treating each subject in an isolated manner, the encouragement of independent thought by the pupil, the comprehensive nature of analyses, problems and rules, the simple and yet very full treatment of Units, Fractions, Properties of Numbers, Miscellaneous Principles, Decimals, Denominate Numbers, Aliquot Parts, Ratio, Practice, Analysis

and its various applications, and new methods of explanation and operation in Compounding (Alligation), Involution, and Evolution.

PART SECOND is devoted chiefly to *Commercial Transactions* and *Computations;* it comprises a digest of fundamental principles, and full explanations of methods of computation with very extensive illustrations.

Aside from clear and exact definitions, concise rules, and lucid explanations, we have endeavored to present a system of *general principles* relating to the different subjects which will enable the student more fully to understand the *nature* and true *theory* of business transactions.

As the value of such a work as this depends greatly upon the character of its problems, we have aimed to present, as far as possible, those occurring in actual business. Many of them were presented to the authors within the last five years, by business men who required their solution for business purposes; especially is this true of the problems in Partnership and Investments.

Experience and observation have taught us, in relation to money, banks, interest, and exchange, that business students need something more than rules, forms, and tables. The various and contradictory opinions upon these subjects set forth by business men of even considerable experience, prove a lack of knowledge of first principles which should incite the student to a very thorough examination for himself.

The nature of Interest, and the principles of Exchange and Balance of Trade are fully explained, and, if found correct, will necessarily expose some radical but popular errors. The problems submitted will be found to contain facts and statistics supporting our views.

Each subject has been discussed according to the best authorities, and practical business men have very kindly afforded their advice and criticisms on the several portions with which they were most familiar.

Equation of Payments and Accounts, and Partnership Settlements are more fully treated than in any other work of the kind. The student is made a *rule to himself*, and by the rules and processes recommended he is led with "open eyes" into the usual perplexities of these subjects. The most ordinary practical problems in Mechanics, Natural Science and Mensuration are also explained.

PART THIRD consists of tables which will be found of great utility in all departments of business. These comprise rates of Interest and Exchange, Values of Foreign Coins, Measures and Weights (U. S. Custom House Standard), Compound Interest and Discount, Annuities,

Present Value of Bonds, and other miscellaneous standard tables not contained in any other single work.

Answers to most of the problems and a compendious Alphabetical Index are appended, to facilitate the use of the book as a work of reference.

3. *Authorship.*—The Authors have been engaged for many years as teachers in the first schools and commercial colleges of the country or in practical business. The *editorial* joint author was for several years instructor of mathematics in one of the oldest and best Institutes in New England, has had practical experience in business, and for some time has been the Principal of one of the largest public Grammar Schools in the West. His extensive acquaintance with professional and business men and leading educators, has enabled him to obtain such information and criticisms as are essential to the production of a work of this kind

The Business Arithmetic contains much of the matter (thoroughly revised) that was comprised in Bryant & Stratton's Commercial Arithmetic, so long and favorably known among business men and commercial students. New articles have been prepared with great care and approved by competent critics. The entire manuscript has been submitted to the leading author, who trusts that he now offers to the public a work of superior merit.

Mr. E. E. White is to be credited with much of the subject-matter and most of the problems taken from the Commercial Arithmetic which was originally prepared by him in connection with Mr. J. D. Merriam, Cashier of the City Bank, Cleveland, Ohio, and Messrs. Bryant & Stratton, Founders of the National Chain of Mercantile Colleges.

The work of revision, preparing or securing new articles, etc., has devolved upon Mr. Stowell, who has edited the entire book. Valuable suggestions have been obtained from many of the best arithmetics published, among which should be specially mentioned Felter's, Greenleaf's National, and the arithmetics of Robinson, Walton, and White. The other most valuable works consulted, are Parson's Laws of Business, Bryant & Stratton's Commercial Law, Bushnell's Legal Directory, McCulloch's Commercial Dictionary (London, 1869), Walker's Science of Wealth, Moran on Money, Merchants and Bankers' Magazine, Encyclopædia of Commerce by I. Smith Homans, the Dictionary of Mathematics by Davies & Peck, the New American Cyclopædia and Annuals, the valuable works of Ogden and of Heyl, adopted as standards by

the United States Treasury and Revenue Departments, and Webster's Counting House Dictionary.

Many thanks are tendered to professional and business men, librarians, editors, and Government officers, who have most courteously responded to queries presented to them, and afforded liberal facilities for obtaining the most valuable and reliable information.

An able instructor in Mathematics and Commercial Law has given special attention to Cube Root, Bankruptcy, Equation of Payments, Averaging Accounts and Partnership.

Annuities and Life Insurance have been treated by a leading Actuary; Investments and tables of the Present Value of Bonds have received the attention of experienced bankers and real estate dealers; Banking, Exchange and Stocks have been treated mainly by a practical banker and a member of the New York Stock Board.

The editor has made every effort to secure the most complete and reliable information, and has labored patiently to produce a work which it is hoped may be found of great service in the cause of sound practical education, and one of permanent value for reference in actual business.

H. B. BRYANT.

CHICAGO, *July 1, 1872.*

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BUSINESS ARITHMETIC.

PART FIRST.

THE SCIENCE OF NUMBERS.

SECTION I.

UNITS AND NUMBERS.

1. A ***Unit*** is a *single one;* as, a tree, a city, a person, an inch, an acre, a dollar, a cent, a degree, one, one, 1, 1, 1, 1.

2. There are *three classes of units:*

1. *Units of things.* These are *individual objects themselves,* and distinct *collections* or *groups* of such objects. The unit including all others is the *Universe.*

2. *Units of quantity.* These are *standards* of *measure* or *computation,* used for determining or indicating *extent, duration, weight, value,* and *intensity.*

3. *Units of number.* These are *expressions* showing *how many* things, quantities, or ones are to be considered. *Arithmetic* deals only with *this class* of units.

3. An ***Integer,*** or an *integral unit,* is a *whole* one. Thus, *one* is an integer, and *two* represents two integers.

4. A *fractional unit* is one of several equal parts into which an integer may be divided. Thus, *one-third* is a fractional unit, and *two-thirds* represents two fractional units.

5. 1. A unit referring to some *thing* or *quantity* is ***Concrete;*** as, one man, one hour.

2. Concrete units of *quantity* are called ***Denominate*** units.

6. A unit considered apart from any thing or quantity, *simply as one,* is ***Abstract.*** *The abstract unit is the basis of all numbers* and of the entire science of them. It may be indefinitely repeated in value, as in integral numbers, and it may be indefinitely divided into parts, as in fractional numbers.

7. A ***Number*** is an *expression* for one or more *units of number*. When the units expressed are integral, the expression is simply called *a number;* when the units are fractional it is called *a fraction;* when integral and fractional units are combined it is called *a mixed number*.

8. A ***Concrete*** or ***Denominate Number*** refers to *concrete* units; as, four hours, six pens.

9. An ***Abstract Number*** refers simply to *abstract* units; as, five, twenty.

10. A ***Simple Number*** expresses units of only *one kind* or denomination; as, sixty, forty-two inches, ninety dollars.

11. A ***Compound Number*** expresses units of *different* denominations but of the same variety, and having a fixed and known relation; as, three pounds, four ounces; two miles, ten rods, seven feet, nine inches.

12. In considering any number it is of the highest importance that the actual or relative *value* of the unit expressed be well understood. This value is the same for all abstract numbers, but varies in every variety of concrete and denominate numbers.

13. A unit of one kind may comprise several units of another kind of the same variety, or may be only a part of another unit.

14. A comprehensive view of the three classes of units is shown in the following Table of

THE UNIT SYSTEM.

	Class.	Including	Species.	Referring to	Variety.	Expressed by	Kind.
UNITS OF	I. THINGS		IMMATERIAL		Minds		*Names.*
					Thoughts		*Sentences.*
					Truths		
			MATERIAL		Animals		*Common names.*
					Vegetables		
					Minerals		
	II. QUANTITY		EXTENT		Distance		*Inches, ft., degs., etc.*
					Area		*Sq. in., etc.*
					Volume		*Cu. in., etc.*
			DURATION		Time		*Seconds, etc.*
			WEIGHT		Gravity		*Grains, oz., etc.*
			VALUE		Currency		*Dollars, etc.*
			INTENSITY		Forces		*Degrees.*
	III. *NUMBER*		CONCRETE		Things		*Words,*
					Quantity		
			Integral and Fractional				*Letters,*
			ABSTRACT		Ones		*or Figures.*

SECTION II.

RELATIONS AND OPERATIONS.

15. Relations.—Different numbers, referring to the same fundamental unit, have *relations of similarity and equality.*

6 is *similar* to 2, both being abstract, and 6 is *equal* to three 2's.

10 inches is similar to 5 inches, and 10 inches is equal to two 5 inches.

16. 1. A ***Problem*** in Arithmetic is a question to be solved or answered by some arithmetical operation. It states the relation between one or more given numbers and a required number, or proposes two or more numbers whose relation is to be found.

2. The *analysis* of a problem is the logical statement of its conditions and of the steps required in its solution.

3. The *answer* is sometimes called the *result.*

4. The *solution* is finding the answer.

17. A ***Rule*** is a statement of a process of solution, or of a method of operation, without indicating the analysis.

18. An ***Operation*** in Arithmetic is any process of *comparing, counting, expressing,* or *combining* units or parts of units.

19. Comparison is the process of determining the relations of numbers by comparing them with respect to kind or magnitude.

Comparison of kind determines whether the numbers are similar or dissimilar.

Comparison of magnitude determines how much greater one is than the other, or how many of one are equal to the other.

20. 1. The relation of *magnitude* is called ***Ratio.***

2. The *difference* between two numbers is called their ***Arithmetical Ratio.***

3. The quotient of one number, divided by another, is called their ***Geometrical Ratio,*** or simply their *Ratio.*

21. Counting is the process of naming in succession several numbers differing by one or more units. Thus, 1, 2, 3, 4, etc.; 9, 8, 7, 6, etc.; 1, 3, 5, 7, 9, etc.; 20, 17, 14, 11, 8, 5, etc.

22. ***Expressing*** numbers is using some word or character to indicate how many units are concerned.

23. Two or more numbers are *United* or **Combined** by counting together the units in both or all of them, and expressing the result in one number called their sum or amount.

24. Relations and operations are indicated by position or signs.

25. Units of greater value are placed at the left of units of less value.

26. ***Signs indicate*** operations or relations.

The following are the signs or symbols in common use:

27. The sign + is the sign of *addition*; it is called ***plus,*** signifying more, and denotes the *combination* or addition of any numbers between which it is placed, to form one number, called their *sum* or *amount.* A number or term before which it is placed is called *positive,* and is to be added.

28. × is the sign of *multiplication,* and denotes the *combination* of several equal numbers in one, called their *product;* it is read "times," "into," or "multiplied by."

29. − is the sign of *subtraction;* it is called *minus,* signifying *less,* and denotes the *comparison* of the number after it with the number before it to determine their *difference,* or Arithmetical ratio.

A number or term before which it is placed is called *negative,* and is to be subtracted.

30. ÷ is the sign of *division,* and indicates the *comparison* of the number before it with the number after it, to determine their *quotient.*

31. : is the sign of *ratio,* and indicates the same as the sign of division.

NOTE.—Several authors say that ratio is the quotient arising from dividing the number after the sign by the number before it.

32. ( ), [ ], —, the parenthesis and vinculum, are signs of *combination* or *aggregation,* and indicate that the *result* of the operations or relations included by them is to be considered as a single term in relation to other terms in an arithmetical combination.

33. $=$ denotes *equality*, and indicates that the term or terms after it are equivalent in value to all that is before it.

34. 5^3 indicates that 5 is to be used as a factor three times, or that its third power is to be found. The ^{3} is called an *index*.

35. $\sqrt{\ }$ is called the *radical sign*, and when used alone over a number indicates that the number is to be resolved into two equal factors, or that the square root is to be found. $\sqrt[3]{\ }$ indicates that the cube root is to be found.

36. $16^{\frac{1}{4}}$, or $(16)^{\frac{1}{4}} = \sqrt[4]{16}$, and indicates that 16 is to be resolved into four equal factors, or that the fourth root is to be found.

37. $\{$ the *bracket*, generally indicates a common relation between the number before it and two or more after it. Sometimes it indicates that a similar operation is to be performed with the number before it and each of the numbers after it.

38. $>$ and $<$ are signs of *inequality*, the larger number being placed near the opening, and the smaller number near the apex of the angle. Thus, $12 > 10$; $8 < 10$.

39. $\sim$ is the *indefinite sign of difference*, and indicates that the difference between two numbers is to be found without indicating which is the greater. Thus, $8 \sim 5 = 5 \sim 8 = 3$.

40. Numbers connected by signs constitute an *Arithmetical combination*; as, $4 + 3$; 2×3; $9 - 5$; $4 \div 2$; $8 : 4$; $5 + 2 \times 2 - 4 : 5$; $10 + 2 \div (2 \times 2) \times 5 - \overline{2 \times 5}$.

41. Any expression of equality is an *equation*. What precedes the sign of equality is the *first member* of the equation, and what follows the sign is the *second member* of the equation. The numbers in each member are the *terms* of the equation. $5 + 3 - 2 = 3 \times 2$; $10 + 2 \div (2 \times 2) \times 5 - \overline{2 \times 5} = 5$.

NOTES.—1. A *formula* is a general expression for an operation, and may be indicated by an equation.

2. The signs $+$ and $-$ are sometimes regarded as separating terms, and $\times$ and $\div$ as uniting into one term the numbers between which they stand, and these principles are generally applied in Algebra and higher mathematics. It is more simple in Arithmetical combinations to regard each number as a separate term unless the ( ) or vinculum be used, and this rule has been followed in this book. Thus, by the first principle stated, $17 - 4 \times 3 + 20 \div 5 = 17 - 12 + 4 = 9$, but according to the *arithmetical* method the value of the same is $11\frac{4}{5}$.

SECTION III.

NOTATION AND NUMERATION.

42. Notation is—1st. *The art of expressing numbers by certain characters,* each of which represents a certain number of units. 2d. *The art of expressing Arithmetical combinations,* or of properly using arithmetical characters and symbols. (Arts. 40, 41.)

NOTE.—Expressing numbers or combinations by *words* may be called *verbal notation.*

43. 1. A **Numerical Expression** is any number expressed by characters.

2. The *numerical value* of any character is the number of units denoted by it.

44. Numeration is the *art of reading numerical expressions.*

45. In Notation and Numeration we mentally *count* or compute the number of units to be expressed, *compare* the values of different units, *write* each in a particular relative position called its order, and *read* the expression verbally.

46. There are two methods of Notation in common use, the *Roman* and the *Arabic.*

ROMAN NOTATION.

47. By the *Roman Notation* numbers are expressed by one or more of seven capital letters. These letters are I, denoting one; V, five; X, ten; L, fifty; C, one hundred; D, five hundred; M, one thousand. These are called *Roman numerals.*

48. 1. If any letter is repeated its value is repeated. III = three, etc.

2. If any letter is annexed to one of greater value than itself, the sum of the two is indicated, but if prefixed to one of greater value, their difference is indicated. VI denotes V + I = six. IV denotes V − I = four. LX = L + X = sixty. XL = L − X = forty.

49. A dash —, or vinculum ——, placed over Roman numerals denotes a thousandfold their original value. $\overline{V}$ = five thousand. $\overline{XXVIII}$ = twenty-eight thousand.

50. The following table contains the principal combinations of Roman numerals:

TABLE.

I	denotes	*one.*	XXI	denotes	*twenty-one.*
II	"	*two.*	XXX	"	*thirty.*
III	"	*three.*	XL	"	*forty.*
IV	"	*four.*	L	"	*fifty.*
V	"	*five.*	LX	"	*sixty.*
VI	"	*six.*	LXX	"	*seventy.*
VII	"	*seven.*	LXXX	"	*eighty.*
VIII	"	*eight.*	XC	"	*ninety.*
IX	"	*nine.*	C	"	*one hundred.*
X	"	*ten.*	CC	"	*two hundred.*
XI	"	*eleven.*	CCC	"	*three hundred.*
XII	"	*twelve.*	CCCC or CD	"	*four hundred.*
XIII	"	*thirteen.*	D	"	*five hundred.*
XIV	"	*fourteen.*	DC	"	*six hundred.*
XV	"	*fifteen.*	DCCCC or CM	"	*nine hundred.*
XVI	"	*sixteen.*	M	"	*one thousand.*
XVII	"	*seventeen.*	MD	"	*fifteen hundred.*
XVIII	"	*eighteen.*	MM	"	*two thousand.*
XIX	"	*nineteen.*	$\overline{XX}$	"	*twenty thousand.*
XX	"	*twenty.*	$\overline{M}$	"	*one million.*

51. The character Ɔ is found in some Latin inscriptions. CIƆ was used for one thousand; CCIƆƆ for ten thousand; CCCIƆƆƆ one hundred thousand, etc., the number of thousands being increased tenfold by prefixing C and annexing Ɔ.

52. Examples.—Express the following in Roman numerals:

1.	24.	*6.*	68.	*11.*	407.	*16.*	2564.
2.	27.	*7.*	73.	*12.*	740.	*17.*	1890.
3.	39.	*8.*	82.	*13.*	909.	*18.*	1871.
4.	45.	*9.*	99.	*14.*	973.	*19.*	50692.
5.	56.	*10.*	159.	*15.*	1040.	*20.*	1,204,076.

Note.—Express the equivalents of the partial products in examples in Multiplication and Division, in Roman numerals.

53. Read the following examples:

1. XCIV.	*8.* $\overline{\text{MMCD}}$XCIX.	*15.* $\overline{\text{L}}$XXXIX.
2. DLX.	*9.* CMLXX.	*16.* $\overline{\text{DCXIV}}$.
3. XLIX.	*10.* DCCXXXVIII.	*17.* $\overline{\text{XLIII}}$.
4. XXVII.	*11.* $\overline{\text{LX}}$CCCII.	*18.* CCXXII.
5. DCLXVI.	*12.* MDCCCLXXII.	*19.* MCM.
6. MXLIII.	*13.* $\overline{\text{IX}}$CC.	*20.* $\overline{\text{VII}}$DCCLXVII.
7. $\overline{\text{XXX}}$XIX.	*14.* CCCCIↃↃↃↃ.	

ARABIC NOTATION.

54. By the ***Arabic Notation*** numbers are expressed by one or more of ten characters or *figures*. These are the *cipher* 0, denoting *naught;* 1, denoting one; 2, two; 3, three; 4, four; 5, five; 6, six; 7, seven; 8, eight; 9, nine. All but 0 are significant figures. These are also called the nine *digits.*

55. The cipher is called zero or naught. It represents no value itself; it is used only in connection with other figures to indicate their proper relative order and value.

56. The ***Decimal System*** of numbers is based on the following principles:

Principle 1. A certain *place* in every number is called the *first order,* and is considered as the place of the fundamental unit, 1. A significant figure occupying this place represents simply a certain number of ones, or units of the first order. The *orders* extend right and left from the first.

Principle 2. The *ratio* (Art. 20, 3) of a unit of any order to a unit of the next lower, or right-hand order, is *ten*. Hence units in this system are said to *increase towards the left,* and *decrease towards the right* in a *tenfold ratio.*

NOTE.—The units in a duodecimal system have a twelvefold ratio. The name of any numerical system is derived from the ratio of the units of that system.

57. The orders at the left of the first are called *second, third, fourth,* etc., and the units of these orders are—2d, *tens,* 3d, *hundreds,* 4th, *thousands,* etc.

58. The orders at the right of the first are the *second decimal order*, the *third decimal order*, the *fourth decimal order*, etc., and the units of these orders are—2d, *tenths*, 3d, *hundredths*, 4th, *thousandths*, etc. The orders of decimals are separated from the first order by a *period*, called the *decimal point.*

59. To express a unit of the second order, 1 is combined with 0; thus 10 = one *ten*, and is equivalent to ten units of the first order.

In a similar manner any number of tens may be expressed; as, two tens is expressed by 20, three tens by 30, etc.

60. A unit of the third order is expressed thus, 100; it is called one *hundred*, and is equivalent to ten tens.

NOTE.—Removing a figure to the left, by annexing a cipher or changing the place of the decimal point, increases its value tenfold. Thus 4 becomes 40; .4 becomes 4.

61. The orders above the third are classified in groups or periods of threes. To the 1st or lowest order in a group the distinguishing name of the group is given; the 2d order in each group is called tens of that group, and the 3d is called hundreds of that group.

62. Combining units of different orders by writing each figure in its proper place, their sum is expressed; thus any number may be formed.

NOTE.—When one number is annexed to another so as to form one new number, it is equivalent to increasing the first number tenfold, a hundredfold, or more, and then combining the second number with it. Thus if to 3 we annex 14, making 314, it is equivalent to increasing 3 a hundredfold by annexing two ciphers, and combining 14 with the 300.

63. The names given to different numbers indicate the units composing them. Thus thir-*teen* means three and ten, fif*ty*-six means five tens and six.

64. For convenience in reading numbers (Art. 44) consisting of several integral figures, the groups are separated by commas.

NOTE.—1 This method of separating into groups of three is the French method, and is used in the United States and in most countries in Europe.

2 Another method, the English, includes six orders in a group, repeating the first six orders of the system with the distinguishing name of the group. This method is used generally in Great Britain and the British Provinces.

65. RULE FOR NUMERATION.

1st. *Determine the order and name of each significant figure in the number to be read.*

2d. *Begin at the left hand, and name the significant figures in each group, together with their relative unit value, and add the name of the group.*

66. The general arrangement of the units of the decimal system, and the names applied to some of the different orders, may be seen from the following form of the

NUMERATION TABLE.

FRENCH.	No. of Group.	4th.			3d.			2d.			1st.			1st Dec.*		2d Dec.		
	Group . .	Billions.			Millions.			Thousands.			Units.			Tenths.		Thousandths.		
	Names . .	Hundred Billions.	Ten Billions.	Billions.	Hundred Millions.	Ten Millions.	Millions.	Hundred Thousands.	Ten Thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten Thousandths.	Hundred Thousandths.
	Orders . .	12th	11th	10th	9th	8th	7th	6th	5th	4th	3d	2d	1st	2d Dec.	3d Dec.	4th Dec.	5th Dec.	6th Dec.
	Units . .	1	1	1	1	1	1	1	1	1	1	1	1.	1	1	1	1	1
ENGLISH.	Names .	Hundred Thousand Millions.	Ten Thousand Millions.	Thousand Millions.														
	Groups	2d Period. Millions.						1st Period. Units.										

* *Dec.* here means *Decimal.*

NOTE.—The figure indicating the number of an order is called the *index* of the order; as, 4 is the index of the 4th order.

67. The name for any group above the third in the French table may be readily derived from the Latin word for the number two less than the number of the group. Thus the name for the seventh period or group is *quintillions*, derived from *quinque*, the Latin word for five; the name for the twelfth group is *decillions*, derived from *decem*, the Latin word for ten.

NOTE.—The Latin words for the first twenty numbers are as follows: *Unus* means one; *duo*, two; *tres*, three; *quatuor*, four; *quinque*, five; *sex*, six; *septem*, seven; *octo*, eight; *novem*, nine; *decem*, ten; *undecim*, eleven; *duodecim*, twelve; *tredecim*, thirteen; *quatuordecim*, fourteen; *quindecim*, fifteen; *sexdecim*, sixteen; *septendecim*, seventeen; *octodecim*, eighteen; *novendecim*, nineteen; *viginti*, twenty.

68. Denominate numbers whose units have a decimal relation, may be expressed like simple numbers. Thus two dollars, four dimes, and five cents may be written \$2.45. All numbers of the *Metric System* can be thus expressed.

NOTE.—When the ratio is the same between every two successive orders, the notation is said to have a *constant scale*, but when the ratio is not the same between every two successive orders, the notation is said to have a *varying scale*.

69. Numbers may be expressed according to the following

RULE FOR NOTATION.

Write each significant figure in the order of units to which it belongs, and write zero in every place not otherwise occupied between the decimal point and the extreme right or left-hand order occupied.

NOTE.—It is generally convenient to begin writing numbers at the left hand, filling every place with a significant figure or a cipher.

70. EXAMPLES IN NOTATION AND NUMERATION.

NOTE.—The numbers following, if not written under one another, will afford better drill.

1. Write figures for three hundred, forty; six hundred, fifty-one; nine hundred, two; one thousand, four hundred, six; eight thousand, forty-seven.

2. Nineteen thousand, twelve; ten thousand, five hundred, eighty; three hundred sixty thousand, thirty-six; two hundred four thousand, ten; four million, nine thousand, two hundred.

3. Ten million, forty thousand, twenty; seven hundred one million, two hundred, fourteen; three hundred twenty million, six hundred thousand, nine; forty billion, two million, eight hundred; four quintillion, eighty trillion, forty.

4. Two units of 4th order, seven of 2d, nine of 1st; eight of 8th order, six of 5th, three of 2d; two of 10th order, five of 7th, four of 3d; six of 12th order, nine of 9th, one of 5th, three of 2d; three of 15th order, four of 11th, one of 7th, two of 5th, eight of 1st.

5. Eighteen billion, forty thousand, ninety-six; four of 3d

order, six of 8th, nine of 5th, two of 1st, three of 6th; forty million, two thousand, fifty; six of 2d order, eight of 7th, nine of 10th, two of 3d, one of 2d; ninety-seven hundred thousand, sixteen hundred, nine.

6. Read or write words for 70906; 40002007; 9090009; 30041; 1900000740003020; 90805000600; 204003000500006000007089001060; 40000500030020190800 7; 321407972896001; 9090099900090090.

7. Read at sight 7040; 90081; 642009; 4720260; 30209; 604005; 2680904; 20005; 6096; 700040; and others similar.

8. Write and read twenty numbers, each consisting of more than six figures, and each containing four or more ciphers differently placed.

71. 1. The ***Simple Value*** of any figure is its value when standing in the *first* order.

2. The ***Local Value*** of any figure is its value when located in any other place than the first.

72. The value of all simple numbers is computed in units of the first order, and such numbers are read as units of this order. Thus *six* hundreds = *six-hundred* units of the 1st order, and 600 is read *six hundred* instead of six *hundreds.* 40 is read *forty* (units of the 1st order) instead of four *tens.*

REDUCTION, ANALYSIS, AND TRANSFORMATION OF NUMBERS.

73. As all units of higher orders have their equivalents in units of lower orders, any number may be transformed by reduction.

74. 1. ***Reduction*** of numbers is computing or counting the value of units of one kind or order, in units of a higher or lower order. This process is based on comparison. (Art. 19.)

2. *Reduction ascending* is reducing numbers to units of higher orders by dividing the number of units of the kind expressed by the number of them equivalent to a unit of the order required. Thus 400 units = 40 tens, or 4 hundreds. 360 inches = 10 yds., or 30 ft.

3. *Reduction descending* is reducing units to numbers of lower orders by multiplying the number expressed by the number of

units of the kind required equivalent to one of the kind expressed. Thus 3 hundreds = 300 units, or 30 tens, or 3000 tenths.

$2. = 200 cts., or 20 dimes, or 2000 mills.

2 yds. = 72 inches, or 6 ft.

75. Transformation of numbers is expressing their equivalent value in units of higher or lower orders.

NOTE.—*Coördinate* units are units of the same rank or order; *subordinate* units are of lower rank than others; *superior* units are of higher rank than others.

76. Analysis of Numbers is separating them into their elements by comparing and expressing the orders or names of the different figures.

NOTE.—In analysis and transformation the orders may be numbered or the names abbreviated. Thus *u* may signify units; *t*, tens; *h*, hundreds; *th*, thousands; *tth*, ten thousands; *hth*, hundred thousands, etc.

77. ILLUSTRATION OF ANALYSIS AND TRANSFORMATION.

By analysis, 41 is found to consist of $\overset{t}{4} + \overset{u}{1}$ (Art. 62), and this may be transformed by reducing the $\overset{t}{4}$ to units, making the number $= \overset{u}{\overline{41}}$. Or $\overset{t}{1}$ may be reduced to units, making the number $= \overset{t}{3} + \overset{u}{11}$.

623 = 6 of 3d order + 2 of 2d + 3 of 1st $= \overset{h}{6}\ \overset{t}{2}\ \overset{u}{3} = \overset{t}{\overline{62}} + \overset{u}{3} = \overset{u}{\overline{623}} = \overset{h}{5} + \overset{t}{12} + \overset{u}{3} = \overset{h}{5} + \overset{t}{11} + \overset{u}{13}$. $\overset{u}{\overline{23}} = \overset{t}{2} + \overset{u}{3}$.

78. A great variety of transformations might be made, but the most common arise from one of the following:

1. Reducing subordinate units to superiors; as, $\overset{u}{60} = \overset{t}{6}$; $\overset{u}{53} = \overset{t}{5} + \overset{u}{3}$; $\overset{u}{127} = \overset{t}{12} + \overset{u}{7} = \overset{h}{1} + \overset{t}{2} + \overset{u}{7}$. This is applied chiefly in combining numbers by addition or multiplication, so as to express the result in the simplest manner.

2. Reducing the entire number of superior units to subordinates; as, $\overset{h}{6} = \overset{t}{60} = \overset{u}{600}$. This is applied chiefly in reading numbers, and in treating partial dividends; as, $\overset{t}{3} + \overset{u}{4} = \overset{u}{34}$; $\overset{th}{1} + \overset{h}{2} + \overset{t}{0} = \overset{t}{120}$.

3. Reducing a single superior unit to subordinates; as, $\overset{t}{3} + \overset{u}{1} = \overset{t}{2} + \overset{u}{11}$. This is applied chiefly in subtraction, when a figure of the minuend is less than its co-ordinate of the subtrahend.

79. The following is a convenient method of indicating the analysis and transformation of any number:

1. 10 10 10 Constant Scale. (Art. 68, Note.)

4th th.	3d h	2d t.	1st u.	
			7421	
		742	1	Analysis and 1st transformation.
	74	2	1	
7	4	2	1	
6	14	2	1	3d transformation.
6	13	12	1	
6	13	11	11	

2. 10 10 10 10 10 Constant Scale.

	hth.	tth.	th.	h.	t.	u.	
501705 units.	5	0	1	7	0	5	1st transformation and analysis.
		50	1	7	0	5	2d transformat'n.
			501	7	0	5	
				5017	0	5	
					50170	5	
						501705	
	4	9	10	16	9	15	3d transformation.

3. 10 10 10 Constant Scale.

	E.	Doll.	Dimes.	Cts.	
5275 cts.	5	2	7	5	Analysis.
		52	7	5	2d transformation.
			527	5	
				5275	
			527.5		1st transformation.
(Common form.)		52.75			
	5.275				
	4	12	7	5	3d transformation.
	4	11	17	5	
	4	11	16	15	

4. 12 20 24 Varying Scale.

	Lb.	Oz.	Pwt.	Gr.	
22150 grs.				22150	
		927	927	2	1st transformation.
	46	46	7	2	
	3	10	7	2	
				22150	2d transformation.
	2	21	26	26	3d transformation.

NOTE.—The transformed number may be written above the original number. Thus $40213 = \overset{tth}{3} + \overset{th}{9} + \overset{h}{11} + \overset{t}{10} + \overset{u}{13}$, or $\left\{\begin{array}{ccccc} 3 & 9 & 11 & 10 & 13 \\ 4 & 0 & 2 & 1 & 3 \end{array}\right.$.

EXAMPLES.

80. 1. Analyze and transform 421; 9188; 17040; 200605; 57420612.

2. Transform 420 cts.; 7265 cts.; $17.50; $2.

3. Transform 3 ft. 2 in.; 48 in.; 36 oz.; 50 pwt.

4. Transform 27 of 2d order + 16 of first order; 10 of 3d order + 10 of 4th order; 7 of 5th order + 21 of 3d order + 10 of 1st order.

5. Analyze and transform the Examples in Article 70.

SECTION IV.

ADDITION.

PROBLEMS FOR MENTAL SOLUTION.

81. 1. How many men are 5 men, 7 men, 11 men, and 8 men?

2. How many dollars in three boxes if the first contain $9, the second $4, and the third $6?

NOTE.—The mark $ signifies dollars of U. S. money. The original form of it was probably a barred 8, $\not{8}$ signifying 8 *Reals*, the divisions of the Spanish dollar, which was taken as the basis of the U. S. coinage.

3. If you should pay $4 for a hat, $12 for a coat, and $10 for some boots, how much would they all cost you?

4. If there are 7 verbenas, 6 daisies, 5 geraniums, and 4 roses in a bouquet, how many flowers in the bouquet?

5. If in a certain hardware store there are 11 ten-pound weights, 9 pound weights, 10 eight-ounce weights, 12 ounce weights, and 10 half-ounce weights, how many *weights* in all?

6. If you should write 7 thirteens, 8 twenties, 5 six-hundreds, and 3 one-thousand-sixty-four's, how many *numbers* would you write in all?

7. If in a certain room there are 7 chairs, 2 sofas, 2 tables, a stand, a book-case, a piano, and a stove, how many articles of furniture in the room?

NOTE.—The pupil may here be shown that units of different *kinds* may be counted together as units of the same *species*. (Art. 14.)

8. If in one house there are 3 men, 4 women, and 2 children; in another 5 men, 4 women and 3 children; and in another 2 men, 2 women, and 1 child; how many men in the three houses? how many women? how many children? how many persons?

9. How many eggs in three baskets, the first containing 2 dozen, the second one-half dozen, and the third 11 eggs?

10. How many *bills* has a man who has a $5 bill, a $2 bill, a $10 bill, a $3 bill, and a $20 bill? How many *dollars* has the same man?

11. How many *what* has a farmer who has 7 sheep, 10 cows, 4 horses, 2 mules, and 4 oxen?

12. How many days in the three winter months? How many in the spring months? How many in the summer months? How many in the autumn months?

13. How many letters in the first ten words on this page?

14. How many panes of glass in the windows of the school-room?

15. How many are $5+6+7+8+9+10+11+12+13+14$?

NOTE.—The pupil should think *eleven*, *eighteen*, etc., and not five, six, seven, eight, nine, ten, *eleven*, for the sum of the first two terms, etc.

16. $8 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7$?

17. $19 + 8 + 8 + 8 + 9 + 9 + 9 + 6 + 6 + 6$?

18. $40 + 12 + 9 + 7 + 13 + 8 + 6 + 12 + 7 + 5$?

19. $59 + 11 + 12 + 10 + 13 + 9 + 14 + 8 + 15 + 7$?

NOTE.—Many other combinations should be given orally by the teacher, until, by frequent and *rapid* practice, the pupil can perform the addition as rapidly as the numbers can be named.

20. If five cents were paid for an orange, ten cents for a pencil, and three cents for some paper, the number of cents paid for all would be the amount of 5 cts., 10 cts., and 3 cts., counted together, and this amount would be expressed in one number, 18 cts., called the *sum* of the three numbers.

This operation would be properly indicated by the equation (Art. 41),

$$5 \text{ cts.} + 10 \text{ cts.} + 3 \text{ cts.} = 18 \text{ cts.}$$

82. *Addition* is the process of uniting or combining several numbers in one sum or amount, equivalent to all the units in all the numbers.

NOTE.—The numbers to be added have been called *addends* by a recent writer.

83. 1. Only coördinate units of the same species or kind can be added.

3 men + 4 men = 7 men; 10 yd. + 6 yd. = 16 yd. 3 men and 10 yd. cannot be added.

12 yd. and 2 ft. cannot be added, but the 12 yd. can be reduced to ft.; 12 yd. = 36 ft., and 36 ft. + 2 ft. = 38 ft.

Six tens and four units cannot be added, but six tens may be reduced to sixty units, and 60 + 4 = 64.

2. In adding large numbers it is convenient to arrange coördinates of the same kind in a single column; otherwise errors would be more likely to occur.

84. It is desirable to express as many superior units as possible, so when several numbers are to be added it is more convenient to add the column of the lowest order first, expressing, in each order of the amount, only such amounts or parts of amounts as are not equivalent to one or more of the next higher order, and combining with the amount of each column the superior units derived from the sum of lower orders.

Thus, if the sum of a column of the 1st order be 37, only 7 is written as units, the 30 being reduced to 3 tens and added with the column of tens. If the sum of a column be 128, the 8 is written in the order of the column, the 120 is reduced to 12 of the next higher order and combined with the figures of that order. If the sum be 27 inches, only 3 is expressed as inches, the 24 being reduced to 2 ft. and expressed as such.

85. The general method of adding is expressed by the following

RULE.

1. *Write the numbers to be added so that coördinate figures shall be arranged in columns.*

2. *Beginning at the top or bottom of the right-hand column, compute and name, in succession, the sum of the first two or more numbers in the column, and the amount of this sum and one or more of the next numbers, until the entire column is added; proceed thus with each column in order.*

3. *Reduce the sum of each column, when first found, to units of the next higher order, writing in the order of the column only such a part of the sum as is not equivalent to a superior unit.*

NOTES.—1. In adding numbers not arranged in columns, as in parts of Bills and Accounts, *care* should be taken to *combine coördinates*, in reading partial amounts.

2. When only two numbers are to be added, it is more convenient to begin with the highest order, writing the amount in regular order from left to right, observing to increase each partial sum by *one*, if the sum of the figures of the next lower order is more than 9.

3. In adding numbers of the *Decimal System*, it is only necessary to express the right-hand figure of the sum of each column in its order, combining the other part of the sum with the next higher order. This is generally called "writing down the units and carrying the tens to the next column."

4. The general Rule given is adapted to the addition of Compound as well as Simple numbers.

86. ILLUSTRATIONS.

1.
```
 431
 709
 262
  44
 700
   2
  31
----
2179
```

Beginning at the top of the right-hand column, read 10, 12, 16, 18, 19, or 10, 16, 19. Write 9 in the 1st order, and combining the (10 of 1st order =) 1 of 2d order with the first number in the next column, read 4, 10, 14, 17, or 4, 14, 17. Write 7 in the 2d order, combine 1 with 4 of 3d order, and read 5, 12, 14, 21, or 5, 14, 21. Write 1 in the 3d order and 2 in the 4th order. The amounts may be read either upwards or downwards, but the pupil should not say one and two are three, three and four are seven, seven and two are nine, nine and nine are eighteen, eighteen and one are nineteen, neither should counting by ones (as on the fingers) *ever* be allowed.

2. Add $\left\{\begin{array}{r} 47999 \\ 81892 \end{array}\right\}$. Beginning at the *left*, write the amount at once.

$$\begin{array}{r} 47999 \\ 81892 \\ \hline 129891 \end{array}$$

3. $23.50 4.375 .25 400. $428.125 — Beginning with the right-hand order, write 5 mills in the 4th dec. order. Read the next column 12 cents. Write 2 in the order of hundredths, and combining the 1 with the next column read 6, 9, *11*. Write 1 dime in tenths' place, and combining (10 dimes =) $1 with the next column, write its sum 8 in 1st order, 2 in 2d order, and 4 in 3d order, making the entire amount four hundred twenty-eight dollars, twelve cents, five mills.

$23.50
4.375
.25
400.
$428.125

4. Add $3.75 ; .25 ; 4.82 ; 9.75 ; 8.48 ; 10.70 ; .37, without arranging coördinates in columns. In such a case each order must be kept distinct in the mind when adding. Beginning with the lowest order of the first number, we may read towards the right, 10, 12, 17, 25, *32*. Writing down the 2 we next read 10, 12, 20, 27, 31, 38, *41*. Writing the 1 in tenths' place, we read 7, 11, 20, 28, *38*. Writing 38 as dollars, we have $38.12 as the entire sum.

87. *Tests.*—The accuracy of the work is generally tested by reading the additions in an opposite direction from that in which the additions were first made, upwards or downwards, or if not in columns, towards the right or left. If the amounts obtained in both readings are the same, it is highly probable that the results are correct, but if they are not equal, there is certainly an error in the first or second operation, and this must be found by carefully reviewing the work in both directions.

88. Another method of testing additions depends on the following principles:

1. The sum of several numbers, divided by any number, will give the same remainder as would be obtained by dividing each of the numbers by the same divisor and then dividing the sum of the remainders by the same.

		Quotient.	Rem.	
Thus	$468 \div 10 =$	46 ;	[illegible]	
	$926 \div 10 =$	92 ;	6	
	$340 \div 10 =$	34 ;	0	
Sum	$1734 \div 10 =$	173 ;	4	These remainders are always equal.
And	$(8 + 6) \div 10 =$	1 ;	4	

		Quotient.	Rem.	
	$64 \div 7 =$	9;	1	
	$23 \div 7 =$	3;	2	
	$95 \div 7 =$	13;	4	
Sum	$182 \div 7 =$	26;	0	Equal remainders.
And	$(1 + 2 + 4) \div 7 =$	1;	0	

2. Any number divided by 9 will give the same remainder as the sum of its digits divided by 9. (Art. 188.)

Thus $17092 \div 9 = 1899$ and 1 rem.,

And $1 + 7 + 0 + 9 + 2, (= 19) \div 9 = 2$ and 1 rem.

NOTE.—The number one less than the base or radix of any system of numbers, holds the same relation to numbers in that system as 9 does to all numbers in the decimal system. This is true of 11 in the duodecimal system, 6 in the septenary, etc.

89. *Test by Nines.*—1. Divide the sum of the digits in each number by 9, and write down the remainders.

2. Divide the sum of the remainders thus found by 9, and write down the remainder.

3. Divide the sum of the digits in the amount by 9, and if the remainder thus found equals the remainder found just before, the amount is correct.

Thus,	76093	$(7+6+0+9+3)\div 9$,	gives rem.	$=7$
	4126	$(4+1+2+6)\div 9$,	"	$=4$
	701045	$(7+0+1+0+4+5)\div 9$,	"	$=8$
	364	$(3+6+4)\div 9$,	"	$=4$
	15	$(1+5)\div 9$,	"	$=6$
	781643	$(7+4+8+4+6)\div 9$,	"	$=2$
		$(7+8+1+6+4+3)\div 9$,	"	$=2$

Observation.—Accuracy must be acquired by careful practice, *rapidity* may be acquired. These two acquirements are the most desirable scientific qualifications of an accountant.

90. *Adding several columns at once.*—1. Considerable practice will enable the accountant to add more than one column at once. This contraction cannot well be extended to more than three or four columns with any practical advantage.

The process in such cases is merely an extended application of the plan of beginning at the left to add. (Art. 85, Note 2.)

Thus in adding $\left\{\begin{array}{r} 84 \\ 79 \end{array}\right\}$ it is not difficult to begin at the left, and,

$$\begin{array}{r} 163 \end{array}$$

observing that 1 of 2d order will be derived from the first column, read 163 at once as the sum of the two numbers.

$$\begin{array}{r} 76 \\ 85 \\ 93 \\ 67 \\ 98 \\ \underline{46} \\ 465 \end{array}$$

2. If 76, 85, 93, 67, 98, and 46, are to be added, we may write them in the usual manner, and, beginning at the top, may read 161, 254, 321, 419, *465*; or, reading upwards, 144, 211, 304, 389, *465*.

3. Add in same manner 83 + 42 + 64 + 50 + 47 + 56 + 89 + 75 + 92 + 45 + 71 + 44 + 53 + 82 + 78 + 95 + 87 + 26.

$$\begin{array}{r} 421 \\ 768 \\ 507 \\ 952 \\ 350 \\ 875 \\ \hline 3873 \end{array}$$

4. Three or more columns may be added in the same manner. Thus we may read downwards 1189, 1696, 2648, 2998, *3873*; or upwards 1225, 2177, 2684, 3452, *3873*.

5. Add in same manner 243 + 465 + 371 + 842 + 607 + 829 + 743 + 623 + 247 + 356 + 465 + 283 + 472 + 574 + 520 + 806 + 943 + 721 + 845 + 297.

$$\begin{array}{r} \$10.75 \\ 3.00 \\ 4.25 \\ .50 \\ 1.20 \\ 207.80 \\ 35.18 \\ .15 \\ .02 \\ 5.00 \\ 11.06 \\ .42 \\ \hline \$279.33 \end{array}$$

6. When columns are incomplete, the operation is not at all difficult, as in some Bill and Ledger columns. Read downwards 13.75, 18, 18.50, 19.70, 227.50, 262.68, 262.83, 262.85, 267.85, 278.91, *279.33*.

7. Add in same manner in *one* sum—

$7.50	$152.25	$40.50
12.35	10.05	12.25
200.00	18.50	160.00
81.15	17.43	70.40
19.60	20.00	45.50
42.08	100.50	50.00
150.25	7.25	200.00

8. This method may be used in adding horizontally. Thus, 4.25, 3.02, .75, 1.50, 17.12, 5.00, 2.18, may be read towards the right, 7.27, 8.02, 9.52, 26.64, 31.64, *33.82*.

91. EXERCISES IN COUNTING AND RAPID COMPUTATION.

1.	Count by *fours*	from	7 to 87.
2.	" *fives*	"	3 to 103.
3.	" *sixes*	"	5 to 125.
4.	" *sevens*	"	4 to 144.
5.	" *eights*	"	3 to 163.
6.	" *nines*	"	2 to 182.
7.	" *twelves*	"	5 to 245.
8.	" *seventeens*	"	6 to 346.
9.	" *eighteens*	"	7 to 367.
10.	" *twenty-threes*	"	8 to 468.

92. EXAMPLES IN ADDITION.

NOTE.—The following additions should be performed by different methods, each result should be tested, and contractions should be practiced till sufficient skill is acquired by the student to render him *accurate* and *rapid* in such computations.

1. Seven thousand 4 hundred, XI + $\overline{\text{VI}}$, nine hundred, twenty + 19 million, 30 thousand, DC + eleven hundred thousand, 17 hundred, nine + two hundred seventy million, XLIX thousand, 64 + ninety-nine million, CMIX thousand, DLXIX.

2. $\overline{\text{XV}}$, sixty + MMDCCLXXVII + fourteen million, sixty-nine thousand, forty + 97 billion, 689 million, 8 thousand, twelve + CXL million, seventy-six thousand, four + twelve hundred seventy-nine thousand, ninety.

3. Ninety-eight million, $\overline{\text{DLV}}$, eighty-seven + $\overline{\text{MMM}}$, forty-nine thousand, CMXLIX + 280 thousand, six hundred LXXIV + nine hundred eight thousand, 98 tens, VII + 79 million, 879 thousand, six hundred, 99.

4. 87 billion, 384 thousand, eighty-four + sixty-seven million, 49 thousand, ten + 907 thousand, twelve hundred seventy + eighty-eight billion, 12 million, 67 thousand + 9 billion, 900 million, 80 thousand, 900.

5. 760 + 400250 + 89997 + 9877 + 786 + 5432 + 10897 + 780040 + 600899 + 8997.

6. 9999 + 88080 + 707007 + 66778 + 890769 + 888 + 7777 + 999999 + 78966 + 987654.

7. 12345 + 67890 + 23456 + 78902 + 34567 + 89023 + 45678 + 90123 + 56789 + 98765.

8. 987654 + 976548 + 965487 + 954876 + 948765 + 876549 + 865497 + 854976 + 849765 + 765498 + 7549876 + 749876 + 654987 + 649876 + 549876 + 456789 + 567894 + 678945 + 789456 + 894567.

9. 11.50 + 2.45 + .75 + 9.68 + 10.25 + 1.12 + .75 + 1.82 + 16.36 + 4.87 + 19.15 + 12.25 + 5.00 + 2.50 + 19.20 + .60 + 3.84 + .48 + 21.00 + 1.13.

10. 120. + 17.48 + 250.75 + 910.17 + 800.50 + 560.92 + 130.25 + 9.00 + 4.87 + 15.62 + 18.00 + 7.40 + 238.92 + 18.25 + 5.00 + 167.80 + 11.69 + 432.70 + 6.48 + 261.15.

11. The expense of the Revolutionary War was estimated to be $135,193,703; of the War of 1812, $107,159,003; of the Mexican War, $66,000,000; of the War of the Rebellion, $3,000,000,000. What was the total estimate for the four wars?

12. Find the total population of the largest ten cities in the United States in 1870, from the following Census Report:

New York	942,541	Baltimore	267,354
Philadelphia	674,022	Boston	250,526
Brooklyn	396,300	Cincinnati	216,239
St. Louis...........	310,864	New Orleans	191,322
Chicago	298,983	San Francisco	149,482

13. Find the total foreign commerce of the United States in American and foreign vessels for the five years mentioned in the following table:

Year	Exports and Imports in American vessels.	Exports and Imports in foreign vessels.	Total for each year.
1850	$239,272,084	$90,746,954	
1855.........	405,485,462	131,139,904	
1860.........	507,247,757	255,040,793	
1865.........	167,402,872	437,010,124	
1869........	289,950,272	586,492,012	

14. Find the total estimated population of Europe from the following statistics:

Belgium..........	4,827,000	Portugal	3,830,000
Denmark.........	1,785,000	Roumania	4,701,000
Germany	38,507,000	Russia	71,633,000

France	38,193,000	Sweden	4,161,500
Greece	1,330,000	Norway	1,699,500
Great Britain	30,836,000	Switzerland	2,517,000
Italy	26,500,000	Servia	1,208,000
Luxemburg	200,000	Spain	16,413,000
Netherlands	3,626,000	Turkey	10,622,000
Austrian Empire	35,978,000		

15. Find the total population of the British Provinces in North America, estimated as follows in 1870:

Ontario and Quebec, 3,201,351; New Brunswick, 302,950; Nova Scotia, 375,511; Prince Edward Island, 93,338; Newfoundland, 146,536; British Columbia and Vancouver, 10,000.

16. Compute the number of miles of railways in the United States, and their total cost, from the following table of statistics in January, 1870:

Sections.	Miles R. R.	Cost.
Northeast	4,274	$179,805,000
Middle-east	10,792	652,619,000
Southeast	5,837	154,000,000
Gulf and Southwest	5,294	180,472,000
West and Northwest	20,828	949,667,000
Pacific and West	1,835	95,850,000

17. Compute the number of acres devoted to raising cotton, and the number of bales produced in the United States in 1870, from the following table:

States.	Acres.	Bales.
North Carolina	451,714	170,000
South Carolina	601,764	220,000
Georgia	1,330,491	495,000
Florida	140,909	50,000
Alabama	1,437,272	510,000
Mississippi	1,644,512	725,000
Louisiana	920,700	495,000
Texas	900,937	465,000
Arkansas	711,734	375,000
Tennessee	526,184	215,000
ther States	218,823	80,900

18. Compute the total number of immigrants who arrived in the United States from 1856 to 1869, inclusive, from the following statistics:

Immigrants in *1856*, 200,436; in *1857*, 251,306; in *1858*, 123,126; in *1859*, 121,282; in *1860*, 153,640; in *1861*, 91,920; in *1862*, 91,987; in *1863*, 176,282; in *1864*, 198,418; in *1865*, 248,120; in *1866*, 318,554; in *1867*, 298,358; in *1868*, 297,215; in *1869*, 352,569.

19. Compute the total number of bushels of the various kinds of grain produced in the United States in 1869, from the following statistics:

Wheat....................	264,146,900	bushels.
Rye	22,227,900	"
Peas and Beans............	15,763,444	"
Oats	298,284,000	"
Buckwheat.................	17,255,500	"
Corn	874,120,005	"
Barley	28,652,200	"

20. Compute the entire population of the United States in 1870, from the following statistics of the U. S. Census Report:

Ala.....	996,992	La.....	726,915	N. Y....	4,382,759	Wis. ...	1,054,670
Ark	484,471	Me.....	626,915	N. C...	1,071,361	Arizona.	9,658
Cal .	560,247	Md. ..	780,894	Ohio ...	2,665,260	Colorado .	39,864
Conn ...	537,454	Mass ...	1,457,351	Oreg ...	90,923	Dakota....	14,181
Del.....	125,015	Mich...	1,184,059	Penn...	3,521,951	D. C......	131,700
Fla	187,748	Minn...	439,706	R. I....	217,353	Idaho.. .	14,999
Ga......	1,184,109	Miss....	827,922	S. C....	705,606	Montana ..	20,595
Ill....	2,539,891	Mo.....	1,721,295	Tenn ..	1,258,520	New Mex.	91,874
Ind.....	1,680,637	Neb....	122,993	Texas ..	818,579	Utah.....	86,786
Iowa. ..	1,194,020	Nev....	42,491	Vt. ...	330,551	Wash.. ..	23,955
Kan....	364,399	N. H....	318,300	Va	1,225,163	Wyoming.	9,118
Ky......	1,321,011	N. J....	906,096	W. Va..	442,014	Total....	

LEDGER COLUMNS.

93. In adding long columns of figures, as in a Ledger, the following method is sometimes used:

Add the columns in order, and place the footings under each other upon a separate piece of paper (testing the accuracy of the

same); point off the right-hand figure (except in the last column), and add the left-hand figure or figures to the next column, thus:

$57.45	
28.75	*Process.*
36.87	4.7
4.56	4.7
98.88	6.1
6.25	29
49.38	
9.63	
$291.77	

The figures, expressing the sum of the left-hand column, together with the figures cut off on the right, *read upwards*, will be the sum total. The advantage of this method is twofold: 1. The partial results being preserved, it is easier to detect errors. Any column may be reädded without the trouble of adding the preceding. 2. The total sum when written is correct, and the page is not defaced by erasures and corrections.

The student should write out long ledger columns on slips of paper, and daily practice in adding them, being as careful to obtain a correct result as he would be in actual business.

The following ledger columns are given merely as examples. The student can easily increase the number of them to any extent:

1.	*2.*	*3.*	*4.*
3.25	32.56	75.50	19.50
8.37	8.15	284.38	23.86
2.50	6.33	3287.15	12.45
12.35	17.09	111.01	14.52
9.00	.90	43.96	25.48
.88	.75	263.55	42.54
.93	3.25	1900.09	8.60
4.65	21.87	1356.63	9.37
5.48	22.20	15.20	8.80
10.12	7.15	7.15	.65
1.20	4.32	13.48	.73
9.15	78.90	3456.38	.38
7.75	18.88	348.54	11.25
18.64	3.33	2.75	.86

SECTION V.

MULTIPLICATION.

94. PROBLEMS FOR MENTAL SOLUTION.

1. How many days in 3 weeks?

2. How many days in 4 weeks and 5 weeks?

3. How much will 6 barrels of apples cost at $3 per barrel?

4. How much will two coats cost at $14 each?

5. How much are a man's expenses for 4 weeks if he pays $7 per week for board and $3 per week for other expenses?

6. How many wings have 6 ducks and 5 geese?

7. How many wheels on 4 wagons, 3 carriages, and 2 dray-carts?

8. If a man buys two cigars every day, paying ten cents for each one, how much will he pay out in four days?

9. How many volumes in three sets of Encyclopædias, each consisting of 10 volumes, and in 2 sets of 16 volumes each?

10. If lead pencils can be bought for 8 cts. each, pens for 9 cts. per dozen, and paper for 18 cts. per quire, how much must be paid for 3 pencils, 2 dozen pens, and 3 quires of paper?

11. If a man spends two hours each day in eating, how many hours would he spend in the month of July in eating?

12. If a man paid fifty cents for each of three meals per day, how much would he pay for his meals in September?

13. How much would a family pay for magazines and papers in four years if they took "Harper's Monthly" at $4.50 per year, "Our Young Folks" at $3, the "Bright Side" at $2, and the "Weekly Post" at $1.50?

14. If a man should start from Washington and travel towards Harrisburgh for 3 days at the rate of 30 miles per day, he would then be 36 miles from Harrisburgh; what is the whole distance between the two places?

15. If a lady bought 6 yards of sheeting @ 12 cts., 3 yds. flannel @ 22 cts., 6 yds. cambric @ 11 cts., and 10 yds. delaine @ 20 cts., and after paying for all had $3 left, how much money had she at first?

NOTE.—The sign @ signifies *at* or *each*, and indicates that the price with which it is used is the price per yard, pound, or other unit of quantity by which the article is commonly measured.

16. If two propellers carry 33,000 bushels of wheat each, and two schooners 13,000 bushels each, how many bushels are carried by the four vessels?

17. If each of three schooners carry 125,000 staves and 200,000 shingles, how many staves and how many shingles are carried by the three vessels?

18. Two schooners brought 171 cds. of wood each, and another one 180 cds.; how many cds. were brought by the three?

19. At the beginning of a certain week there were 700,000 bushels more of wheat in store than was required for city use for 6 days at 5,400 bu. for each day. How many bushels were in store?

20. $3 + 2 \times 5 + 5 \times 3 \times 2 \times 3 + 60 \times 12$?

NOTE.—Other combinations should be given orally and rapidly.

95. 1. If a man have four five-dollar bills, he will have in all $\$5 + \$5 + \$5 + \$5 = \$20$. In such a case the man is said to have *four fives*, and \$20 is said to be *four times* five dollars. When a series of such equal numbers is to be combined, *one* of the equal numbers is said to be repeated or taken as many times as there are equal numbers in the series; or, it is said to be *multiplied* by the number indicating how many equal terms are to be combined.

2. In ten gross of buttons there are $(144 + 144 + 144 + 144 + 144 + 144 + 144 + 144 + 144 + 144 =)$ 1,440 buttons; that is, ten 144's = 1440, or $144 \times 10 = 1440$.

3. In one year there are 365 days; in 12 years there are twelve 365's, or $365 \times 12 = 4380$ days.

In multiplying large numbers, as it is practically impossible to perform the entire operation at once, we may consider each part of the number separately upon the principle that a whole is equal to all its parts.

Thus 365×12 = (twelve 5's of 1st order) + (twelve 6's of 2d order) + (twelve 3's of 3d order).

Taking each of these parts twelve times, and adding the results, we shall have—

Twelve 5's of 1st order	. . . . =	60
" 6's of 2d "	. . . . =	720
" 3's of 3d "	. . . . =	3600
And the sum of these	. . . =	4380

4. In multiplying 462 by 23, we may, for convenience, write them thus:

```
  462
   23
 1386
 924
10626
```

Then we say three 2's = 6, and as the 2 is of 1st order, the result will be of the 1st order, and may be written as such. Then three 6's of 2d order = 18 of 2d order, of which we write the 8 in the 2d order, and reserve the 10 = 1 of 3d order, to combine with the units of that order. Then three 4's = 12, and the 1 added makes the entire number in 3d order = 13, which is expressed in the usual manner. Next we multiply by the 20 or 2 of 2d order. Twenty 2's = 40 of 1st order, or 4 of 2d order, which therefore is written under the 8. Then twenty 6's of 2d order = 120 of 2d order, or 12 of 3d order; so the 2 is written under the 3 and the 10 reduced to the next higher order. Then twenty 4's of 3d order = 80 of 3d order, or 8 of 4th order, and the 1 added makes the entire number in 4th order = 9. Then combining the two partial results by addition, we obtain 10626 as the result produced by multiplying 462 by 23.

96. *Multiplication* is the process of finding the amount of a certain number of equal numbers, or of a certain number of equal parts of some number.

Thus $42 \times 6 = 42 + 42 + 42 + 42 + 42 + 42 = 252$.

$18 \times \frac{5}{6} = (\frac{1}{6} \text{ of } 18) \times 5 = 3 + 3 + 3 + 3 + 3 = 15$.

$\frac{4}{7} \times 3 = \frac{4}{7} + \frac{4}{7} + \frac{4}{7} = \frac{12}{7}$.

$\frac{8}{11} \times \frac{3}{4} = (\frac{1}{4} \text{ of } \frac{8}{11}) \times 3 = \frac{2}{11} \times 3 = \frac{2}{11} + \frac{2}{11} + \frac{2}{11} = \frac{6}{11}$.

97. 1. The number of which the whole or a part is to be repeated or multiplied is the ***Multiplicand.***

2. The number indicating how many numbers equal to the multiplicand, or how many parts of it are to be taken, is the ***Multiplier.***

3. The amount obtained by multiplying is the ***Product.***

NOTES.—1. The Multiplicand and Multiplier are called *factors* of the Product, and the Product is a *Multiple* of each of its factors.

2. When the multiplier consists of several figures, the products of each figure of the multiplier and the multiplicand are *partial products.*

98. The product of any two factors is the same, whichever of them is regarded as the multiplicand. Thus three 4's, or $4 \times 3 = 12$, and four 3's, or $3 \times 4 = 12$.

99. The product is of the same kind as the multiplicand.

Even if, for convenience, the factors be reversed, yet the product may still be regarded as of the same kind as the original multiplicand. If we would compute the number of hours in 365 days, the most natural reasoning leads us to express the operation by $\overset{\text{hrs.}}{24} \times 365$, but by a little longer process of reasoning we may express the operation properly by $\overset{\text{hrs.}}{365} \times 24$; for if there were but 1 hr. in a day, in 365 days there would be 365 hrs., but as there are 24 hours in a day, there are 24 times 365 hrs. in 365 days.

100. The multiplier is properly regarded as an *abstract number*, and yet what we may call the *logical multiplier* may, for convenience, be used as the *arithmetical multiplicand*, the numerical result being the same. (Art. 98.)

101. By writing the first figure of each partial product directly under the figure of the multiplier producing it, figures of the same order will be arranged in columns, and can readily be added.

NOTE.—This arrangement is not essential, but convenient, and is properly a part of the *art* of multiplication.

102. By observing the orders of figures in factors and products, the following law of relation between them has been discovered:

LAW OF THE ORDER OF FIGURES IN PRODUCTS.

NOTE.—When two figures are both integral or both decimal, they may be said to be of *similar* orders; but when one is integral and the other decimal, they may be said to be of *dissimilar* orders.

1. *The order of the product of any two figures of a* ***similar*** *order is* ONE LESS THAN THE SUM *of the indices of the* ***orders.***

2. *The order of the product of any two figures of* ***dissimilar*** *orders is one more than the difference of the indices of the orders.*

NOTE.—When the orders are dissimilar the product will be integral if the greater index be that of an integral order, and decimal if the greater index be that of a decimal order.

ILLUSTRATIONS.

1. $400 \times 20 = 8000$; that is, (4 of 3d order) $\times$ (2 of 2d order) = 8 of 4th order. Orders. $(3d + 2d) - 1 = 4th$.

2. $.003 \times .02 = .00006$; that is (3 of 4th dec. ord.) $\times$ (2 of 3d dec. order) = 6 of 6th dec. order. $(4th + 3d) - 1 = 6th$.

3. $40 \times .002 = .08$; that is (4 of 2d order) $\times$ (2 of 4th dec. order) = 8 of 3d dec. order. $(4th - 2d) + 1 = 3d$.

4. $.03 \times 300000 = 9000$; that is (3 of 3d dec. order) $\times$ (3 of 6th order) = 9 of 4th order. $(6th - 3d) + 1 = 4th$

103. GENERAL RULE.

1. *Write the multiplier under the multiplicand in the most convenient order.*

2. *Beginning with the highest or lowest order of the multiplier, multiply each figure in the multiplicand (beginning with its lowest order) by each significant figure of the multiplier in succession, writing the coördinate figures of the several partial products in the same column, and reducing to superior orders, as in addition.*

3. *Add the partial products to obtain the complete product.*

NOTES.—1. When there are ciphers at the right of either factor, or both of them, it is most convenient to write the significant figure of the lowest order in the multiplier under the significant figure of the lowest order in the multiplicand.

2. Having determined the order of the first figure of the first partial product (Art. 102), the first figure of each succeeding partial product may be written under the order to which it belongs, or the first figure of each partial product may be written under the figure of the multiplier producing it, care being taken, when there are ciphers at the right of either factor, to indicate the proper order of the lowest significant figure in the complete product, by annexing one or more ciphers.

3. Sufficient practice will enable the pupil to use two figures of the multiplier at once when they are not large.

104. TESTS.

First Method.—Reverse the factors.

Second Method.—Divide the product by one of the factors, and if the work be correct the quotient will equal the other factor.

Third Method.—1. Find the excess of *nines* in each of the factors, whether two or more.

2. Find the excess in the product of the excesses in the factors; this should equal the excess in the entire product.

Fourth Method.—Find the *sum* of as many numbers equal to the multiplicand as there are units in the multiplier.

105. MULTIPLICATION TABLE.

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300

NOTES.—1. To find the product of any two factors in the table, look for one of them in the column at the left, and on the same line, toward the right, under the other factor, to be found at the top of the table, will be found the product.

2. The entire portion of the Multiplication Table above, enclosed in heavy lines, should be learned perfectly by all who desire to compute with rapidity.

106. EXAMPLES.

1. Multiply 3464 by 306.

Multiplicand	3464
Multiplier	306
	20784
	10392
Product	1059984

2. Multiply 23045 by 70800.

		Test by Nines.
	23045	5 Excess.
	70800	6 "
	184360	30 } 3 "
	161315	
Product	1631586000	3 "

3. Multiply 405678 by 34006.
4. Multiply 38674506 by 30080.
5. Multiply 46923000 by 46702.
6. Multiply 83400607 by 33000.
7. Multiply 843464 by 30706.
8. Multiply 708000 by 4700.

9. How many feet would a horse travel in 109 days at the rate of 35 miles per day? (A mile contains 5,280 feet.)

10. How much can 508 men earn in 65 days, if each man receives 3 dollars per day?

11. Multiply 365 by 243, and write the equivalents of the partial products in Roman numerals.

	Test by Nines.
365	5 Excess.
243	0 "
1095 = MXCV.	$5 \times 0 = 0$ "
1460 = $\overline{\text{XIV}}$DC.	
730 = $\overline{\text{LXXIII}}$.	
88695	0 "

Note.—Partial products should be read until the pupil can give the proper value to each without any hesitation.

12. 76943×87659.
13. 123456×78901.
14. 234567×89012.
15. 345678×90123.
16. 897498×986589.
17. 34090×78009.
18. 85042×59700.
19. 24352×15.
20. 65230×16.
21. $379 \times 485 \times 896$.
22. $109600 \times 408000 \times 870$.
23. 1148×230476.
24. $347 \times 347 \times 347$.
25. $869 \times 869 \times 869$.
26. Find the sum of 7056×73, and 984×21.
27. $(493 \times 89) + (9070 \times 46) + (864 \times 930)$.

28. $(63 \times 10)+(63 \times 8)+(63 \times 5)+(63 \times 21)+(63 \times 17)$.
29. $375 \times 12 \times 20 \times 4$.
30. $375 \times (12 \times 20 \times 4)$.

CONTRACTIONS.

107. Contractions are of two kinds—

1. The abbreviation of written solutions by mental computations.

2. The substitution for ordinary processes of others more brief but producing the same results.

The object of all contractions is to secure accurate rapidity in computing, by writing only a part of the solution, and in many cases only the result.

Care should be taken to use the most appropriate and best contraction in each particular case.

108. To multiply by a unit of any integral order; as, 10, 100, 1000, etc.

To the multiplicand annex as many ciphers as are in the multiplier, or remove the decimal point to the right as many places, less one, as the order of the unit in the multiplier. (Art. 60, Note.)

1. $3. \times 10 = 30.$ $17. \times 100 = 1700.$
2. $450. \times 1000 = 450000.$
3. $\$7.42 \times 10 = \$74.20.$
4. $\$11.50 \times 100 = \$1150.$
5. $\$9.25 \times 1000 = \$9250.$
6. $9680 \times 10000.$
7. $1447 \times 1000.$
8. $\$150. \times 100.$
9. $\$220.75 \times 1000.$
10. $14.372 \times 100.$

NOTE.—Other similar examples should be solved by the pupil, writing only the results.

109. To find the product of several factors; as, $317 \times 8 \times 7$; $9280 \times 10 \times 40$; $369 \times 12 \times 12$; $7 \times 7 \times 7 \times 7$.

Compute the product of two or more of the factors mentally, if convenient, and multiply this by another factor, or by the product of two or more of the other factors.

1. $317 \times 8 \times 7 = 317 \times 56$, or 2536×7.

2. $9280 \times 10 \times 40 = 9280 \times 400$, or 92800×40, or (as $40 = 4 \times 10$), 928000×4.

3. $369 \times 12 \times 12 = 369 \times 144$, or 4428×12.

4. $7 \times 7 \times 7 \times 7 = 49 \times 49$, or 343×7.

5. $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 16 \times 16 \times 16$, or 64×64.

NOTES.—1. Any number that can be produced by multiplying together two or more integral factors, is called a *Composite* number.

2. The process of finding the product of several *equal* factors is *Involution*, and the product of such factors is a certain *Power* of one of them. Thus the product of $3 \times 3 \times 3 = 27$, is the third power of 3, and to obtain it 3 is *involved* to the third power, or is used three times as a factor, and the equation for this is $3^3 = 27$. (Art. 34.)

6. $7091 \times 4 \times 9$.

7. \$$125.15 \times 10 \times 70$.

8. 2240 lbs. $\times 7 \times 80$.

9. $608700 \times 6 \times 5 \times 10 \times 10$.

10. $480 \times 600 \times 7 \times 10$.

NOTE.—It may be observed that when there are ciphers at the right of two or more of several factors, as many ciphers as all of them may be annexed to the product of the significant figures of the factors; as, $7200 \times 50 \times 8 \times 200$ = the product of $72 \times 5 \times 8 \times 2$ ($= 72 \times 80 =$), 2160, with five ciphers annexed = 216000000.

110. To find partial products when one part of the multiplier is a multiple of another part; as, 356×84; 8 being equal to two 4's; 921×312; 12 being equal to four 3's.

Multiply first by the smaller part or figure, and then multiply the partial product thus obtained by the ratio of one or more of the other figures of the multiplier to the part used, to obtain the partial product by them.

EXAMPLES.

1.
```
  356
  8|4
 ----
 1424
2848
-----
29904
```

Multiply the first partial product by 2 to obtain the partial product by 8.

2.
```
    921
   3|12
  ----
  2763
 11052
 ------
287352
```

Here multiply first by 3, then multiply *this* partial product by 4 to get the partial product by 12.

NOTE.—Be careful to write each partial product in its proper order.

3. $8924 × 42|6. (42 = 7 × 6.)
4. 70429 × 72800. (72 = 9 × 8, or 28 = 4 × 7.)
5. 649600 × 84|2. (84 = 7 × 12.)
6. 40987 × 639.
7. 56140 × 8729.
8. 12345 × 8020.
9. 80974 × 90153.
10. 52863 × 14412.

111. To use the multiplicand as a partial product, when only two partial products are required, and one figure in the multiplier is 1; as, multiplying by 14, 16, 21, 104, 501, 12001, etc.

Write the multiplicand as the first partial product by 1, then multiply THIS *by the other part of the multiplier, being careful to write the figures of the second partial product under the coördinate figures of the first partial product.*

EXAMPLES.

1.
```
 763  × 19.
 6867
 -----
14497
```

2.
```
128 × 401.
512
-----
51328
```

3.
```
10796     × 10012.
   128552
---------
108088552
```

4. 56098 × 61.
5. 12560 × 7001.
6. 496 × 1201.
7. 8900 × 10080. (Art. 109, Note.)
8. 5241 × 136. (Art. 110.)
9. 5280 × 4801.
10. 1728 × 124100.

112. To find an equivalent product, when the multiplier is one less than an exact number of tens; as, 19, 59, 99, 999, etc.

Multiply by the nearest exact number of tens, (using contraction), *and from the product subtract the multiplicand.*

1. 763×59 = one less than sixty 763's, hence $= (763 \times 60) - 763 = 45780 - 763 = 45017$.
2. 4208×89.
3. 61804×119.
4. 56×99. (Art. 108.)
5. 1728×999.
6. 63×9999.
7. 76438912×99999.
8. 897×990.
9. 2610×9900.
10. 7600×999000.

NOTE.—Other contractions may be found in Articles 181, 183, 184, and others still may be invented by the student or the accountant in practice.

SECTION VI.

SUBTRACTION.

113. PROBLEMS FOR MENTAL SOLUTION.

1. How many hours in the forenoon of a day when the sun rises at five o'clock?

2. If a boy receive 25 cts. from his father, and spend 18 cts., how much has he still?

3. If a man should buy two dozen eggs, and should find seven broken, how many whole ones would there be?

4. If a lady buy some thread for 10 cts., some needles for 5 cts., some cloth for 20 cts., and some braid for 8 cts., and give the clerk 50 cts., how much change should he return?

5. If a grocer sells 2 lb. of butter @ 28 cts., 2 chickens @ 40 cts., and 2 qt. of berries @ 12 cts., and receives a two-dollar bill, how much change should he return?

6. If your lesson consists of 15 articles each day, and you learn all but 2 articles, how many articles would you learn in 5 days, and how many would remain unlearned to trouble you?

4

7. If you were allowed 2 hours in which to learn and recite a lesson, and the recitation were to occupy 40 minutes, how much time could you have for study?

8. Which is better, and how much, to work 6 days at $1.50 per day, or to receive $2.20 per day and remain without work 2 days in 6?

9. Which merchant makes more in one day, and how much more, he who sells 82 yd. of cloth @ 20 cts., which he bought @ 18 cts., or he who sells 52 yd. of the same kind of cloth @ 21 cts.?

10. On one day of July, 1870, there were 37,000 bushels of wheat shipped from Chicago, and on the corresponding day in 1871, 89,000 bushels were shipped. What was the excess of the latter over the former?

11. If a man buys 2000 bu. barley @ 78 cts., and sells the same @ 80 cts., how much does he make?

12. In one day the cotton sales in New York amounted to 815 bales, of which 210 bales were for exportation; how many were for home use?

13. At the close of a certain day there were 98,000 bu. oats stored at Buffalo, 44,000 bu. having been received during the day. How many bushels were in store on the morning of the same day?

14. In two days $800,000 in specie was shipped from Boston. The first day $450,000 was shipped; how much was shipped the second day?

15. If at New Orleans the stock of cotton be 57,084 bales, and at Galveston 25,058 bales, how many more bales at the former place than at the latter?

16. If a man buy 2000 lb. wool @ 52 cts., and sell the same @ 58 cts., and also sell 3000 lb. @ 54 cts. that cost him 57 cts., does he gain or lose by the two transactions, and how much?

17. $91-7\times2-60-(8\times12)\times6-30$?

18. $43+13-48\times11+50-120\times5+17$?

19. $17000+41000-38000-1400\times9$?

20. $\$4.50+\$6.50\times8-\$60-\$14-\$14$?

NOTE.—Other combinations should be given orally and rapidly.

21. If a man who has $12 expend $3 he will have 9 left. This $9 is the ***Remainder*** of $12—$3.

22. If A own 9 lots and B 5, A owns 4 more lots than B. This 4 lots is the ***Difference*** between 9 lots and 5 lots; $9\sim5$.

23. The difference between two numbers is equal to the remainder that is found by taking from the larger as many units as are expressed by the smaller.

NOTE.—This statement refers only to numbers as expressed in Arithmetic as positive terms, and not to the comparison of positive terms with negative.

24. How much more cloth in a piece 10 yds. long than in one 7 yds. long?

If the shorter piece were applied as a measure to the longer, it would be found that the longer contained 3 yds. more than the shorter.

114. ***Subtraction*** is the process of comparing numbers to find their difference. The larger number is called the ***Minuend,*** because it is to be diminished; the smaller number is called the ***Subtrahend,*** because, in a certain sense, it is to be taken away from the other.

115. In subtracting it is generally more convenient to write the figures of the subtrahend under the coördinate figures of the minuend, and the remainder below a horizontal line drawn under the subtrahend.

NOTE.—Sometimes it is more convenient to reverse the positions of one or two terms. Thus, if a man have \$75, and pay for 3 T. coal @ \$18, we could most conveniently find how much he had left by the following arrangement of terms, writing the minuend, \$75, under the subtrahend, \$54.

$$\begin{array}{r} \$18 \\ \underline{3} \\ 54 \\ \underline{75} \\ \$21 \end{array}$$

116. When each figure of the subtrahend is not larger than the coördinate of the minuend, the comparison is very readily made, and the result may be written at once from *left to right;* as in—

$$\begin{array}{r} 78401629 \\ \underline{43201426} \\ 35200203 \end{array}$$

117. When one or more figures of the subtrahend exceed the coördinates of the minuend, and the numbers are so large as not to be readily and exactly compared in the mind at once, such a

transformation is made as to separate the minuend into several parts larger than the corresponding parts of the subtrahend. Thus in $\left.\begin{array}{r}730210\\ -\underline{456789}\end{array}\right\}$ the minuend may be transformed into

hth	tth	th	h	t	u	
6,	12,	9,	11,	10,	10,	from which we can take
4	5	6	7	8	9	leaving the
Rem. 2	7	3	4	2	1.	

It may be observed that it is only necessary to reduce a single unit of any particular order to subordinates.

	hth	tth	th	h	t	u
827092 =	7,	12,	6,	10,	8,	12
− 734264 =	7,	3.	4,	2,	6,	4
Remainder =		9	2	8	2	8

This process of transformation is sometimes called "borrowing tens."

118. Another method of comparison, called "adding tens," is used by many. This is based on the principle that the difference between any two numbers is the same as the difference between them if equally increased or diminished. Thus $15-12=3$; $(15+6)-(12+6)=3$, and $(15-6)-(12-6)=3$.

In case a transformation is required, it may be avoided by adding *ten* to any figure of the minuend that is too small, and then adding *one* to the next higher order in the subtrahend—

Thus in $\left\{\begin{array}{r}321\\ -\underline{136}\\ (185)\end{array}\right\}$ ten is added to 1, making 11, from which 6 is subtracted, leaving 5; 1 ten is then added to 3, making it 4, and ten is added to 2, making 12, the difference (12−4) being 8; then 1 hundred is added to 1, making 2, and the difference (3−2) 1 is found. It may now be observed that we have added to the minuend 10 tens + 10 units, and to the subtrahend the equivalents 1 hundred + 1 ten; that is, the

	h	t	u
minuend has been *changed* to	3,	12,	11
and the subtrahend " "	2,	4,	6
making the Remainder =	1	8	5

which is equal to the remainder found by transforming the

	h t u
minuend into	2, 11, 11, and
subtracting	1 3 6
	1 8 5

119. RULE.

1. *Write the figures of the subtrahend under the coördinate figures of the minuend.*

2. *Begin at the right hand, and subtract each figure of the subtrahend from the figure over it.*

3. *If any minuend figure be less than the coördinate figure of the subtrahend, increase it by the ratio of a unit of the next higher order to a unit of that order, and diminish the next superior figure of the minuend by 1; or add to* IT *the ratio of the next order, and add 1 to the next superior figure of the subtrahend; then subtract.*

NOTE.—In the decimal notation 10 is the ratio to be added.

120. *Test.*—Add the true remainder to the subtrahend, and the sum will equal the minuend.

121. EXAMPLES.

1. From 3084 take 2793.

	2 10 18 4	Minuend changed in form.	
	3084	Minuend.	
Add	2793	Subtrahend.	Test.
	291	Remainder.	
	3084		

2. From 406309 take 347278.
3. From 100102 take 90903.
4. From 5000050 take 80432.
5. From one billion take one million and one.
6. From 32670804 take 3867498.
7. From 30006070 take 4906007.
8. From 40 hundreds take 25 tens.
9. From 205 tens take 264 units.
10. From 230 tens take 12 hundreds.
11. From 2 millions take 2 thousands.

12. From 16 tens take 75 units.
13. From 101 thousand take 56 hundreds.
14. From 1 ten take 8 units.
15. $7980-(120+5040)$.
16. $(186908+3789)-(67403+80247)$.
17. $\overline{(365\times 63)+(879\times 99)}\sim(172098\times 1470)$.
18. $\overline{9\times 111111\times 10-111111}-8888$.
19. $4027804-\overline{\text{MMLIX}}\text{DXC}-28$ hundred.
20. $(8982\times 15)-(8982\times 12)$.

SECTION VII.

DIVISION.

122. MENTAL PROBLEMS.

1. How many lots of 4 acres each in 24 A.?

2. How many weeks in 35 days?

3. How many oranges at 5 cts. each can be bought for 15 cts.?

4. How many patterns of 5 yd. each can be cut from a piece of cloth containing 43 yd., and how many yd. will remain?

5. How many cases of canned fruit, each holding 12 cans, would contain 120 qt. of fruit, if each can contained 2 qt.?

6. How many 7's in 28? In 49? In 91?

7. In how many days can a man walk 96 miles if he walk 4 miles an hour and 8 hr. each day?

8. If 15 men can pave a street in 8 days, in how many days could 10 men pave the same?

9. If a man earn \$9.60 in 6 days by working 8 hr. a day, how many hours a day must he work to earn \$12 in 6 days at the same rate per hour?

10. How many qt. of berries, at 8 cts. a quart, must a man give for 14 lb. of sugar at 12 cts. a lb.?

11. If in 4 lots there are 24 acres, how many A. in one lot?

12. If in 35 days there are 5 weeks, how many days in one week?

13. If 3 oranges cost 15 cts., what is the cost of one orange?

14. If a piece of cloth, containing 43 yd., be cut into 5 equal patterns, how many yd. in each pattern?

15. If 4 boxes soap, each containing 30 lb., cost $7.20, what is the cost per lb.?

16. What is one-eighth of 32? 72? 120?

17. If a man buys 20 bushels of oats from one man, 15 bu. from another, 35 bu. from another, and 42 bu. from another, at 60 cts. per bu., and then sells an equal number of bushels to each of 8 men at 63 cts. per bu., how much profit does he receive from each man, if all the grain be sold?

18. If from a barrel containing $31\frac{1}{2}$ gallons of syrup $8\frac{1}{2}$ gallons be drawn out, and what remains be put into 5 jugs of equal size, how many gal. in each jug?

19. How many miles per hour must a horse travel to go as far in 10 days, traveling 8 hr. each day, as a locomotive would go in 40 hr. at 24 miles per hour?

20. How many in each of 9 equal parts of $(17 + 31 \times 3 - 82 + 10 \times 3)$?

Observe 1st—That in the first ten problems we are required to compare two numbers of the same kind in respect to their magnitude; that is, to find the *Ratio* of the two numbers.

Observe 2d—That in the last ten problems we are required to find how many units in each of several equal parts of a certain number.

123. Division is the *process* of finding either—

1. How many equal parts of a certain value in a given number, that is a Ratio; or,

2. How many units in each of several equal parts of a given number.

124. The number expressing *how many* in each case is called the ***Quotient.***

125. The number measured or divided into parts is called the ***Dividend.***

126. The number used as a measure, or denoting the given number of equal parts, is called the ***Divisor*** or ***Measure.***

127. 1. Any part of the Dividend left undivided is called the ***Remainder.*** Thus 27 divided by 4 gives a quotient 6 and a remainder 3.

2. An *exact quotient* is found by annexing to the integral quotient an expression for the ratio of the remainder to the divisor.

128. When the dividend is abstract the quotient is abstract; when the dividend is concrete, and the divisor concrete, the quotient is abstract; when the dividend is concrete, and the divisor is abstract, the quotient is like the dividend.

The remainder is always a part of the dividend.

129. Division is indicated in various ways, viz.: $24 \div 6$; $\frac{24}{6}$; $24:6$; $6|\underline{24}$; $6)\underline{24}$. The forms $\frac{24}{6}$ and $24:6$ indicate *Ratio;* the form $\frac{24}{6}$ is also called a *Fraction.*

Divisor. Dividend.

$$17)\underline{59} \qquad\qquad 17)\underline{59}$$

Quotient $=$ 3 Remainder $=$ 8. Exact quotient $= 3\frac{8}{17}$

130. ILLUSTRATION OF PROCESS.

Short Division.

$$\begin{array}{r} 12)\underline{97049} \\ 8087\frac{5}{12} \end{array}$$

Long Division.

$$\begin{array}{l} 12)97049(8087\frac{5}{12} \\ \quad\ \underline{96} \\ \quad\ \ 104 \\ \quad\ \ \ \underline{96} \\ \quad\ \ \ \ 89 \\ \quad\ \ \ \ \underline{84} \\ \quad\ \ \ \ \ 5 \text{ Remainder.} \end{array}$$

Beginning at the highest order of the dividend, we first consider such a part of the dividend as will contain the divisor, and write the corresponding quotient figure. Thus one-twelfth of 97 thousand is 8 thousand, and there is a remainder of 1. This 1 is reduced to 10 subordinate units, and as this does not contain 12, a cipher is written in the quotient. Then the 10 is reduced to the next order and combined with 4 tens, making 104 tens, one-twelfth of which is 8 tens, with a remainder of 8. This 8 is reduced and combined with 9 units, making 89 units, one-twelfth of which is 7 units, with a remainder of 5, which may be expressed as a remainder, or the ratio of it to the divisor may be indicated and annexed to the quotient.

131. When the partial products and reductions are not written, the process is called ***Short Division.*** When the partial products and reductions are written, the process is called ***Long Division.***

NOTE.—The pupil should be required to read the partial products and dividends, or to express them in Roman numerals.

132. In measuring one number by another, we may either consider how many times one may be subtracted from the other, or what factor combined with one would produce the other. Thus $54 \div 18 = 3$; that is, $54 - 18 - 18 - 18 = 0$, or $18 \times 3 = 54$. $68 \div 13 = 5\frac{3}{13}$; that is, $68 - 13 - 13 - 13 - 13 - 13 = 3$, or $(13 \times 5) + 3 = 68$. Hence Division may be regarded as based on Subtraction, or as the *reverse* of Multiplication.

133. LAW OF THE ORDER OF QUOTIENT FIGURES.

NOTE.—This corresponds to the law in Multiplication. (Art. 102.)

1. *The index of the order of the quotient figure is one more than the difference of the indices of the orders of the dividend and divisor when the latter orders are similar.*
2. *The index of the order of the quotient figure is one less than the sum of the indices of the orders of the dividend and divisor if the latter orders are dissimilar.*

NOTE.—The order of the quotient figure will be integral if the order of the dividend be of a rank superior to that of the divisor, and it will be decimal if the order of the dividend be of a rank subordinate to that of the divisor. (Art. 75, Note.)

ILLUSTRATIONS.

1. $800 \div 20 = 40$; (3d order − 2d ord.) + 1 = 2d order.
2. $.009 \div .03 = .3$; (4th dec. ord. − 3d dec. ord.) + 1 = 2d dec. ord.
3. $8 \div 200 = .04$; (Dividend subordinate.)
4. $.09 \div .003 = 30.$; (Dividend superior.)
5. $6000 \div .2 = 30000.$
6. $80 \div .004 = 20000.$
7. $.09 \div 30 = .003.$

134. GENERAL RULE.

1. *Write the divisor at the left of the dividend.*

2. *Consider how many numbers equal to the divisor are contained in the first left-hand figure or figures of the dividend, and write the number expressing the same as the first figure of the quotient below the figures of the dividend used, or at the right of the dividend.*

3. *From the part of the dividend considered, subtract the product of the divisor by the quotient figure.*

4. *Reduce the remainder to the next lower order; combine the result with the dividend figure of that order; divide this amount by the divisor, and write the result as the next right-hand figure of the quotient. Thus proceed till the division is carried to the required extent.*

5. *If there be a remainder at last, write it near the quotient as a* REMAINDER, *or to the quotient annex the expression for the ratio of the remainder to the divisor.*

NOTES.—1. Many, in practice, prefer to place the divisor at the right of the dividend, and the quotient directly under the divisor.

2. If any remainder equals or exceeds the divisor, the last quotient figure must be increased; if any product is greater than the partial dividend used, the quotient figure must be diminished.

3. After the first quotient figure is found, if the divisor is not contained in the number obtained by one reduction and combination, a cipher must be written in the next order of the quotient, and the partial dividend reduced to the next lower order and combined with the figure of that order.

4. When the division is completed, if the last figure of the quotient does not occupy the correct order, ciphers must be annexed to the quotient to indicate the proper integral order, or the decimal point must be removed to the left to indicate the proper decimal order.

5. If the divisor and dividend are concrete, they must be of the same kind.

135. TESTS.

First Method.—Multiply the divisor by the quotient, adding in the remainder, if any. The result should equal the dividend.

Second Method.—Add the excess of nines in the remainder to the excess of nines in the product of the excess in the divisor, multiplied by the excess in the quotient (without the remainder or

fractional part). The excess of nines in this sum of excesses should equal the excess of nines in the dividend.

Other methods may be invented, based upon the relation of division to subtraction.

Ex.	193)74020(383	*Test.*
	579	383
	1612	193
	1544	1149
	680	3447
	579	383
	101	101 = Rem.
		74020 = *Dividend.*

Test.—Excess of nines in Rem. . . . . = 2
Excess in divisor . . . = 4
" quotient . . . = 5
" product (5 × 4) . . . = 2
Sum of excesses . . . . . . = 4
Excess in dividend . . . . . = 4

Suggestion.—Make 1 or 19 the first trial divisor, and 7 or 74 the first trial dividend.

136. EXAMPLES AND PROBLEMS.

1. 70432658 ÷ 648.
2. 870420030 ÷ 1429.
3. 114007000 ÷ 8795.
4. 100100010001 ÷ 477777.
5. 9704006704200 ÷ 1592740.
6. 48000 ÷ 600.
7. 304608 ÷ 304.
8. 6743207 ÷ 6200.
9. 780569825 ÷ 15. (Use Short Division.)
10. 6070890100 ÷ 14. (Short Division.)
11. $1240672 ÷ 25. What kind of units in the quotient?
12. $70.80 ÷ 30.
13. $6. ÷ 8.
14. 7280 yds. ÷ 18 yds. What kind of units in the quotient?
15. 942768 ft. ÷ 5280 ft.

16. 1100960 cu. ft. ÷ 128 cu. ft.

17. \$18000 ÷ \$6.50.

18. (360 miles × 43) ÷ (180 × 86).

19. (11286 ÷ 32) ÷ (11286 ÷ 8).

20. (80963500 ÷ 24) – (80963500 ÷ 3 ÷ 8).

21. The total immigration to the United States in 21 years was 7,556,007. What was the average number per year?

22. The total cost of four wars of the United States is estimated at \$3,308,352,706. What was the average cost of each?

23. The population of the United States in 1860 was estimated at 31,443,321, and in 1870 at 38,538,180. What would be the quotient of 1000 times the difference of these two estimates, divided by the estimate for 1860?

24. If the national debt amount to \$2,466,806,497., and it be decreased \$191,154,747. in 21 months, in how many months could it all be paid at that rate?

25. What is the ratio of 7200 to 240? 667600 to 3540?

26. What is the ratio of the population of the State of Illinois, estimated at 2,539,638, to that of California, estimated at 556,615?

27. If the cost of 4,707 miles of railroads in Illinois be \$217,560,000, what is the average cost per mile?

28. The number of persons employed in manufacturing in Chicago in 1870 was 20,156, and the total amount of wages paid was \$10,283,000. What was the average amount paid to each person?

29. The distance from New York to Chicago is 911 miles; from Chicago to San Francisco 2,442 miles. How many days would be required for the trip, at the rate of 479 miles per day?

30. How many tons of coal, at \$8.50 per ton, would pay for seventeen thousand feet of lumber, at \$35. per thousand?

137. CONTRACTIONS.

To divide by a unit of any integral order; as, 10, 100, 1000, etc.

From the right of the dividend cut off as many figures as there are ciphers in the divisor, or remove the decimal point as many places toward the left, less one, as the order of the unit of the divisor.

1. $17000 \div 100 = 170$.
2. $\$9280. \div 1000 = \$9.280$.
3. 40265 lb. $\div$ 100 = 402 lb., and Rem. 65 lb.
4. $5289 \div 100 = 52\frac{89}{100}$.
5. $\$704200 \div 100$.
6. $189234 \div 1000$.
7. $2820000 \div 100$.
8. $6482007 \div 1000$.
9. $14262017 \div 100$.
10. $116200300045 \div 100000$.

138. To divide by a composite number; as, $24 = 3 \times 8$, $3600 = 4 \times 9 \times 100$.

Divide first by one of the factors, and the resulting quotient by another factor, and proceed thus till all the factors have been used.

1. $693 \div 63 = (693 \div 9) \div 7 = 11$.
2. $1438200 \div 84600 = (1438200 \div 100) \div 846 = 17$.
3. $63810000 \div 709000 = (63810000 \div 1000) \div 709 = 990$.

NOTE.—When ciphers are found at the right of both dividend and divisor, an equal number can be cut off from each, and the division made with the remaining figures.

4. $809970000 \div 812000 =$
   $809970 \div 812 = 750$.
5. $224000000 \div 250000$.

139. When remainders arise in dividing by the factors of a composite number, the several remainders must be reduced to the same unit value and combined.

1. If 7924 be divided by 300, using the factors 100 and 3, the first quotient will be 79, with a remainder 24, and $79 \div 3$ will give a quotient 26, with a remainder 1, and this is one of the original 79 hundreds, hence equals 100 units, which, combined with the first remainder, 24, gives a complete remainder of 124, hence $7924 \div 300 = 26$, and 124 Rem.

2. In 21078 gills how many gallons, and how many gills over, there being 32 gills in a gallon? *Ans.* 658 gal. and 22 gills.

Divide 21078 by 32, using the factors 4, 2, and 4.

4	21078
2	5269 pt. and 2 gills rem.
4	2634 qt. and 1 pint "
	658 gals. and 2 quarts remaining.

2 qt.$=(2\times2\times4)=16$ gills; 1 pt.$=(1\times4)=4$ gills; and 2 gills $+4$ gills$+16$ gills$=22$ gills, as the entire remainder.

3. $21073\div96$, using factors 8, 4, and 3.

8	21073			
4	2634	Rem. 1	. . =	1 unit of dividend.
3	658	Rem. 2	$(2\times8)=$	16 units of "
	219	Rem. 1	$(1\times4\times8)=$	32 " "
			Hence true Rem. =	49

4. $786571\div48$ (using three factors).
5. $142375913\div128$ (using four factors).

SECTION VIII.

MISCELLANEOUS PRINCIPLES.

140. If each of several numbers, whose sum is found, be multiplied by any number, the sum of the products of the changed numbers will equal the product of the sum of the original numbers multiplied by the same number; if each be divided, the sum of the quotients will equal the quotient of the original sum divided by the same number.

$$\begin{array}{rl} 1470; & \times 2 = 2940; \ \div 2 = 735 \\ +\ 842; & \times 2 = 1684; \ \div 2 = 421 \\ +\ 394; & \times 2 = 788; \ \div 2 = 197 \\ =2706; & \times 2 = 5412; \ \div 2 = 1353 \end{array}$$

141. If the minuend and subtrahend be multiplied or divided by the same number, the difference of the products or quotients

will equal the product or quotient of the original difference, multiplied or divided by the same number.

$$\begin{array}{rl} 41748; & \times 2 = 83496; \ \div 2 = 20874 \\ -12484; & \times 2 = \underline{24968}; \ \div 2 = \underline{6242} \\ = 29264; & \times 2 = 58528; \ \div 2 = 14632 \end{array}$$

142. If any factor of a product be multiplied or divided by any number, the product of the changed factor and the other factor or factors will equal the product or quotient of the original product, multiplied or divided by the same number.

$$\begin{array}{l} 24\times6 = 144 \\ (24\times3)\times6 = 432 = 144\times3 \\ 24\times(6\times3) = 432 = 144\times3 \\ (24\div3)\times6 = 48 = 144\div3 \\ 24\times(6\div3) = 48 = 144\div3 \end{array}$$

143. If one factor be multiplied or divided by any number, and another factor be divided or multiplied by the same number, the product of the changed factors will equal the product of the original factors.

$$\begin{array}{l} 24\times6 = 144 \\ (24\times3)\times(6\div3) = 144 \\ (24\div3)\times(6\times3) = 144 \end{array}$$

144. If any factor be multiplied or divided by any number, and the product of the changed factors be divided or multiplied by the same number, the result will equal the product of the original factors.

$$\begin{array}{l} 24\times6 = 144 \\ \overline{(24\times3)\times6}\div3 = 144 \\ \overline{(24\div3)\times6}\times3 = 144 \\ \overline{24\times(6\times3)}\div3 = 144 \\ \overline{24\times(6\div3)}\times3 = 144 \end{array}$$

145. If a dividend be multiplied, or a divisor divided by any number, the quotient of the changed terms will equal the quotient of the original terms, multiplied by the same number.

$$\begin{array}{l} 84\div6 = 14 \\ (84\times2)\div6 = 28 = 14\times2 \\ 84\div(6\div2) = 28 = 14\times2 \end{array}$$

146. If the dividend be divided, or the divisor be multiplied by any number, the quotient of the changed terms will equal the quotient of the original terms, divided by the same number.

$$\begin{aligned} &84 \div 6 = 14 \\ &(84 \div 2) \div 6 = 7 = 14 \div 2 \\ &84 \div (6 \times 2) = 7 = 14 \div 2 \end{aligned}$$

147. If both dividend and divisor be multiplied or divided by the same number, the quotient will not be changed.

$$\begin{aligned} &84 \div 6 = 14 \\ &(84 \times 2) \div (6 \times 2) = 14 \\ &(84 \div 2) \div (6 \div 2) = 14 \end{aligned}$$

148. If a divisor be multiplied or divided by any number, and the quotient of the changed terms be multiplied or divided by the same number, the result will equal the original quotient.

$$\begin{aligned} &84 \div 6 = 14 \\ &\overline{84 \div (6 \times 2)} \times 2 = 14 \\ &\overline{84 \div (6 \div 2)} \div 2 = 14 \end{aligned}$$

149. If a dividend be multiplied or divided by any number, and the quotient of the changed terms be divided or multiplied by the same number, the result will equal the original quotient.

$$\begin{aligned} &84 \div 6 = 14 \\ &\overline{(84 \times 2) \div 6} \div 2 = 14 \\ &\overline{(84 \div 2) \div 6} \times 2 = 14 \end{aligned}$$

150. In any arithmetical combination, consisting only of positive and negative terms, if the sum of the negative terms be subtracted from the sum of the positive terms, the result will be the same as if each positive term were added, and each negative term subtracted separately.

$$27 - 9 + 10 + 2 - 13 + 8 - 4 + 18 - 2 - 6 = (27 + 10 + 2 + 8 + 18) - (9 + 13 + 4 + 2 + 6) = 31.$$

151. In any combination consisting only of factors and divisors, if the product of the factors be divided by the product of the divisors, the result will be the same as if each factor and divisor were used separately.

$$14 \times 3 \div 7 \times 8 \div 16 \times 15 \times 2 \div 5 = (14 \times 3 \times 8 \times 15 \times 2) \div (7 \times 16 \times 5) = 18.$$

$$42 \div 14 \times 50 \times 2 \div 60 \times 17 \div 5 = \frac{42 \times 50 \times 2 \times 17}{14 \times 60 \times 5} = 17.$$

152. CANCELLATION.

In any combination consisting of factors and divisors, the solution may be shortened by dividing the product of the factors and the product of the divisors, or one factor and divisor, by one or more common measures or factors. (Art. 147, and Art. 163, Note 2.)

NOTES.—1. A common factor of two or more numbers or combinations is a factor that is contained in each of them.

2. When two terms are divided by a common factor, a mark is drawn across each of them, and the quotient of the term divided by the common factor is written just above or below the term.

When the quotient is 1 it is not generally written.

1. $\frac{4 \times 15 \times 84 \times 7}{14 \times 12 \times 9}$. In order to abbreviate the work most, we divide by the greatest common factors, 4, 3, 14.

Dividing the terms 4 and 12 by 4; 15 and 9 by 3; 84 and 14 by 14, we have

$$\frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{15}} \times \overset{6}{\cancel{84}} \times 7}{\underset{1}{\cancel{14}} \times \underset{3}{\cancel{12}} \times \underset{3}{\cancel{9}}}$$

and then dividing 6 and 3 by 3, we have,

$$\frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{15}} \times \overset{\overset{2}{\cancel{6}}}{\cancel{84}} \times 7}{\underset{1}{\cancel{14}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{12}} \times \underset{3}{\cancel{9}}}, \text{ or simply } \frac{\cancel{4} \times \overset{5}{\cancel{15}} \times \overset{\overset{2}{\cancel{6}}}{\cancel{84}} \times 7}{\cancel{14} \times \underset{\cancel{3}}{\cancel{12}} \times \underset{3}{\cancel{9}}} = \frac{1 \times 5 \times 2 \times 7}{1 \times 1 \times 3} = \frac{70}{3} = 23\tfrac{1}{3}.$$

The process is called *Cancellation*, from the method of marking out or cancelling the divided terms.

2. $89 \times 400 \div 60 \times 18 \div 74 \div 8 \times 126$.

3. A man sold 217 lb. butter at 20¢ per lb., and with the proceeds he bought coffee @ 42¢; he sold the coffee @ 45¢, and bo't soap @ 15¢; he sold the soap @ 18¢, and b't cheese @ 12¢; sold the cheese @ 14¢, and b't sugar @ 15¢. How many lb. sugar did he receive?

4. A man paid $96 for coal @ $8; exchanged the coal for candles @ 12¢; exchanged the candles for dried berries @ 32¢; exchanged the berries @ 34¢ for almonds @ 40¢; exchanged the

almonds for sugar @ 14¢, and sold the sugar @ 15¢. How much did he gain or lose?

5. $(74+16)\times 42\div\overline{(37+8)\times 126}\div 2\times 56\div 14.$

153. ALIQUOT PARTS.

The value of one of the equal parts of a number (A. 123.2) is called an aliquot part of that number. Thus 3 is one of the two equal parts of 6, and is called an aliquot part of 6; this particular part is called one-half, and 3 is said to be one-half of 6; $\frac{1}{2}$ of $6=3$.

154. In multiplication, sometimes the product may be found most readily by multiplying the multiplicand by the most convenient number of which the multiplier is an aliquot part, and dividing the result by the number of parts in the whole.

$642\times 25=\overline{642\times(25\times 4)}\div 4=642\times 100\div 4=64200\div 4=16050.$ (Art. 108 and Art. 144.)

NOTE.—*Write* only the final result.

155. In division, sometimes the quotient may be found most readily by dividing the dividend by the most convenient number of which the divisor is an aliquot part, and multiplying the result by the number of parts in the whole.

$14250\div 25=\overline{14250\div(25\times 4)}\times 4=14250\div 100\times 4=142.50\times 4=570.$ (Arts. 137, 148.)

156. TABLE OF ALIQUOT PARTS.

The following table contains most of the aliquot parts commonly used in practice with numbers of the decimal scale:

1	*10*	*100*	*1000*	*10000*	*50*	*70*	[illegible]
$\frac{1}{2}$	5	50	500	5000	25	35	45
$\frac{1}{3}$	$3\frac{1}{3}$	$33\frac{1}{3}$	$333\frac{1}{3}$	$3333\frac{1}{3}$	$16\frac{2}{3}$	$23\frac{1}{3}$	30
$\frac{1}{4}$	$2\frac{1}{2}$	25	250	2500	$12\frac{1}{2}$	$17\frac{1}{2}$	$22\frac{1}{2}$
$\frac{1}{5}$	2	20	200	2000	10	14	18
$\frac{1}{6}$	$1\frac{2}{3}$	$16\frac{2}{3}$	$166\frac{2}{3}$	$1666\frac{2}{3}$	$8\frac{1}{3}$	$11\frac{2}{3}$	15
$\frac{1}{7}$	$1\frac{3}{7}$	$14\frac{2}{7}$	$142\frac{6}{7}$	$1428\frac{4}{7}$	$7\frac{1}{7}$	10	$12\frac{6}{7}$
$\frac{1}{8}$	$1\frac{1}{4}$	$12\frac{1}{2}$	125	1250	$6\frac{1}{4}$	$8\frac{3}{4}$	$11\frac{1}{4}$
$\frac{1}{9}$	$1\frac{1}{9}$	$11\frac{1}{9}$	$111\frac{1}{9}$	$1111\frac{1}{9}$	$5\frac{5}{9}$	$7\frac{7}{9}$	10
$\frac{1}{12}$	......	$8\frac{1}{3}$	$83\frac{1}{3}$	$833\frac{1}{3}$	$4\frac{1}{6}$	$5\frac{5}{6}$	$7\frac{1}{2}$
$\frac{1}{16}$	......	$6\frac{1}{4}$	$62\frac{1}{2}$	625	$3\frac{1}{8}$	$4\frac{3}{8}$	$5\frac{5}{8}$

157. EXAMPLES.

1. 1728×125.
2. $870569 \times 1428\frac{4}{7}$.
3. $17000 \times 166\frac{2}{3}$.
4. $89786 \div 125$.
5. $\$421.35 \times 83\frac{1}{3}$.
6. $\$6.25 \times 3333\frac{1}{3}$.
7. $52800 \div 11\frac{1}{9}$.

NOTE.—If the remainder be required, divide the decimal part of the final result by the number of aliquot parts. $89.786 \times 8 = 718.288$; $288 \div 8 = 36$ for the *Remainder*, or the exact quotient will be $718\frac{36}{125}$ or $718\frac{36}{125}$.

8. 7280 ft. $\div 16\frac{2}{3}$ ft.
9. \$84 $\div$ 25 cts.
10. $\$185 \div \$1.25$.
11. $892 \times 28\frac{4}{7}$.

NOTE.—A multiplier or divisor that is more than one aliquot part of one number, is only one aliquot part of some other number, and the latter should be used.

Thus $28\frac{4}{7} = \frac{2}{7}$ of $100 = \frac{1}{7}$ of 200.

12. $18796 \div 66\frac{2}{3}$.
13. 11487×450.
14. $728025 \div 23\frac{1}{3}$.
15. $94 \times 83333\frac{1}{3}$.

REVIEW PROBLEMS.

158. 1. In two stores are goods to the amount of \$726,942. The value of the goods in one is \$497,800. What is the value of the goods in the other?

2. In 12 cars are 864 sheep. How many in each car?

3. In four grain elevators are 976,110 bushels of grain. In three are 32,000 bu., 46,180 bu., and 4,920 bu. How many in the other one?

4. Santa Fé is 1262 miles farther from Washington than is Chicago, which is 844 miles from Washington. How far from Washington to Santa Fé?

5. San Francisco is 3283 miles from Washington, and Denver is 1442 miles nearer. How far from Washington is Denver?

6. The cost of the construction and equipment of railroads in California and Oregon up to 1870 was \$52,350,000; the difference in the cost to each State was \$40,950,000. What was the expense for each of these States?

NOTE.—Observe that the difference of two numbers added to their sum equals double the larger number, and their difference subtracted from their sum equals double the smaller number.

7. If in 306 towns, each containing an equal number of inhabitants, there are 1,059,984 people, how many in each town?

8. In how many miles are there 5,227,200 feet, there being 5,280 ft. in one mile?

9. If the men in a certain place were taxed \$78,150 for improvements, each to pay \$12.50, and only 6,202 paid their tax, how many would there be who did not pay?

10. What power of 15 is 11,390,625? (Art. 109, Note 2.)

11. How many hours would it take you to count 1,154,800 words if you count four each second?

12. In a certain number of bushels of grain there were 21 lots of 360 bushels each, and 42 bushels more. How many bushels in all?

13. If 1746 bunches of shingles were shipped in lots of 75 bunches each, and 21 bunches were retained, how many lots were shipped?

14. If the ratio of the population of Cleveland to that of Harrisburg be 4, and the population of the former place be 86,016, what is the population of the latter place?

15. What should be the first term in the following: ( ) $\times$ $7+41-117\times25+104\div109=6$?

159. REVIEW QUESTIONS.

1. How find one of two numbers when the other and their amount are known?

2. How find one of several equal numbers whose amount and number are known?

3. How find one of several unequal numbers when the amount and the other numbers are known?

4. How find the smaller of two numbers when the larger and their difference are known?

5. How find larger of two numbers when the smaller and their difference are known?

6. How find two numbers whose sum and difference are known?

7. How find the multiplicand?

8. How find the multiplier?

9. How find a part of the multiplicand or multiplier when the other parts and the entire product are known?

10. How find the number of equal factors when their product and one of them are known?

11. How find one of several factors when their product and the others are known?

12. How find the dividend?

13. How find the divisor?

14. How find one of two numbers when the other and their ratio are known?

15. How find the first term of a combination when all the other terms and the final result are given?

160. PROBLEMS FOR MENTAL SOLUTION.

1. $84-11+20-30\div7+120-16+27\div70\times38+12-8\div16\times170+250$.

2. $18-7\times(96\div12)+(14\times2)\div4\times2+7\div5$.

3. $450-20-19-18-17-16-15-14\times2-13-12-11-9-8\div3-7-6-5$.

4. $18\times2+12+9-1\div7\times12+16-82\times6-14\div2+9-11+13-12-9-8\times4$.

5. $79-18\times4-100\div(63\div7)\times8\div4\times5\times3+480\div120\times(14-5\times10)$.

NOTE.—Other combinations should be given orally and rapidly by the teacher.

6. How many times can \$18 be taken from a drawer containing \$112, and how many dollars will remain?

7. If publishers send to one merchant 14 Readers, 25 Spellers, 20 Arithmetics; to another 12 Readers, 12 Spellers, 24 Arithmetics; to another 18 Readers, 6 Spellers, 12 Arithmetics, how many books of each kind are sent out?

8. A man having placed \$85 in a drawer, observed that \$3 was missing each day for 14 days; how much then remained in the drawer?

9. A man bought 52 bbls. apples at \$3 a bbl., and 12 cases of grapes at \$8 a case; how much did he pay for all?

10. A man walked 14 miles one day, 12 miles the next, and the next he rode three times as far as he had walked in the two days; how far did he travel in the three days?

11. At what price per acre can a man afford to buy land if he has but \$3840, and wants 60 acres?

12. If one business man in every 16 succeeds, how many of 352 would succeed?

13. If a young man spends 10 cts. a day for cigars, in how many days might he save enough money to make one-half of a quarterly payment on a lot that costs \$160?

14. If a clerk receive \$16 per week, and he pay \$8 per week for board and \$3 for other expenses, in how many weeks can he save enough to pay a debt of \$75?

15. A man whose property is worth \$72,000., divides the same among his 5 children, giving to each of his three sons \$16,000, and the remainder to his 2 daughters in equal portions; how much does each daughter receive?

16. A boy sold 48 papers at 5 cts. each, and thereby gained 96 cts.; at what rate did he buy the papers?

17. A man bought 40 qt. of berries, and he sold them for \$6, which was \$1.20 more than he paid for them: what was the cost per qt.?

18. A man sold a horse that cost him \$80 at a gain of \$30, and with the money purchased sheep at \$5 each: how many sheep did he purchase?

19. If a man supply 40 families with ice at three dollars per month, how much will he make in 4 months if the cost of procuring the stock of ice be \$100, and the monthly expense of delivering it \$45?

20 Two men own 96 lots, and one owns 16 lots more than the other: how many lots does each man own?

21. If a lady can copy 5 pages an hour, in how many hrs. could she earn \$6.40, if she receive 8 cts. a page?

22. Two men engage in business, each man having \$1600; they continue in business 15 months, one man making \$80 per month, and the other losing \$20 per month. How much more has the first than the second at the end of the time?

23. If one of 17 equal parts of the population of Oregon in 1860 was equal to one of 30 equal parts of the population in 1870, what was the population in 1860 if in 1870 it was 90,000?

24. A man bought two loads of potatoes, one containing 25

bushels, and the other 18 bu. Finding 7 bushels poor, he sold the rest at $3 a bushel: how much did he receive for them?

25. A ship sailed directly east one day 24 miles, the second day south 12 miles, the third day west 8 miles, the fourth day north 10 miles, the fifth day east 10 miles, the sixth day north 10 miles, and the seventh day west 9 miles: how many miles, and in what direction from the meridian and parallel, drawn through the place of starting, was the vessel then?

161. PROBLEMS FOR WRITTEN SOLUTION.

1. If in one month a man receive and expend, at different times, the following amounts, what will be the amount of cash on hand at the end of the month?

Receipts—$1646.10; $50; $100; $50; $50; $50; $2250.

Expenses—$17.48; $130.50; $59.25; $91; $60; $7.80; $13.40; $95.50; $24; $779.10?

2. How many more bushels of wheat were raised in 1869 in the first three of the following States than in the last three?

Illinois, 29,200,000; Indiana, 20,600,000; Iowa, 25,000,000; Ohio, 20,400,000, Pennsylvania, 16,500,000; Kentucky, 5,500,000.

3. How much must a man make per year to pay his household expenses of $3275, allow his wife $870, each of his two daughters $500, his son $750 for personal expenses, and pay his own personal expenses of $1125, and $520 interest on borrowed money?

4. What is the total weight of 242 clerks at 137 lbs. each, 34 bakers at 142 lbs. each, and 135 tailors at 135 lbs. each?

5. The number of persons employed in manufacturing in St. Louis in 1870 was 33,551, and the amount of wages paid was $15,903,174: how much more did this allow each person than was allowed in 1860, when 11,737 persons were employed, and $4,377,901 paid?

6. What was the difference in the receipts and expenditures of the Chicago Board of Education in 1869, as shown by the following extract from their Report:

Received from	School-Tax Fund	$454,902.55
"	State Fund	35,724.59
"	Rents and Interest	41,774.75

Expended for	Salaries	$414,655.70
"	Rents	12,234.49
"	Incidentals	113,085.90
"	Improvements	11,007.57

7. At the close of 1870 there were 1627 national banks in operation, with a total capital of $436,478,000, and a circulation of $299,729,000. If equally distributed, what would have been the capital and circulation of each?

8. How much corn was raised in 1869, at 25 bushels to the acre, in Kentucky on 2,060,000 acres, and in Louisiana on 674,000 acres?

9. What is the population of China, which contains 3,741,878 sq. mi., and 119 inhabitants per sq. mi., and of Japan, containing 149,399 sq. mi. and 234 inhabitants per sq. mi.?

10. What was the total number of depositors, and the amount deposited in the savings banks of the New England States in 1870, as shown by the following statistics:

States.	Depositors.	Amount of Deposits.
Maine	39,527	$10,490,368
New Hampshire	71,536	18,759,461
Vermont (1868)	14,295	2,037,934
Massachusetts	431,769	112,119,016
Rhode Island	67,239	27,067,072
Connecticut	165,692	47,904,834

11. From the 1st of Jan., 1868, to the 30th of Nov., 1870, the amount of money paid into the Treasury of the United States as the proceeds of fines, etc., by customs officers at New York, was $847,613.55; of which the collector received $96,861.99, the surveyor $96,837.48, naval officer $28,602.12, and informers the remainder. How much did the last receive?

12. The value of the real estate in Virginia is estimated at $276,023,367, and of the personal property at $85,387,600. What would be the average wealth of each person if this were equally distributed, the population being 1,224,830?

13. In the manufacturing establishments of San Francisco, in 1870, 15,228 persons were employed, and material, valued at $18,600,000, was converted into merchandise valued at $41,000,000. If the operatives received two dollars per day for 300 days in the

year, and other expenses besides material amounted to $2,104,200, what was the amount of gain?

14. The valuation of property, real and personal, in California in 1860, was $139,654,667, and in 1870, $269,644,088. What was the average gain per year?

15. What is the average daily income of a Life Insurance Company that receives $7,184,344.13 in a year?

16. If passenger cars can be manufactured for $1960, and sold for $2450 each, how much would be made on a sale amounting to $100,450?

17. If a man gain $4248 by selling 472 acres of land at $53 per acre, at what price did he buy it?

18. Two men travel from the same place in opposite directions for 6 days, one 33 miles per day, the other 29 miles per day. In how many days could they meet, returning at the rate of 30 miles each per day?

19. An estate was divided among four children, the first receiving $7,580, the second $8,650, the third $1,290 less than the first, and the fourth $283 more than the second. What was the value of the estate?

20. How long will it take a man to save enough to buy a lot of 50 ft. front at $245 per foot, if he earn $180 per month, and pay $35 per month for rent, and $87 for other expenses?

SECTION IX.

PROPERTIES OF NUMBERS.

NOTE.—Only integral numbers are referred to in this Section (excepting Arts. 183-185), although some of the principles here stated may be applied to fractions also.

162. The ***Properties*** of a number are its peculiar relations to some other number or numbers, expressed by some description of it, and not by the difference or ratio between it and the other number or numbers.

163. A ***Measure*** of a number is any number by which it can be exactly measured without a remainder. Thus 3 inches is a measure of 15 inches, but not of 17 inches.

NOTE 1.—Every *Measure* of a number is an exact *Divisor* of that number.

1. Name three measures of 24 qts.
2. Name four measures of $60.
3. Name some number that will measure 18, 30, and 54.
4. What divisor of 72 is also a measure of 45?

NOTE 2.—A number which measures each of several numbers is called their *Common Measure* or *Common Divisor*, and is also a *Common Factor* of them.

5. Name five numbers of which both 3 and 4 are measures.
6. Name all the divisors of 18.

164. 1. A ***Factor*** of a number is any number which, being multiplied by some other number, will produce the given number. (Art. 97, 3, Note 1.) Thus 6 is a factor of 24, because $6 \times 4 = 24$.

2. Every *Factor* of a number is also a *Measure* of it.

3. Every number is a factor of itself, and 1 is a factor of every number.

4. *The Factors composing a number* are those numbers whose product equals the given number.

Thus each of the numbers 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, is a Measure and a Factor of 60; but *the Factors composing* 60 can only be 2 and 30, or 3 and 20, or 4 and 15, or 5 and 12, or 6 and 10, etc., and *not all* of these combined.

5. Finding the factors that compose a number is called *resolving the number into its factors*, or *factoring* it.

Ex. 1. Name every factor of 32.

2. Name the factors *composing* 32; those composing 48; 96; 120.

3. Name three factors of 54; of 60; 80; 90.

4. Name a factor of 28 which is also a factor of 40.

NOTE.—A number which is a factor of each of two or more numbers is called a *Common Factor* of them.

5. Name three numbers of which 7 is a common factor.

6. What is the smallest number of which 4, 5, and 8 are factors?

165. An ***Even Number*** is a number of which 2 is a measure.

1. Name ten even numbers.

2. Name ten even numbers of which 3 is a measure.

166. An ***Odd Number*** is a number of which 2 is not a measure.

1. Name ten odd numbers.

2. Name ten odd numbers of which 3 is a measure.

NOTES.—1. The product of any two or more factors, one or more of which are even, is even.

2. The product of odd factors is odd.

167. A ***Prime Number*** is a number whose only factors are itself and 1; as, 3, 19, 29.

1. Name all the prime numbers from 1 to 100.

2. Name all the prime numbers from 100 to 200.

168. A ***Composite Number*** is a number of which some other numbers besides itself and 1 are factors; as, 8, 15, 27.

NOTE.—Factors may be either prime or composite.

1. Name the composite numbers from 1 to 100.

2. Name the composite numbers from 100 to 200.

169. A ***Multiple*** of a number is any number of which it is a factor. (Art. 97, 3, Note 1.)

NOTE 1.—Every composite number is a multiple of each of its factors.

1. Name three multiples of 7; of 4; of 25.

2. Name three multiples of 15; of 8; of 20.

3. What multiple of 4 is also a multiple of 6?

NOTE 2.—Every prime number is a multiple of itself and of 1.

4. Name the smallest two multiples of 13.

5. What is the smallest number that is a multiple of 7, of 8, and of 12?

NOTE 3.—A measure or divisor of any number is also a measure or divisor of every multiple of that number.

170. A ***Common Multiple*** of two or more numbers is a number of which each is a factor or measure.

1. Name a common multiple of 3, 6, and 8.

2. Name a common multiple of 3, 2, and 5.

3. Name two common multiples of 4, 7, and 10.
4. Name a common multiple of 13 and 4.

NOTE.—C. M. is sometimes used as the abbreviation for the Common Multiple.

5. Name the smallest number that is a C. M. of 2, 3, and 4.
6. Name the smallest number that is a C. M. of 7 and 21.

NOTE.—Every number is a C. M. of itself and 1.

7. Name a C. M. of the number of which 2, 3, and 4 are the factors, and the number of which 3 and 5 are the factors.
8. Name a C. M. of three prime factors.
9. Name a C. M. of three composite factors.
10. Name a C. M. of two composite factors that are prime to each other.

171. The *Least Common Multiple* of several numbers is the *least* number of which each of them is a factor. (Art. 169, Ex. 5.)

NOTE.—L. C. M. is the abbreviation for the Least Common Multiple.

1. How many multiples of any number may there be?
2. How many C. M. of several numbers may there be?
3. How many L. C. M.?
4. Can the L. C. M. of 8, 10, and 40 be less than 40? How many of the prime factors of 40 must it contain? Will any number containing some prime factor not in the numbers considered be the *Least* C. M.?
5. What are the prime factors of 12? of 40? Why must the L. C. M. of 12 and 40 be more than 40?

What prime factor of 12 is not comprised in the prime factors of 40?

Will a number composed of the prime factors of 40 and this different prime factor of 12 contain each of them?

Examine $\mathrlap{\overbrace{5 \times 2 \times 2 \times 2}^{40} \times 3}5 \times 2 \times \underbrace{2 \times 2 \times 3}_{12} = 120 = \overbrace{5 \times 2}^{10} \times \underbrace{2 \times \overbrace{2 \times 2 \times 3}^{12}}_{24} =$

$\mathrlap{\underbrace{5 \times 2 \times 2}_{20} \times \underbrace{2 \times 3}_{6}}5 \times \overbrace{2 \times 2 \times 2}^{8} \times 3.$ L. C. M. of 6, 8, 10, 12, 20, 24, 40, is 120.

6. Indicate by brackets that 21, 12, 8, and 5, are contained in the following combination of prime factors, and find the product of these factors:

$$7 \times 3 \times 2 \times 2 \times 2 \times 5 \times 1.$$

Indicate three *different* numbers of which the product of these same factors is the L. C. M.

7. Factor 84 and indicate the numbers of which it is the L. C. M.

172. The L. C. M. of several numbers is composed of their prime factors, each factor being taken the greatest number of times that it occurs in either of the numbers.

NOTES.—1. With the prime factors of the largest number may be combined such other factors as are needed to compose the L. C. M.

2. If one number be a measure of another, it may be omitted in finding the L. C. M. Why?

173. Either of the following methods may be used in finding the L. C. M.:

1. L. C. M. of 18, 20, 24, 96.

$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$$
$$20 = 2 \times 2 \times 5$$
$$18 = 2 \times 3 \times 3$$

Combining with the factors of 96 one 5 comprised in 20, and a second 3 comprised in 18, and not comprised in 96, we have,

$$2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 3 = 1440 \text{ as L. C. M.}$$

Arrange these factors so as to group them by brackets.

NOTE.—The pupil may write a rule for this first method.

2. Select the prime factors which are common, and combine with the other factors, as follows:

$$\begin{array}{r|llr} 2 & 18, & 20, & (24), \ 96. \\ \hline 2 & 9, & 10, & 48. \\ \hline 3 & 9, & 5, & 24. \\ \hline & 3, & 5, & 8. \end{array}$$

$$2 \times 2 \times 3 \times 3 \times 5 \times 8 = 1440, \text{ L. C. M.}$$

174. The second method may be described by the following

RULE.

1. *Arrange the numbers on a horizontal line.*

2. *Enclose in a parenthesis, and omit any number that is a measure of any other number mentioned.*

3. *At the left write some prime number which is a common factor of two or more of the numbers, and in a second line write the quotient of each number divided by this factor, and the numbers of which it is not a factor. Proceed in the same manner until the numbers in the last line are prime to one another.*

4. *Find the product of the prime factors at the left, and the numbers in the last line.*

175. EXAMPLES.

1. L. C. M. of 18, 8, 20, 27.
2. " 120, 84, 36, 90, 18.
3. " 4, 5, 6, 7, 8, 9.
4. " 40, 25, 60, 35, 80, 45.
5. " 20, 50, 30, 70, 40, 90.
6. " 19, 42, 76.
7. " 144, 72, 30.
8. " \$12, \$20, \$32, \$60.
9. " 2 ft., 8 in., 10 in., 1 ft.
10. " 7, 9, 11, 23.

176. The ***Greatest Common Measure,*** or ***Greatest Common Divisor,*** of two or more numbers is the greatest number that will exactly measure each of them. Abbreviation, G. C. D.

1. What is the G. C. D. of 24, 32, 80?
2. " " " 40, 60, 100?
3. " " " 28, 42, 56?
4. " " " 32, 48, 80?
5. " " " 90, 150, 300?
6. Factor 36, 48, and 60, and group the *common factors.*

$$36=\overbrace{2\times2\times3}\times3$$

$$48=2\times2\times\overbrace{2\times2\times3}$$

$$60=\overbrace{2\times2\times3}\times5$$

What is the G. C. D. of these numbers?

177. The G. C. D. is equal to the greatest common factor, or the product of all the prime factors that are common.

1. Factor and find G. C. D. of 18, 20, 24, 96.
2. Factor and find G. C. D. of 36, 54, 81.

NOTE.—Let the pupil write a *rule* for this method of finding the G. C. D.

3. Factor 24, 36, and 60, and combine their factors so as to indicate their product.

What is the quotient of their product divided by their L. C. M.? What is the quotient of their product divided by their G. C. D.?

4. What is the G. C. D. of 24 and 48?
5. What is the G. C. D. of 32 and 64?

What is the G. C. D. of any two numbers whose difference is equal to the smaller number?

6. What is the G. C. D. of 32 and 40?
7. What is the G. C. D. of 72 and 81?

Can the G. C. D. be *greater* than the difference between two numbers? When will it be less?

8. Name two numbers whose difference is their G. C. D.
9. Name two numbers whose difference is greater than their G. C. D.
10. Name two numbers whose difference is a multiple of the smaller number.

What is the G. C. D. of these numbers?

11. What is a measure of 18, 24, and of their *sum?*
12. Show that a common measure of any numbers is also a measure of their sum.
13. What is a measure of 24, 40, and of their difference?
14. Show that a common measure of any two numbers is also a measure of their difference.

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80)176(2
   160
    16)80(5
       80
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178. Comparing 80 and 176 by division, we find that 176 = (two 80's) + 16, hence 80 is not a common divisor of 80 and 176.

But 16 is a measure of 80; 80 = five 16's. Now 176, which is equal to two 80's and one 16, is therefore equal to *twice* five 16's + *one* 16 = ten 16's + one 16 = eleven 16's; hence 16 is a *common* divisor of 80 and 176. Since the G. C. D. of 80 and 176 must also be a measure of 2×80 or 160, and if of 160 and 176, then of their

difference 16, and since no number greater than 16 can measure itself, 16 *is* the *greatest* common divisor.

179. When numbers are not readily factored, their G. C. D. may be found by the following

RULE.

1. *Divide the larger number by the smaller.*

2. *Divide the first divisor by the first remainder, the first remainder by the second remainder, and thus continue till there is no remainder. The last divisor will be the G. C. D.*

NOTE.—When the G. C. D. of more than two numbers is to be found by this method, the G. C. D. of any two of them may be first found, then the G. C. D. of this G. C. D. and another number, and so on.

180. EXAMPLES.

1. Find G. C. D. of 80 and 144.
2. Of 84, 320, 64.
3. Of 56, 98.
4. Of 69, 161.
5. Of 168, 392.
6. Of 126, 294.
7. Of 148, 296.
8. Of 16, 32, 86.
9. Of 92, 138, 161.
10. Of 2048, 2560.

COMPLEMENTS.

181. The ***Complement*** of a number is the difference between it and a unit of the next higher order. Thus the complement of 7 is $10-7=3$; of 95, $100-95=5$; of 220, $1000-220=780$, etc.

182. In some cases of multiplication it is more convenient to use a larger number than one of the factors as a multiplier, and from the product subtract the product of the other factor and the difference between the correct and the assumed factor. This contraction may be used to advantage when the complement of one of the factors is a convenient multiplier.

1. Thus $7896 \times 98 = 7896 \times 100 - (7896 \times 2)$.
2. $98724 \times 999 = 98724 \times 1000 - (98724 \times 1)$.
3. $1786543 \times 9980 = 1786543 \times 10000 - (1786543 \times 20)$.
4. $960 \times 1728 = 1728 \times 1000 - (1728 \times 40)$.
5. $9920 \times 4567 = 4567 \times 10000 - (4567 \times 80)$.

EXAMPLES.

1. 123456 × 999.
2. 8972047 × 9998.
3. 74284 × 970.
4. 9980 × 145280.
5. 140809 × 999999.
6. 5280 × 975.

183. RECIPROCALS.

NOTE.—To understand this Article some knowledge of Fractions is required, and if the instructor think best it may be omitted till some progress has been made in Fractions.

1. The ***Reciprocal*** of a number is 1 divided by that number. Thus the reciprocal of 5 is $\frac{1}{5}$; of 12, $\frac{1}{12}$; of 240, $\frac{1}{240}$, etc.

2. The Reciprocal of any integer is expressed by the fraction whose numerator is 1, and whose denominator is the given number.

Ex. Name the reciprocals of 7, 15, 29, 41, 50, 100, 476, 975, 1040.

3. The Reciprocal of a fraction is expressed by the fraction inverted. Thus the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$; of $\frac{8}{5}$, $\frac{5}{8}$; $\frac{12}{11}$, $\frac{11}{12}$, etc.

Ex. Name the reciprocals of $\frac{4}{7}$, $\frac{9}{10}$, $\frac{15}{3}$, $\frac{20}{14}$, $\frac{18}{27}$, $\frac{19}{30}$, $\frac{40}{10}$, $\frac{1}{4}$, $\frac{1}{8}$.

4. The Reciprocal of a Ratio is expressed by the Ratio inverted; hence it is sometimes called an *Inverse Ratio.*

Thus the reciprocal of 12 : 4 is 4 : 12; of 24 : 6, 6 : 24; of 20 : 60, 60 : 20; of $\frac{42}{7}$, $\frac{7}{42}$; of $\frac{9}{18}$, $\frac{18}{9}$; 1 : 6, 6 : 1, etc.

Ex. 1. Name the reciprocals of 21 : 42; 100 : 50; 72 : 9; $\frac{6}{24}$; $\frac{18}{6}$; 2 : 1.

2. What is the *value* of the reciprocals of 12 : 24; 36 : 9; 8 : 80; 80 : 16?

184. If any number be *divided by the reciprocal of a multiplier*, the quotient will equal the product sought. Thus $4 \times 2 = 4 \div \frac{1}{2} = 8$; $16 \times \frac{1}{4} = 16 \div 4 = 4$; $20 \times 5 = 20 \div \frac{1}{5} = 100$, etc.

185. 1. If any number be *multiplied by the reciprocal of a divisor*, the product will equal the quotient sought. Thus $18 \div 3 = 18 \times \frac{1}{3} = 6$; $6 \div \frac{1}{4} = 6 \times 4 = 24$.

2. The product of two mutual reciprocals is 1.

Thus $\frac{1}{7}$ is the reciprocal of 7,
and 7 " " $\frac{1}{7}$,
and $\frac{1}{7} \times 7 = \frac{7}{7} = 1$.

NOTE.—Reciprocals may frequently be used to advantage when the multiplier or divisor is a fraction, and when one ratio is to be made equal to another ratio, as in Proportion.

Use reciprocals in the following

EXAMPLES.

1. $784 \times \frac{1}{3}$.
2. $1728 \div \frac{4}{8}$. (Read "ratio of 4 to 8.")
3. $4090 \div \frac{1}{10}$.
4. $960 \div \frac{3}{27}$.
5. $1184 \times \frac{7}{42}$.

AVERAGES.

186. The average of two numbers is one-half their sum. Thus 6 is the average of 7 and 5; 8 and 4; 9 and 3; 10 and 2; 11 and 1.

The difference between the average of two terms and each of them is the same. Thus $7 \sim 6 = 5 \sim 6$; $8 \sim 6 = 4 \sim 6$; $9 \sim 6 = 3 \sim 6$, etc.

187. The average of several terms is the quotient of their sum divided by the number of terms. Thus the average of 3, 8, 11, 9, and $14 = (3+8+11+9+14) \div 5 = 9$. The average of 14, 20, 25, and 17 is $(14+20+25+17) \div 4 = 19$.

EXAMPLES.

1. If a man travel 27 miles one day, 32 another, and 36 another, what is the average distance per day?

2. Suppose a man expends $60 for clothing for himself, $90 for his wife, $40 for each of his two daughters, and $35 for each of his two sons; what is the average expense of clothing for each member of that family?

3. What is the average price of gold for any week when the quotations are 110, 111, 112, 113, 111, 112?

4. What is the average temperature for three days, during which the thermometer stands at 82°, 84°, and 83° on each different day?

PECULIAR NUMBERS.

188. 1. *Nine.*—The relation of the number 9 in the decimal system of notation is such that any number divided by 9 will leave the same remainder as the sum of its digits divided by 9. The

remainder in this case is called the *excess of nines.* Thus $75 \div 9 = 8$, *Rem.* 3, and $(7+5) \div 9 = 1$, *Rem.* 3.

10 consists of	1 *nine*, and	$(1+0)=1$ unit.	
12 "	1 "	$(1+2)=3$ units.	
23 "	2 *nines*, and	$(2+3)=5$ units.	
96 "	10 "		6 units.
695 "	77 "		2 units, etc.

2. This may be explained as follows: Take any number, as $347=300+40+7$.

$300=33$	*nines* +		3 units.
$40=\ 4$	" +		4 units.
$7=\ 0$	" +		7 units.
$347=37$	" +		14 units, and
$14=\ 1$	" +	$(1+4)$ or	5 units, and
$347=38$	" +		5 units.

It will be observed that the first three remainders are the same as the digits of the number, and that the excess of nines in the number equals the excess in the sum of these digits.

3. Every number consists of a certain number of nines, plus the sum of its digits.

NOTES.—1. This property of 9 affords short and accurate tests for ordinary operations. (Arts. 80, 101, 3, 135, 2.)

2. Other properties of 9, and of 3, 7, and 11 are omitted here as of but little practical utility.

189. MISCELLANEOUS PROBLEMS.

1. What is the least sum of money with which you could purchase sugar @ 12 cts., coffee @ 32 cts. or dried peaches @ 20 cts.?

2. What is the greatest length of equal sections of a cable that can be cut from coils containing 120 ft., 340 ft., and 520 ft.?

3. What is the greatest length of curb-stones that can be used without cutting, on either of three blocks, 360 ft., 384 ft., and 420 ft. long?

4. What is the average length of a month in a year of 365 days?

5. What is the product of the complement of 80, and the reciprocal of $\frac{1}{10}$?

SECTION X.

COMMON FRACTIONS.

190. 1. If any unit, as an orange, a farm, a mile, or simply *one*, be divided into two equal parts, each part would be one-half of the unit divided.

2. If the unit be divided into five equal parts, what would one part be called? What would two parts be called? Three parts? Four parts? Five parts?

3. Since a unit is broken in obtaining such parts, every such part, or collection of such parts, is called a ***Fraction*** (*fractum* meaning *broken*).

4. One of the equal parts is a *Fractional Unit;* as, *one*-half, *one*-sixth, etc.

5. The whole of which the fractional unit is a part is the *Unit of the Fraction.* If one-fourth of an *orange* be taken, one orange is the *unit of the fraction.* If one-half of a hundred be taken, then *one hundred* is the *unit of the fraction,* and in this case the value of the fractional unit $\frac{1}{2}$ is 50 units of the first order. If $\frac{1}{2}$ of eight-ninths be taken, then eight-ninths ($\frac{8}{9}$) is the unit of the fraction or the number divided, the value being $\frac{4}{9}$.

191. A *Fraction,* then, or a *Fractional Number,* is an expression for one or more fractional units.

NOTE.—The fractional unit has the same varied application as the integral (Art. 14), and is abstract unless otherwise designated.

192. The elements of every fraction are the *unit* of *the fraction,* the *fractional unit,* and the *number of fractional units,* considered or expressed.

193. The name of the fractional unit is derived from the number of equal parts in the whole. Thus if the whole be divided into three equal parts, the parts are called thirds; if into ten, tenths; if into twenty, twentieths, etc.

194. 1. Fractions are commonly expressed by writing the number showing how many parts in the whole under the number showing how many parts are taken, with a line between them; as, $\frac{4}{10}$, $\frac{5}{7}$, etc.

2. The lower term is called the ***Denominator***, because it *names* the fractional units; the upper term is called the ***Numerator***, because it *numbers* the fractional units.

$$\text{Thus} \left\{ \begin{matrix} 9 & \text{Numerator.} \\ 11 & \text{Denominator.} \end{matrix} \right\} \text{Terms.}$$

NOTE.—Abbreviations—N. may signify Numerator; D. Denominator.

195. 1. Since $\frac{3}{4}=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$, it is evident that if we take $\frac{1}{4}$ of each of *three ones* the same value will be expressed as in three-fourths of *one*.

2. The N. may be regarded as the number divided, and the D. as the number by which it is divided. Thus every common fraction may be regarded as *an expression of division*, the value of which is the quotient of the N. divided by the D.; as,

$\frac{5}{9}$ = one-ninth of five, in which case 5 is the unit of the fraction, and $\frac{1}{9}$ of 5 the fractional unit.

$\frac{3.8}{4}$ = one-fourth of three and eight-tenths, or of thirty-eight tenths.

196. 1. A *Common Fraction* is a fractional number, both of whose terms are expressed.

2. Common fractions are divided into three classes—*simple*, *compound*, and *complex*.

3. A *Simple Fraction* is a *single* fraction, each of whose terms is a simple integer; as, $\frac{5}{6}$, $\frac{7}{8}$.

4. A *Compound Fraction* is a fraction of a fraction, the unit of the compound fraction being itself a fraction; as, $\frac{1}{3}$ of $\frac{9}{10}$, $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{2}{5}$ of $\frac{6}{7}$ of $2\frac{1}{2}$.

5. A *Complex Fraction* is a fraction having a fraction for one or both of its terms; as, $\frac{\frac{2}{3}}{\frac{10}{11}}$, $\frac{\frac{9}{5}}{4}$, $\frac{5}{\frac{6}{7}}$.

NOTE.—It may be observed here that every compound fraction may be expressed in the form of a complex fraction, having a fraction for the numerator Thus the compound fractions above become $\frac{\frac{9}{10}}{3}$ $(=\frac{3}{10})$; $\frac{2 \times \frac{3}{4}}{3}$, or $2 \times \frac{\frac{3}{4}}{3}$; and the last becomes $2 \times \frac{6 \times \frac{2\frac{1}{2}}{7}}{5}$.

197. 1. *Simple Fractions* are divided into *proper* and *improper*.

2. A *Proper Fraction* is one whose numerator is less than its denominator; as, $\frac{5}{9}$, $\frac{7}{11}$, $\frac{99}{100}$.

3. An *Improper Fraction* is one whose numerator equals or exceeds its denominator; as, $\frac{6}{6}$, $\frac{8}{8}$, $\frac{9}{7}$, $\frac{14}{10}$.

198. A ***Mixed Number*** consists of a whole number and fraction combined; as, $2+\frac{1}{2}$, $10+\frac{6}{7}$, $25+\frac{11}{12}$; but these are expressed without the sign of addition; thus $2\frac{1}{2}$, $10\frac{6}{7}$, $25\frac{11}{12}$, and are read "two and one-half," etc.

NOTE.—In reading such a mixed number as $200\frac{7}{9}$, it is necessary to make an emphatic pause after the integer, or to designate the fraction as such, in order to distinguish it from an improper fraction, the verbal expression of which would be similar.

199. 1. It is evident from the very nature of a fraction (Art. 195, 2) that—1st. If both terms be multiplied or divided by the same number, the value will not be changed. (Art. 147.)

$$\frac{2}{6}=\frac{2\times2}{6\times2}=\frac{4}{12};\ \frac{1}{4}=\frac{1\times2}{4\times2}=\frac{2}{8}.$$

$$\text{and } \frac{8}{12}=\frac{8\div4}{12\div4}=\frac{2}{3};\ \frac{4}{8}=\frac{4\div4}{8\div4}=\frac{1}{2}.$$

2. If the N. be multiplied by any number, the value of the fraction is multiplied by the same number. (Art. 145.)

$$\frac{1}{3}\times3=\frac{1\times3}{3}=\frac{3}{3}=1.$$

3. If the D. be divided by any number, the value of the fraction is multiplied by the same number. (Art. 145.)

$$\frac{1}{3}\times3=\frac{1}{3\div3}=\frac{1}{1}=1.$$

4. If the N. be divided by any number, the value of the fraction is divided by the same number. (Art. 146.)

$$\frac{4}{8}\div2=\frac{4\div2}{8}=\frac{2}{8}.$$

5. If the D. be multiplied by any number, the value of the fraction is divided by the same number. (Art. 146.)

$$\frac{1}{2}\div2=\frac{1}{2\times2}=\frac{1}{4}.$$

200. The value of a fraction may be *increased*—

1st. By increasing the Numerator (or dividend) by addition or multiplication.

2d. By diminishing the Denominator (or divisor) by subtraction or division.

201. The value of a fraction may be *decreased*—

1st. By diminishing the Numerator (or dividend) by subtraction or division.

2d. By increasing the Denominator (or divisor) by addition or multiplication.

REDUCTION OF FRACTIONS.

202. As a unit of one rank or denomination is equivalent to several of lower rank, and is only a part of a unit of higher rank (Art. 13), so also one fraction has a similar relation to other fractions and to integral units. Thus $\frac{1}{2}$ is equivalent to two *fourths*, and $\frac{8}{4}$ is equivalent to two *units*. $\frac{8}{10}=\frac{80}{100}$, and $\frac{80}{10}=8$. 1 doz. = 12 units, and 1 unit = $\frac{1}{12}$ doz. Now if 1 doz. be the unit of a fraction, and the fractional unit be *one-third*, then the true expression for the fraction is $\frac{1 \text{ doz.}}{3}$. If two such fractional units be taken, we have $\frac{2}{3}$ doz. or $\frac{2 \text{ doz.}}{3}$. It is very evident that the fraction $\frac{1}{3}$ doz. is equivalent to 4 units of the first integral order, and 4 is said to be *one-third* of 12; but four *is not one* any more than 12 *is one*, hence 4 is not *the fraction*, but it is the *value* of the fraction, even as 12 is not *one* dozen, but is the *value* of one dozen in units of the first order, and 8 would be the value of $\frac{2}{3}$ doz.

203. In the reduction and transformation (Art. 73) of fractional numbers for purposes of comparison and combination, it is desirable to express their value in integers, or in the largest fractional units possible.

204. When integers and fractions are to be combined or compared, it is frequently necessary to reduce the integers to equivalent fractions, or fractions to equivalent integers. 1. Thus if 2 is to be combined with $\frac{3}{4}$ in a simple number, 2 must be reduced to fourths. One is equivalent to $\frac{4}{4}$, hence two $=\frac{8}{4}$, and combining $\frac{3}{4}$ and $\frac{8}{4}$ by addition we have $\frac{11}{4}$, their sum as a simple fractional number.

2. If the same numbers are compared by subtraction, the result would be $\frac{8}{4}-\frac{3}{4}=\frac{5}{4}$.

3. If compared by division to find their ratio, the result would be $\frac{8}{4}\div\frac{3}{4}=\frac{8}{3}$.

205. *Any integer may be reduced to a fraction by multiplying it by the number of fractional units required in an integral unit.* (Art. 74, 3.) $5=\frac{15}{3}$, $7=\frac{63}{9}$, $12=\frac{120}{10}$, etc.

206. A mixed number is easily reduced by reducing the integer to the same units as the fraction indicates, and combining the two numbers of fractional units in one sum.

Thus $5\frac{2}{3} = \frac{15}{3}+\frac{2}{3}=\frac{17}{3}$.

$4\frac{2}{10}=\frac{40}{10}+\frac{2}{10}=\frac{42}{10}$.

$19\frac{7}{8}=\frac{152}{8}+\frac{7}{8}=\frac{159}{8}$, etc.

RULE.

Multiply the whole number by the denominator of the fraction; to the product add the numerator, and under the result place the denominator.

EXAMPLES.

207. 1. Reduce $19\frac{2}{7}$ to a fraction.

2. Reduce $44\frac{2}{3}$ to an improper fraction.
3. Reduce $3\frac{60}{105}$ to an improper fraction.
4. Reduce $56\frac{3}{4}$ to an improper fraction.
5. Reduce $1236\frac{9}{20}$ to an improper fraction.
6. Reduce $5\frac{3}{720}$ to an improper fraction.
7. Reduce $23\frac{9}{18}$ to an improper fraction.
8. Reduce $133\frac{13}{15}$ to an improper fraction.
9. Reduce $563\frac{1}{3}$ to an improper fraction.
10. Reduce $8006\frac{13}{17}$ to an improper fraction.
11. Reduce 24 to fourths.
12. Reduce 35 to twentieths.
13. Reduce 312 to twelfths.
14. Reduce 19 to twenty-fifths.
15. Reduce 1008 to ninths.

208. Any improper fraction may be reduced to an integer or mixed number. (Art. 74, 2.)

Ex. 1. How many units (integral) does $\frac{12}{3}=$? $\frac{16}{4}$? $\frac{200}{10}$?

2. What mixed number is equivalent to $\frac{7}{3}$? $\frac{14}{5}$? $\frac{28}{5}$? $\frac{77}{10}$? $\frac{100}{99}$? $\frac{199}{100}$?

3. Write a *Rule* for this process.

209. EXAMPLES.

1. Reduce $\frac{19}{5}$, $\frac{25}{4}$, $\frac{57}{19}$.
2. Reduce $\frac{126}{12}$, $\frac{121}{17}$, $\frac{315}{95}$.
3. Reduce $\frac{75}{8}$, $\frac{87}{17}$. $\frac{161}{9}$.
4. Reduce $\frac{7.4}{2}$, $\frac{.42}{3}$, $\frac{8.75}{25}$.
5. Reduce $\frac{7420}{84}$, $\frac{8091}{125}$, $\frac{12345}{1250}$.

210. 1. Any fraction may be reduced to smaller fractional units, or to *higher terms.*

Thus $\frac{2}{3}$ may be reduced to ninths. Since $1=\frac{9}{9}$, $\frac{1}{3}$ would $=\frac{3}{9}$, and $\frac{2}{3}=\frac{6}{9}$; or we may say $\frac{2}{3}=\frac{2\times 3}{3\times 3}=\frac{6}{9}$. (Art. 199, 1.)

It may be observed that the 3 by which each term is multiplied is the ratio of the new denominator, 9, to the original denominator, 3; i. e., $\frac{9}{3}=3$.

2. RULE.

Multiply both terms of the fraction by the ratio of the required denominator to the given denominator.

What is the proper title for this Rule?

EXAMPLES.

1. Reduce $\frac{4}{9}$ to 27ths.
2. Reduce $\frac{1}{3}$ to 60ths.
3. Reduce $\frac{5}{16}$ to 80ths.
4. Reduce $\frac{2}{3}$, $\frac{5}{8}$, $\frac{7}{6}$, and $\frac{11}{12}$ to 24ths.
5. Reduce $\frac{3}{5}$, 2, $\frac{2}{17}$, and $\frac{1}{5}$ to 85ths.

NOTE.—When the denominators of several fractions are the same, the fractions are said to have a *Common Denominator.*

6. Reduce $\frac{3}{7}$ to 63ds.
7. Reduce $\frac{5}{9}$ to 27ths.
8. Reduce $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{2}{3}$ to 24ths.
9. Reduce $\frac{5}{7}$, $\frac{3}{5}$, and $\frac{3}{14}$ to 70ths.
10. Reduce $\frac{7}{12}$, $\frac{5}{8}$, $\frac{3}{4}$, and $\frac{9}{16}$ to 48ths.

211. Any fraction, whose terms are not prime to each other, may be reduced to larger fractional units, or to *lower terms.*

Thus $\frac{6}{8}$ may be reduced to fourths. Since $1=\frac{4}{4}=\frac{8}{8}$, $\frac{1}{4}$ would $=\frac{2}{8}$, and there are as many fourths in $\frac{6}{8}$ as $6\div 2$, or $\frac{3}{4}$; or we may say $\frac{6}{8}=\frac{6\div 2}{8\div 2}=\frac{3}{4}$. (Art. 199, 1.)

2. A fraction is in its *lowest terms* when the terms are prime to each other.

NOTE.—When a fraction is to be reduced to a *certain* higher *denomination*, the Rule in Art. 210, 2, may be applied. Thus if $\frac{8}{12}$ is to be reduced to thirds, each term may be *multiplied* by $\frac{1}{3}$, which is the ratio of 4 to 12, or the terms may be *divided* by 3, the reciprocal of $\frac{1}{3}$. (Art. 183.)

3. A fraction may be reduced to its lowest terms by the following

RULE.

Divide both terms of the fraction by their greatest common divisor.

EXAMPLES.

Reduce to lowest terms—

1. $\frac{63}{84}$.
2. $\frac{16}{128}$.
3. $\frac{14}{91}$.
4. $\frac{45}{105}$.
5. $\frac{72}{168}$.
6. $\frac{150}{720}$.
7. $\frac{96}{144}$.
8. $\frac{360}{1032}$.
9. $\frac{4158}{9702}$.
10. $\frac{504}{1386}$.
11. Reduce $\frac{12}{84}$ to 14ths.
12. Reduce $\frac{20}{75}$ to 15ths.
13. Reduce $\frac{32}{80}$ to 5ths.
14. Reduce $\frac{84}{108}$ to 9ths.
15. Reduce $\frac{49}{70}$ to 10ths.

212. In finding the value of a compound fraction, each part of it, excepting the first, may be regarded, in succession, as the unit of the fraction.

Thus, in $\frac{2}{3}$ of $\frac{15}{17}$, if $\frac{15}{17}$ be regarded as the unit of the fraction, and $\frac{2}{3}$ of *it* be taken, the result will be $\frac{10}{17}$. In $\frac{2}{5}$ of $\frac{10}{13}$ of $\frac{26}{30}$; we may say $\frac{2}{5}$ of $\frac{10}{13}=\frac{4}{13}$, and $\frac{4}{13}$ of $\frac{26}{30}=\frac{8}{30}$. In $\frac{3}{4}$ of $\frac{7}{8}$ we may take $\frac{1}{4}$ of $\frac{7}{8}$ by taking only one-fourth as large parts (Art. 199, 5), giving $\frac{7}{32}$, and $\frac{3}{4}$ of $\frac{7}{8}$ is $\frac{7}{32}\times 3=\frac{21}{32}$. (Art. 199, 2.) In this last case we have the result from $\frac{3}{4}\times\frac{7}{8}$, or $\frac{3\times 7}{4\times 8}=\frac{21}{32}$. The other two cases may be reduced in the same form.

$\frac{2}{3}\times\frac{15}{17}=\frac{30}{51}$, which reduced to lower terms becomes $\frac{10}{17}$ as before.

$$\frac{2}{5}\times\frac{10}{13}\times\frac{26}{30}=\frac{520}{1950}=\frac{8}{30}.$$

213. From the cases mentioned we may deduce the following

RULE.

Multiply all the numerators together for the numerator of the simple fraction, and multiply all the denominators together for the denominator of the simple fraction.

NOTES.—1. Whole or mixed numbers, in connection with a compound fraction, should be reduced to a fractional form.

2. Contract by cancellation (Art. 152), and thus obtain the lowest terms.

What is the correct title of the above *Rule?*

EXAMPLES.

1. Reduce to a simple fraction $\frac{4}{5}$ of $\frac{5}{6}$?
2. Reduce $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{3}{9}$ to a simple fraction.
3. Reduce $3\frac{1}{3}$ of $2\frac{1}{2}$ of $\frac{3}{10}$ to a simple fraction.
4. Reduce $2\frac{1}{9}$ of $1\frac{9}{16}$ of $\frac{4}{5}$ to a simple fraction.
5. Reduce $\frac{4}{5}$ of $\frac{7}{8}$ of $2\frac{1}{2}$ to a simple fraction.
6. Reduce $\frac{4}{5}$ of $1\frac{1}{2}$ of $\frac{3}{7}$ of $2\frac{1}{3}$ to a simple fraction.
7. Reduce $\frac{3}{8}$ of 6 to a simple fraction.
8. Reduce $\frac{2}{3}$ of $2\frac{1}{2}$ of 3 to a simple fraction.
9. $\frac{4}{5}$ of $4\frac{1}{5}$ of $\frac{9}{14}$ of $\frac{7}{8}$.
10. $\frac{6}{7}$ of 42 of $\frac{2}{3}$ of $7\frac{2}{5}$.
11. $\frac{3}{4}$ of $\frac{8}{9}$ of $\frac{9}{7}$ of $2\frac{1}{3}$.
12. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $1\frac{1}{6}$.

$$\frac{\cancel{2} \times \cancel{3} \times \cancel{4} \times \cancel{5} \times \cancel{6} \times \cancel{7}}{\cancel{3} \times \cancel{4} \times \cancel{5} \times \cancel{6} \times \cancel{7} \times \underset{3}{\cancel{6}}} = \frac{1}{3}\ \textit{Ans.}$$

NOTE.—1 remains as a factor in the numerator.

13. $\frac{4}{5}$ of $4\frac{1}{5}$ of $\frac{9}{14}$ of $\frac{7}{8}$.
14. $\frac{6}{9}$ of $2\frac{1}{16}$ of $\frac{7}{25}$ of $\frac{2}{3}$.
15. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{4}{10}$ of $12\frac{4}{7}$.

214. Fractions may be reduced to others having a *common denominator* (Art. 210, Note) by taking for a common denominator any common multiple of the several denominators; but it is generally desirable to reduce to the lowest terms possible, hence the *least common denominator* is commonly required.

215. Fractions may be reduced to the least common denominator, or the largest common unit, by the following

RULE.

Find the least common multiple of the several denominators; this will be the least common denominator. Then reduce each fraction as in Art. 210, 2.

NOTE.—Each fraction should first be expressed in its lowest terms.

EXAMPLES.

Reduce to the least common denominator—

1. $\frac{3}{4}$, $\frac{5}{8}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{7}{12}$.
2. $\frac{3}{4}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{5}{14}$, $\frac{9}{28}$.
3. $\frac{5}{6}$ and $\frac{7}{16}$.
4. $\frac{1}{4}$, $\frac{3}{5}$, $\frac{1}{3}$, $\frac{5}{6}$.
5. $\frac{2}{3}$, $\frac{3}{7}$, $\frac{5}{6}$, $\frac{1}{2}$.
6. $\frac{1}{2}$ of $\frac{2}{3}$, $2\frac{1}{2}$, $\frac{5}{6}$.
7. $\frac{2}{5}$ of $\frac{5}{6}$, $\frac{3}{5}$, $2\frac{1}{3}$.
8. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$.
9. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$.
10. $\frac{3}{4}$, $\frac{7}{8}$, $\frac{6}{7}$, $\frac{5}{9}$.

ADDITION OF FRACTIONS.

216. PROBLEMS FOR MENTAL SOLUTION.

1. What part of a farm is equivalent to $\frac{1}{4}$ and $\frac{3}{4}$ of it?

2. If you walk $\frac{2}{7}$ of a mile in 20 minutes, and $\frac{4}{7}$ in 40 minutes, how far would you walk in an hour?

3. What is the sum of $\frac{3}{8}$ yd. and $\frac{2}{8}$ yd.?

4. Add $\frac{7}{12}$ of a dollar and $\frac{3}{12}$ of a dollar.

5. What is the amount of $\frac{1}{2}$ an orange and $\frac{1}{4}$ an orange?

217. 1. If the sum of $\frac{6}{8}$ and $\frac{2}{4}$ be sought, since only units of the same kind can be added, the $\frac{6}{8}$ may be reduced to the larger denomination of fourths (Art. 211), or the $\frac{2}{4}$ may be reduced to the smaller denomination of eighths (Art. 210), and then added. Thus $\frac{6}{8}=\frac{3}{4}$, and $\frac{3}{4}+\frac{2}{4}=\frac{5}{4}=1\frac{1}{4}$; or $\frac{2}{4}=\frac{4}{8}$, and $\frac{4}{8}+\frac{6}{8}=\frac{10}{8}=1\frac{1}{4}$.

2. Sometimes two reductions are necessary. Thus in $\frac{2}{3}$ yd. + $\frac{4}{9}$ ft., $\frac{4}{9}$ ft. = ($\frac{1}{3}$ of $\frac{4}{9}$) yd. = $\frac{4}{27}$ yd.; $\frac{2}{3}$ yd. = $\frac{18}{27}$ yd., and $\frac{4}{27}$ yd. + $\frac{18}{27}$ yd. = $\frac{22}{27}$ yd.

218. RULE.

Add the numerators of fractions expressing units of the same value, or having a common denominator.

NOTES.—1. If the fractions express units of different values, first reduce each fraction to its simplest form, then reduce the several fractions to the largest common unit. (Art 215.)

2. Integers may be combined with the sum of the fractions when found.

219. EXAMPLES.

1. $\frac{2}{3}+\frac{3}{4}+\frac{5}{9}+\frac{7}{8}$.
2. $(\frac{4}{5}$ of $\frac{3}{4})+(\frac{5}{6}$ of $\frac{6}{7})+2\frac{1}{4}$.
3. Add $\frac{3}{4}$, $\frac{2}{5}$, $\frac{2}{3}$, and $\frac{1}{2}$.
4. Add $\frac{2}{3}$ of $\frac{3}{8}$ and $\frac{9}{16}$.
5. Add $\frac{2}{3}$, $\frac{6}{8}$, and $\frac{4}{7}$.
6. Add $5\frac{1}{4}$, $3\frac{2}{5}$, $5\frac{3}{8}$.
7. Add $\frac{2}{3}$ of $2\frac{1}{2}$ and $\frac{3}{7}$ of $2\frac{1}{3}$.
8. Add $1026\frac{3}{20}$, $1875\frac{3}{4}$, and $5634\frac{2}{3}$.

Suggestion.—First add the fractions.

9. Add $37\frac{1}{2}$, $18\frac{3}{4}$, $33\frac{1}{3}$, and $81\frac{1}{4}$.
10. Add $\frac{2}{3}$, $\frac{9}{34}$ of $4\frac{1}{4}$, $563\frac{5}{8}$, and $\frac{3}{4}$ of $3\frac{5}{9}$.
11. Add $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$.
12. Add $\frac{2}{5}$, $\frac{6}{8}$, $\frac{3}{10}$, and $\frac{4}{7}$ of $1\frac{1}{5}$.
13. Add $\frac{2}{3}$ of $\frac{3}{5}$, and $12\frac{1}{2}$.
14. Add $105\frac{2}{3}$ and $98\frac{1}{12}$.

NOTE.—When the numerators are large, for convenience they may be written as in simple addition, with the denominator at the right, thus $\frac{480}{720}+\frac{1356}{720}+\frac{2004}{720}+\frac{4460}{720}$, may be written

$$\left.\begin{array}{r}480\\1356\\2004\\4460\end{array}\right\}\text{720ths}$$

MULTIPLICATION OF FRACTIONS.

220. 1. If 4 three-sevenths were to be combined; as, $\frac{3}{7}+\frac{3}{7}+\frac{3}{7}+\frac{3}{7}$, the result would be $\frac{12}{7}$ or $\frac{3}{7}\times 4$, that is, we take four times as many fractional units as at first.

2. If we wish to obtain a number equal to four times $\frac{3}{8}$, we may take (as before) four 3's of the same unit, equal to $\frac{12}{8}$, or we may take 3 fractional units, each equal to $\frac{4}{8}$. Now $\frac{4}{8}=\frac{1}{2}$, and 3 of this $\frac{3}{2}$. In the first case the number of units is increased fourfold, in the second the size of the units is increased fourfold. (Art. 199, 2 3.)

221. *To multiply a fraction* use the

RULE.

Multiply the numerator or divide the denominator.

EXAMPLES.

1. $\frac{5}{12}\times 4$.	6. $\frac{18}{27}\times 9$.
2. $\frac{7}{64}\times 16$.	7. $\frac{1728}{2640}\times 12$.
3. $\frac{16}{21}\times 3$.	8. $\frac{420}{728}\times 6$.
4. $\frac{56}{81}\times 9$.	9. $\frac{4}{9}\times 210$.
5. $\frac{28}{1000}\times 25$.	10. $\frac{51}{70}\times 365$.

NOTE.—Contract by cancellation; as, $\frac{5}{66}\times 99=\frac{5\times\overset{9}{\cancel{99}}}{\underset{6}{\cancel{66}}}=\frac{45}{6}=7\frac{1}{2}$.

222. *To multiply by a fraction.*—1. If 16 is to be multiplied by $\frac{3}{4}$, we may consider that $\frac{3}{4}$ can be resolved into two factors, viz., the fractional unit and the number taken; thus $\frac{3}{4}=\frac{1}{4}\times 3$, the same as 3 yds.$=$1 yd.$\times 3$. Now, if any multiplicand be multiplied by the factors of the multiplier, the correct product is obtained; hence $16\times\frac{3}{4}=16\times 3\times\frac{1}{4}$, and as the reciprocal of $\frac{1}{4}$ may be used as a divisor (Art. 184), it may be written $16\times 3\div 4=\frac{16\times 3}{4}=\frac{\overset{4}{\cancel{16}}\times 3}{\cancel{4}}=$ 12. Or since either factor may be used as the multiplier, $16\times\frac{3}{4}$ $=\frac{3}{4}\times 16=\frac{48}{4}=12$.

223. RULE.

1. *Multiply as in Art. 221; or,*

2. *Use the numerator as a multiplier, and the denominator as a divisor, in a combination consisting of the multiplicand and the multiplier.*

EXAMPLES.

1. $15\times\frac{3}{4}=15\times 3\div 4=\frac{15\times 3}{4}=\frac{45}{4}=11\frac{1}{4}$.

2. $56\times\frac{5}{8}=\frac{\overset{7}{\cancel{56}}\times 5}{\cancel{8}}=35$.

3. $5280\times\frac{3}{50}$.

4. $329\times 5\frac{1}{2}$.

NOTE.—Multiply first by the fraction, then by the integer, and add the partial products.

5. $435\times 16\frac{2}{3}$.	8. $5000\times\frac{12}{30}$.
6. $768\times\frac{4}{192}$.	9. $2200\times\frac{7}{12}$.
7. $70245\times\frac{3}{7}$.	10. \$11520. $\times\frac{80}{365}$.

224. *To multiply a fraction by a fraction.*—Multiply $\frac{3}{4}$ by $\frac{7}{8}$.

$$\tfrac{3}{4}\times\tfrac{7}{8}=\tfrac{3}{4}\times 7\times\tfrac{1}{8}=\tfrac{3}{4}\times 7\div 8=\frac{3\times 7}{4\times 8}=\frac{21}{32},$$

(Art. 199, 2 and 5), hence the

RULE.

Find the product of the numerators, and write it over the product of the denominators.

NOTES.—1. Contract by cancellation.

2. Express compound fractions by using the sign of multiplication; express integers and mixed numbers as improper fractions.

EXAMPLES.

1. $\frac{3}{7}\times 18\times 2\frac{3}{4}\times\frac{21}{5}\times\frac{14}{36}\times(\frac{40}{55}\text{ of }\frac{17}{20})$.

$$\textit{Solution.}—\frac{3}{\cancel{7}}\times\frac{\cancel{18}}{1}\times\frac{\cancel{11}}{\cancel{4}}\times\frac{\overset{3}{\cancel{21}}}{\cancel{5}}\times\frac{\overset{7}{\cancel{14}}}{\underset{\cancel{2}}{\cancel{36}}}\times\frac{\overset{\overset{\cancel{4}}{\cancel{8}}}{\cancel{40}}}{\underset{5}{\cancel{55}}}\times\frac{17}{\underset{10}{\cancel{20}}}=\frac{1071}{50}=21\tfrac{21}{50}.$$

2. Multiply $\frac{3}{5}$ by $\frac{5}{6}$.
3. Multiply $\frac{7}{12}$ by $\frac{8}{21}$.
4. Multiply $\frac{5}{8}$ by $\frac{7}{9}$.
5. Multiply 56 by $\frac{7}{8}$.
6. Multiply $\frac{9}{16}$ by 24.
7. Multiply $8\frac{1}{2}$ by $7\frac{2}{3}$.
8. Multiply 111 by $9\frac{2}{3}$.
9. Multiply $\frac{3}{4}$ of $\frac{16}{27}$ by $\frac{8}{10}$ of $\frac{25}{32}$.
10. Multiply $\frac{5}{9}$ of $\frac{9}{10}$ by $\frac{9}{11}-\frac{3}{4}$.
11. Multiply $\frac{3}{4}$ of 8 by 9 times $\frac{2}{3}$.
12. Multiply $8\frac{3}{4}-6\frac{7}{8}$ by $9\frac{2}{3}+\frac{4}{5}$.
13. Multiply 256 by $12\frac{5}{16}$.
14. Multiply $12\frac{1}{2}$ by $16\frac{2}{3}$.
15. $74\frac{3}{7}\times 28\frac{3}{5}$.

NOTE.—First $74\frac{3}{7}\times\frac{3}{5}$, then $\frac{3}{7}\times 28$, then 74×28, and add the partial products.

$$\begin{array}{r}74\frac{3}{7}\\ \underline{3}\\ 5)\underline{223\frac{2}{7}}\\ 44\frac{22}{35}\end{array}\qquad\qquad\begin{array}{r}28\\ \underline{3}\\ 7)\underline{84}\\ 12\end{array}\qquad\qquad\begin{array}{r}74\frac{3}{7}\\ \underline{28\frac{3}{5}}\\ 44\frac{22}{35}\\ 12\\ 592\\ \underline{148}\\ 2128\frac{22}{35}\end{array}$$

16. $11703\frac{5}{12} \times 450\frac{2}{3}$.

$$\begin{array}{r} 11703\frac{5}{12} \\ \underline{450\frac{2}{3}} \\ 7802\frac{5}{18} \\ 187\frac{1}{2} \\ 58515\phantom{\frac{1}{2}} \\ \underline{46812\phantom{0\frac{1}{2}}} \\ 5274339\frac{7}{9} \end{array}$$

17. $4527\frac{3}{11} \times 841\frac{4}{5}$.

18. $1000\frac{4}{9} \times 200\frac{5}{8}$.

19. $999\frac{3}{8} \times 777\frac{2}{5}$.

20. $4080\frac{12}{17} \times 3417\frac{3}{4}$.

SUBTRACTION OF FRACTIONS.

225. Ex. 1. From $\frac{6}{7}$ take $\frac{2}{3}$.

$$\frac{6}{7}-\frac{2}{3}=\frac{18}{21}-\frac{14}{21}=\frac{4}{21}\ \textit{Ans.}$$

Ex. 2. From $2\frac{1}{2}$ take $\frac{3}{4}$ of 2.

$$2\frac{1}{2}=\frac{5}{2},\ \frac{3}{4}\text{ of }\frac{2}{1}=\frac{6}{4}=\frac{3}{2}$$
$$\frac{5}{2}-\frac{3}{2}=\frac{2}{2}=1\ \textit{Ans.}$$

RULE.

Reduce the fractions to a common denominator; then find the difference of their numerators, and under the result place the common denominator.

NOTE.—When the minuend is a whole or mixed number, the same principle may be applied as in simple Subtraction. (Art. 119, 3.)

Thus $3-\frac{5}{7}=2\frac{7}{7}-\frac{5}{7}=2\frac{2}{7}$; $13\frac{2}{3}-9\frac{3}{4}=\begin{cases}13\frac{2}{3}=12\frac{5}{3}=12\frac{20}{12}\\ 9\frac{3}{4}=9\frac{3}{4}=\underline{9\frac{9}{12}}\end{cases}$

$$3\frac{11}{12}$$

3. Subtract $\frac{2}{3}$ from $\frac{3}{4}$.

4. Subtract $\frac{2}{11}$ from $\frac{2}{7}$.

5. From $\frac{1}{3}$ of $\frac{3}{4}$ take $\frac{2}{5}$ of $\frac{2}{11}$.

6. From $\frac{3}{4}$ of $\frac{4}{9}$ take $\frac{1}{3}$.

7. From $\frac{1}{3}$ of $\frac{3}{4}$ take $\frac{3}{10}$ of $\frac{1}{4}$.

8. From $820\frac{2}{3}$ subtract $56\frac{3}{4}$.

9. From $250\frac{1}{5}$ subtract $225\frac{4}{7}$.

10. From $993\frac{3}{4}$ take $546\frac{7}{8}$.

11. From $\frac{3}{4}+\frac{7}{8}+\frac{4}{9}$ take $\frac{3}{4}$ of $\frac{8}{9}+\frac{9}{10}$ of $\frac{5}{6}$.

12. From $\frac{2}{3}$ of 12 take $\frac{3}{4}$ of 9.

13. From 1000 take $156\frac{2}{3}$.

14. From 9 take $\frac{1}{2}$ of $\frac{3}{4}$.

15. From $56\frac{7}{12}+89\frac{3}{4}$ take $5\frac{4}{5}+81\frac{7}{18}$.

16. From $\frac{1}{2}$ of 13 take $\frac{4}{5}$ of 8.
17. From $3\frac{1}{2}$ of 5 take $2\frac{1}{3}$ of 7.
18. From $\frac{5}{7}$ of 42 take $\frac{2}{7}$ of 48.
19. From $\frac{2}{3}$ of $19\frac{1}{2}$ take $\frac{2}{5}$ of $7\frac{2}{3}$.
20. From $875\frac{2}{3}$ take $599\frac{7}{8}$

DIVISION OF FRACTIONS.

226. 1. How many $\frac{4}{5}$'s in 4?

Reducing to same units for comparison, $4=\frac{20}{5}$, and

$$\frac{20}{5}-\frac{4}{5}-\frac{4}{5}-\frac{4}{5}-\frac{4}{5}-\frac{4}{5}=0,$$

hence there are five $\frac{4}{5}$'s in 4.

2. Divide 2 by $\frac{3}{7}$.

$$2=\frac{14}{7}, \text{ and } 14\div3=\frac{14}{3}=4\frac{2}{3}.$$

3. $\frac{3}{4}\div5$. $5=\frac{20}{4}$, and $3\div20=\frac{3}{20}$.
4. $\frac{2}{3}\div\frac{4}{5}$. $\frac{2}{3}=\frac{10}{15}$, and $\frac{4}{5}=\frac{12}{15}$; $10\div12=\frac{10}{12}=\frac{5}{6}$.

227. GENERAL RULE.

Reduce dividend and divisor to the same unit value, and compare the number of units in each.

EXAMPLES.

1. $\frac{5}{9}\div6$; $\frac{7}{8}\div10$; $\frac{4}{10}\div7$.
2. $8\div\frac{2}{3}$; $15\div\frac{6}{10}$; $20\div\frac{3}{5}$.
3. $\frac{4}{5}\div\frac{2}{5}$; $\frac{7}{12}\div\frac{3}{8}$; $\frac{9}{15}\div\frac{5}{6}$.
4. $\frac{7}{9}\div\frac{2}{3}$ of 9.
5. $18\div\frac{5}{6}$ of $\frac{1}{3}$.
6. $\frac{4}{7}$ of $9\div\frac{3}{7}$ of 6.

228. RECIPROCAL RULE.

Multiply the dividend by the reciprocal of the divisor. (Art. 185.)

NOTE.—The pupil may be required to give the correct analysis for the solution of each case by this rule, and also to apply cancellation.

EXAMPLES.

1. Work the six examples in Art. 227 by this rule.
2. $\frac{9}{10}\div3$; $\frac{27}{32}\div9$; $\frac{14}{25}\div7$; $6\frac{1}{2}\div9$.
3. $6084\frac{2}{3}\div5=1216\frac{14}{15}$. *Ans.* $1216\frac{14}{15}$.

Solution.— $5\overline{)6084\frac{2}{3}}$

1216 Rem. $4\frac{2}{3}$.

$$4\frac{2}{3}\div5=\frac{14}{3}\div5=\frac{14}{3}\times\frac{1}{5}=\frac{14}{15}.$$

4. $308\frac{2}{3}\div 12$; $32006\frac{1}{5}\div 9$.
5. $1000\frac{5}{12}\div 5$; $1728\frac{2}{3}\div 12$.
6. $16\div\frac{3}{9}$; $256\div\frac{16}{21}$; $225\div 12\frac{1}{2}$.
7. $30864\div\frac{2}{3}$; $50\div 6\frac{1}{4}$; $48\div 3\frac{1}{3}$.
8. $284\div\frac{2}{3}$ of $\frac{3}{8}$.
9. $\frac{3}{5}\div\frac{1}{2}$; $\frac{2}{3}\div\frac{4}{9}$; $7\frac{1}{2}\div 8\frac{1}{3}$.
10. $4\frac{1}{2}\div 6\frac{1}{4}$; $18\frac{3}{4}\div 15\frac{1}{5}$.
11. $\frac{2}{3}$ of $\frac{3}{4}\div\frac{1}{2}$ of $5\frac{1}{2}$.
12. $\frac{1}{4}$ of $4\frac{1}{2}\div\frac{1}{3}$ of $5\frac{1}{2}$.
13. $\frac{3}{5}$ of $8\div\frac{2}{3}$ of 7.
14. $12\frac{1}{2}$ of $\frac{1}{2}\div 8\frac{1}{3}$ of $\frac{1}{3}$.
15. $(\frac{9}{20}+4\frac{4}{5})\div(4\frac{7}{8}-3\frac{11}{12})$.
16. $(\frac{9}{16}$ of $\frac{16}{27}-\frac{1}{3}$ of $\frac{7}{14})\div(5\frac{1}{2}-4\frac{7}{8})$.
17. $(125\frac{3}{4}-62\frac{4}{5})\div 37\frac{1}{2}$.
18. $(4\frac{1}{2}+6\frac{2}{3})\div\frac{5}{11}$.
19. $(9\frac{1}{10}+4\frac{1}{2}\times\frac{5}{8})\div 6\frac{3}{8}$.
20. $140\times\frac{5}{8}\div 2\frac{2}{3}+56\frac{1}{2}\div 16\div\frac{5}{22}$.

229. REDUCTION OF COMPLEX FRACTIONS.

1. Since every fraction is an expression of division, it follows that complex fractions may be reduced by dividing the numerator of the complex fraction by the denominator.

EXAMPLES.

1. $\dfrac{\frac{4}{5}}{\frac{8}{9}}=\frac{4}{5}\div\frac{8}{9}=\frac{\cancel{4}}{5}\times\frac{9}{\underset{2}{\cancel{8}}}=\frac{9}{10}$.

2. Reduce $\dfrac{\frac{5}{8}}{\frac{7}{12}}$ to a simple fraction.

3. Reduce $\dfrac{\frac{2}{3}}{\frac{3}{4}}$. 4. Reduce $\dfrac{3\frac{1}{3}}{33\frac{1}{3}}$. 5. Reduce $\dfrac{6\frac{2}{3}}{11\frac{1}{9}}$.

230. 1. To find the Least Common Multiple of several fractions—

Reduce them to their lowest terms; then write the L. C. M. of their numerators over the G. C. D. of their denominators.

2. To find the Greatest Common Divisor of several fractions—

Reduce them to their lowest terms, and write the G. C. D. of the numerators over the L. C. M. of the denominators.

231. REVIEW OF FRACTIONS.—MENTAL PROBLEMS.

1. A man having $\$\frac{7}{8}$ gave $\$\frac{3}{5}$ to his son: how much had he left?

2. If $\frac{6}{11}$ be added to a certain fraction, the amount will be $\frac{25}{33}$; what is the fraction?

3. A woman pays $\frac{4}{7}$ of her wages for board, and $\frac{1}{6}$ for clothing; what portion remains?

4. A man traveled $\frac{1}{4}$ of his journey the first day, $\frac{1}{5}$ the second, $\frac{1}{6}$ the third, and the remainder the fourth day: what portion did he travel the fourth day?

5. A man bought a horse, paying $\frac{2}{5}$ of the price down, $\frac{1}{4}$ at the end of one month, $\frac{1}{6}$ at the end of two months, and the rest at the end of three months: what part did he pay last?

6. A man is 56 years of age, and $\frac{5}{14}$ of his age equals the age of his son: how old is the son?

7. A man bought a wagon for \$86., and sold it for $\frac{5}{8}$ of what it cost: how much did he lose?

8. What will $\frac{4}{7}$ of a yd. velvet cost at $\$8\frac{3}{4}$ per yd.?

9. Katie is $16\frac{2}{3}$ yrs. old, and Lizzie's age is $\frac{3}{5}$ as much: how old is Lizzie?

10. A man owning $\frac{5}{8}$ of a steamboat, sells $\frac{2}{3}$ of his share: what part does he still own?

11. Henry bought $\frac{7}{8}$ of a pound of candy and gave his brother $\frac{3}{8}$ of a pound, and his cousin $\frac{2}{3}$ of what remained: how much had he left?

12. A woman is 42 yrs. of age, which is $\frac{6}{7}$ the age of her husband: how old is he?

13. Seven-eighths of a certain number, less five-sixths of it is 5: what is the number?

14. A grocer sold 64 bbls. of apples, which was $\frac{8}{15}$ of what he bought: how many barrels remained?

15. When James is $\frac{3}{4}$ older than he is now he will be 21 yrs. old: how old is he now?

16. A man sold $\frac{4}{7}$ of his peaches for \$728.: what was the value of all?

17. A man sold $\frac{5}{9}$ of his farm, and had 32 acres left: how much had he at first?

18. A man sold a watch for \$144., which was $\frac{1}{5}$ more than it cost him: how much did it cost him?

19. A boy paid 60 cts. for a Reader, which was $\frac{2}{5}$ more than he paid for a History: how much did he pay for the History?

20. $\frac{5}{8}$ of 72 is $\frac{3}{7}$ of what number?

21. A man has \$48., and $\frac{5}{8}$ of this is $\frac{7}{8}$ of what his wife has: how much has she?

22. John is $\frac{3}{5}$ as old as Mary, and Mary, who is 15 years old, is $\frac{3}{5}$ as old as Charles: how old are John and Charles?

23. A man gave \$180. for a horse and harness, and the harness cost $\frac{4}{5}$ as much as the horse: what was the cost of each?

24. If to A's money there be added $\frac{2}{5}$ and $\frac{3}{4}$ of the same, the sum will be \$86.: how much has he?

25. A farmer has loaned money to four persons: to the first $\frac{3}{8}$ of the whole amount loaned, to the second $\frac{1}{5}$, to the third $\frac{3}{7}$, and to the fourth \$30: how much has he loaned, and how much to each man?

26. A coat cost \$56., and $\frac{3}{8}$ of the cost of the coat was $\frac{7}{8}$ the cost of a vest: what was the cost of the vest?

27. If $6\frac{1}{2}$ lb. sugar cost 91 cts., what will $14\frac{2}{7}$ lb. cost?

28. If 8 men can do a piece of work in $11\frac{1}{4}$ days, how long would it take 6 men?

29. If A can build a wall in 8 days, and B in 10 days, how long will it take both working together?

30. 1. How is the value of a proper fraction changed by adding the same number to both terms? By subtracting the same number?

2. How is the value of an improper fraction changed by adding the same number to both terms? By subtracting the same number?

Illustrate each case.

WRITTEN PROBLEMS.

NOTE.—Write *only* such operations as cannot be performed mentally.

31. What is the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{12}$?

32. What is the difference between $\frac{7}{8}$ and $\frac{2}{3}$?

33. Multiply $\frac{2}{3}$ by $3\frac{1}{2}$.

34. Divide $\frac{2}{3}$ by $3\frac{1}{2}$.

35. What are the sum, difference, product, and quotient of $3\frac{1}{2}$ and $2\frac{1}{4}$?

36. What will be the cost of $15\frac{1}{3}$ pounds of butter at $16\frac{2}{3}$ cents a pound?

37. At \$$4\frac{1}{2}$ per tub, how many tubs may be bought for \$$11\frac{4}{7}$.

38. At $28\frac{4}{7}$ cents per bushel, how many bushels of oats may be bought for $16\frac{2}{3}$ cents?

39. How many pounds in four bags, the first containing $360\frac{7}{8}$, the second $580\frac{4}{5}$, the third $296\frac{3}{4}$, and the fourth $375\frac{9}{10}$?

40. In 5 hogsheads of sugar containing, respectively, $945\frac{1}{2}$, $1054\frac{9}{10}$, $963\frac{4}{7}$, $901\frac{23}{30}$, and $899\frac{5}{9}$, how many pounds?

41. A man has 4 lots; the first containing $320\frac{14}{15}$ acres, the second $225\frac{5}{8}$, the third $160\frac{3}{4}$, and the fourth $278\frac{5}{8}$: how many acres in all?

42. A man owes the following sums: to A \$$32.56\frac{1}{4}$, to B \$$44.95\frac{9}{10}$, to C \$$32.72\frac{7}{8}$, to D \$$53.31\frac{3}{20}$, to E \$$192.05\frac{7}{15}$. How much does he owe in all?

43. A farm is divided into 5 fields, containing, respectively, as follows: $20\frac{1}{3}$, $56\frac{9}{11}$, $36\frac{4}{5}$, $9\frac{4}{9}$, and $102\frac{13}{24}$ acres. How many in all?

44. A man purchased $\frac{7}{8}$ of a yard of velvet at the rate of \$$3.62\frac{1}{2}$ per yard; what did it cost him?

45. A man owned $\frac{3}{4}$ of a boat, and sold $\frac{1}{2}$ of $\frac{3}{8}$ of his share for \$2400. At that rate what was the whole of it worth?

46. James has $\frac{3}{4}$ of an orange. He gives Horace $\frac{1}{4}$ of this amount, and then divides the remainder equally between three boys. What part does each of the three boys receive?

47. If $\frac{3}{8}$ of a barrel of flour costs \$5, how much will 2 bags of flour cost, one containing $\frac{2}{3}$ of a barrel, and the other $\frac{5}{8}$ of a barrel?

48. Bought $\frac{4}{5}$ of $\frac{3}{8}$ of $5\frac{1}{2}$ yards of broadcloth, at the rate of \$3.50 per yard. Required the cost of it.

49. What will be the cost of $7\frac{1}{2}$ yards of muslin at $12\frac{1}{2}$ cents per yard, and $12\frac{1}{2}$ yards of gingham at $18\frac{3}{4}$ cents per yard?

50. I purchased 7 loads of coal, each containing $15\frac{3}{4}$ bushels, at $12\frac{1}{2}$ cents per bushel. Required the cost.

51. A owns $\frac{5}{8}$ of a vessel, and sells $\frac{4}{7}$ of his share to B for \$45000. What part of the vessel has he left, and what is it worth at that rate?

52. A owns $\frac{5}{8}$ of a ship. He sells $\frac{3}{7}$ of his share to B for a certain sum, and $\frac{1}{4}$ of what he then owns to C for \$5,000. What was the value of the whole ship at C's rate of purchase?

53. A owns $\frac{5}{12}$ of an acre of land, and B $\frac{3}{8}$ of an acre. How much does A own more than B? How many times more? How much do they both own?

54. I have \$1000 and wish to lay out \$346$\frac{5}{9}$ of it in sugar at 8$\frac{1}{3}$ cents per pound, and the remainder in coffee at 11$\frac{3}{4}$ cents per pound. How many pounds of coffee can I buy?

55. A merchant directed his agent to lay out $\frac{5}{8}$ of \$2354 in wheat at 87$\frac{1}{2}$ cents per bushel; $\frac{3}{10}$ of it in rye at 56$\frac{1}{4}$ cents per bushel; and the remainder in oats at 31$\frac{1}{4}$ cents per bushel. How many bushels of each did he purchase?

56. What will 8$\frac{1}{3}$ pounds of sugar cost at 18$\frac{3}{4}$ cents per pound?

57. A has 6$\frac{2}{3}$ acres in one lot and 7$\frac{3}{4}$ in another; B has 5$\frac{4}{5}$ times as much as A. How many acres of land has he?

58. What will $\frac{2}{5}$ of $\frac{5}{6}$ yards of cloth cost at $\frac{3}{7}$ of $\frac{5}{9}$ dollars per yard?

59. A merchant owns $\frac{7}{8}$ of a mercantile establishment worth \$64,000. He sells $\frac{3}{5}$ of his share to B, and $\frac{1}{2}$ the remainder to C. How much does he receive from B and C respectively, and what part has he remaining?

60. A merchant has 33$\frac{7}{10}$ yards of cloth, from which he wishes to cut an equal number of coats, pants, and vests. What number of each can he cut if they contain 3$\frac{3}{4}$, 2$\frac{7}{8}$, and 1$\frac{1}{5}$ yards respectively?

61. A merchant owned $\frac{9}{16}$ of a stock of goods; $\frac{3}{4}$ of the whole stock was destroyed by fire, and $\frac{7}{10}$ of the remainder damaged by water. What part of the whole stock remained uninjured? How much did the merchant lose, provided the uninjured goods were sold at cost for \$5400, and the damaged at half cost?

62. What number is that from which if you subtract $\frac{1}{9}$ of $\frac{5}{7}$ of 1, and to the remainder add $\frac{2}{5}$ of $\frac{7}{8}$ of 1, the sum will be 10?

63. Suppose a man's family expenses are \$1274$\frac{7}{8}$, and that this is $\frac{7}{9}$ of his profits in business, how much does he save?

64. If a man buys $\frac{8}{11}$ of a farm valued at \$96420., and divides it equally among his sons, giving to each $\frac{3}{15}$ of it, how many sons has he, and what is the value of each one's share?

65. A man invests $\frac{1}{3}$ of his money in land, $\frac{1}{4}$ in cotton, $\frac{2}{13}$ in grain, and the remainder, which is \$2534., in mining stock: what is the amount of each investment, and of all of them?

SECTION XI.

DECIMAL FRACTIONS.

232. 1. If a unit be divided into ten equal parts, what is one of the fractional units called? Three of them?

2. What fractional unit is the same part of one-tenth that one-tenth is of one?

3. What is $\frac{1}{10}$ of $\frac{1}{100}$?

4. What is the ratio of $\frac{1}{10}$ to $\frac{1}{100}$? Of $\frac{1}{100}$ to $\frac{1}{1000}$?

5. What is the ratio of 4 to $\frac{4}{10}$? $\frac{1}{10}$ to $\frac{1}{100}$?

233. A ***Decimal Fraction*** is a fraction whose denominator is some power of ten; as, 10, 100, 1000, or a unit of any higher order of the decimal scale of notation; as, $\frac{7}{10}$, $\frac{4}{1000}$, $\frac{437}{100000}$, etc.

234. Decimal Fractions are sometimes expressed as common fractions, of which they are a species, but generally the denominator is omitted, and the denomination of the fractional units is indicated by writing them in the lower or *decimal* orders of the decimal scale of Notation (Art. 66), and such expressions are called *Decimals.*

NOTE.—The term *decimals* is generally applied only to decimal fractions, but integers in the decimal system are also really decimals.

235. The *Decimal Point* is a period used to separate integers from decimals. When no integers are expressed, the decimal point is placed at the left of tenths' order.

236. Decimals are expressed in units of inferior orders, which are reciprocals of the units in the integral orders, equally distant from the place of the fundamental unit of the scale. The *latter* should *always* be regarded as the *first order.* Thus the 2d superior order is *tens,* the second inferior order is *tenths,* and $\frac{1}{10}$ is the reciprocal of 10. So also .01 is the reciprocal of 100, etc.

237. The decimal orders are also called *decimal places, each* order being counted as one place. Thus in .0223 there are *four decimal places,* although 3 is of the *fifth decimal order.*

238. 1. Since the value of decimal orders decreases towards

the left in a tenfold ratio, every cipher placed between decimal figures and the decimal point, or the removal of the decimal point one place towards the left, diminishes their value *tenfold*, or divides the decimal by ten.

2. The removal of the decimal point one place towards the right increases the value of the number *tenfold*, or multiplies by ten.

239. NOTATION OF DECIMALS.

In writing decimals it is only necessary to have them so arranged with reference to the *first* order that the extreme right-hand significant figure shall be of the order whose unit is the reciprocal of the denominator. Thus $\frac{24}{1000}$ becomes .024; that is, the 4 must occupy the order of thousandths, a unit of which, $\frac{1}{1000}$, is the reciprocal of the denominator, 1000.

Ex. Express $\frac{10505}{1000000}$ as a decimal.

Write the numerator as a whole number, thus, 10505; then place the decimal point so that the right-hand figure, 5, may be in the order of millionths, filling the vacant order with a cipher; thus, .010505.

240. RULE.

Write the decimal as a whole number, and place the decimal point so that the right-hand figure shall be of the lowest decimal order to be expressed.

EXAMPLES.

Write as decimals—

1. $\frac{25}{1000}$, $\frac{25}{10000000}$, $\frac{25}{100}$, $\frac{205}{10000}$, $\frac{205}{100000000}$.
2. $\frac{20005}{10000000}$, $\frac{20004}{100000}$, $650\frac{37}{1000}$, $1\frac{1}{10000000}$.
3. $5000\frac{5}{10000}$, $\frac{2506}{100}$, $\frac{9015035}{10000000}$.
4. $\frac{8040}{1000000000}$, $\frac{1000001}{10000000}$, $40000\frac{15}{100000}$.
5. $900\frac{2}{10000}$, $\frac{16\frac{2}{3}}{100}$, $21000\frac{80}{1000}$.
6. $\frac{7}{100000}$, $\frac{37}{100000}$, $\frac{208}{1000000}$, $\frac{3056}{100000}$, $\frac{38045}{100000}$.
7. $\frac{29}{1000000}$, $\frac{341}{10000000}$, $\frac{5096}{10000000}$, $\frac{60409}{1000000}$, $\frac{421005}{1000000}$.
8. 7 tenths, 24 hundredths, 65 millionths, 40056 millionths.
9. 42 ten-thousandths, 4008 ten-thousandths, 65 hundred-thousandths.

10. 85 thousandths, 54008 hundred-thousandths.
11. Four hundred and seven thousandths.
12. Ninety-five ten-thousandths, 5064 millionths.
13. Six hundred and forty-four ten-thousandths.
14. Seven thousand, and eighty-two ten-thousandths.
15. Fifty-seven hundred-thousandths.
16. Fifty-seven-hundred thousandths.
17. Seven hundred and eight hundred-thousandths.
18. Nine thousand, and forty-eight hundred-thousandths.
19. Seven thousand six hundred and forty-three millionths.
20. Forty thousand and sixty-three millionths.

241. NUMERATION OF DECIMALS.

In reading a decimal the entire decimal is regarded as reduced to units of the lowest order expressed, and the name of this order is applied to the entire number of decimal units. Thus .12 is read *twelve hundredths.*

NOTE.—If decimals were read on the same principle as integers, that is, *towards the first order*, .12 might be read *two* hundredths, *one* tenth; but this would be inconvenient, and is unnecessary.

242. In reading a decimal it is necessary—1st. To determine how many units are expressed. 2d. To ascertain the denomination of the lowest order. 3d. To read the expressed numerator and name the indicated denomination. Thus to read .000753, we observe that 753 units are expressed; that these are reduced to the 7th decimal order or millionths; and accordingly read *seven hundred fifty-three millionths.*

NOTE.—The denomination of any figure in the decimal system may easily be determined by considering a unit of the first order to have as many ciphers annexed as there are decimal or integral orders above or below the first order. Thus $\left.\begin{matrix}.007053\\1000000\end{matrix}\right\}$ shows the denomination of the three to be millionths, and $\left.\begin{matrix}7053000\\1000000\end{matrix}\right\}$ shows the denomination of the 7 to be millions.

243. In reading a mixed number expressed decimally, care must be taken to distinguish the two parts. In many cases there is no caution needed—as, 7.3 is read seven, and three-tenths, but 300.042 read in a similar manner would not be distinguished from .342. Confusion can be avoided by naming either the integral

part as *units*, or the other part as *decimal;* thus 300.042 may be read 300 units, 42 thousandths; or 300, *decimal* 42 thousandths; or, by a more common method, 300, *decimal* naught, four, two.

244. A decimal with a common fraction annexed, as $.13\frac{2}{7}$, is generally called a *Mixed Decimal*, but is more properly a *Complex Decimal.* Thus $.13\frac{2}{7}=\frac{13\frac{2}{7}}{100}$.

245. Decimals may be read according to the following

RULE.

Read the figures as in whole numbers, and add the name of the lowest decimal order expressed.

NOTE.—See Caution in Art. 243.

EXAMPLES.

1. Read *at sight* twenty decimals of higher rank than the 7th decimal order; as, .04, .026, .0201, .00012, etc.
2. Read .01012305; .000027; .500006.
3. 207.0084; 7080.00607008; .006.
4. .002005505; 600.06; 1000.001.
5. 25000000.000250; 206.000000206.
6. .030056; 7051.013005; 7400.0056.
7. $.753\frac{1}{3}$; $16.6\frac{2}{3}$; 5002.875.
8. 4003020050006.04003020001; 400.00011.
9. .16041; 1900.0909009; 40004.50002.
10. $.162359847\frac{1}{7}$; 40056.019008; .001409.

DECIMAL REDUCTION, ANALYSIS, AND TRANSFORMATION.

(Articles 73–79.)

246. 1. A whole number may be changed to a mixed decimal, or a decimal to an equivalent decimal of a lower order, *by annexing ciphers.* Thus: .025=.025000, and 325=325.000. This is, in effect, multiplying both terms of a fraction by the same number.

2. A mixed number may be reduced to an improper decimal fraction by removing the decimal point and writing the denominator, thus: $205.025=\frac{205025}{1000}$. The following examples will make the student familiar with these changes:

1. Reduce .49 to tenths. *Ans.* $4\frac{9}{10}$.
2. Reduce .07 to thousandths. *Ans.* .070.
3. Reduce .034 to hundredths.
4. Analyze .421; .09188; 170.40.
5. Transform 13 of 2d decimal order.
6. Transform .43.
7. Reduce 5 to hundredths.
8. Reduce 1.7 to thousandths.
9. Reduce $.2433\frac{1}{3}$ to hundredths.
10. Reduce, analyze, and transform any of the examples in Art. 240.
11. Reduce .205 to millionths.
12. Reduce .0225 to ten-millionths.
13. Reduce $.1\frac{3}{4}$ to hundred-thousandths.
14. Reduce .0205 to billionths.
15. Reduce .02301 to billionths.
16. Reduce .5 to millionths.
17. Reduce 25. to thousandths.
18. Reduce 404. to hundredths.
19. Reduce 4. to millionths.
20. Reduce 40. to ten-thousandths.
21. Reduce 62.5 to thousandths.
22. Reduce 6.02 to millionths.
23. Reduce 4.506 to billionths.
24. How many *tenths* in 40 *units?*
25. How many millionths in 5 thousandths?
26. How many thousandths in 62.304?
27. How many millionths in 36.0304?
28. How many hundredths in 400?
29. How many tenths in 6 tens?
30. How many millionths in one million?

247. Decimals may be reduced to common fractions simply by omitting the decimal point and writing the denominator under the numerator. (Art. 242, Note.) Thus $.2=\frac{2}{10}$, $.0073=\frac{73}{10000}$, $.014\frac{2}{3}=\frac{14\frac{2}{3}}{1000}$, $7.05=\frac{705}{100}=7\frac{5}{100}=7\frac{1}{20}$.

NOTE.—Decimal common fractions may sometimes be reduced to lower terms, or to mixed numbers.

EXAMPLES.

Reduce to common fractions of the lowest terms, or to mixed numbers—

1. .25, .250, .20506, .75.
2. .125, .0075, .0125, 62.25.
3. 6.225, 80.025, 8.0375, 15.02.
4. 120.0125, .1625, .5625, .01254.
5. .3525, 3.525, 37.75, 62.025.
6. $.18\frac{2}{3}$, $.004\frac{1}{6}$, $2.120\frac{2}{7}$, $.10\frac{1}{3}$.

NOTE.—Reduce a mixed decimal first to a complex fraction, and then reduce this to a simple fraction. Thus $.20\frac{1}{11}=\frac{20\frac{1}{11}}{100}=\frac{221}{1100}$.

248. Common fractions may be reduced to decimals by *reducing the numerator to decimals and dividing the result by the denominator.*

Thus $\frac{4}{5}$ indicates one-fifth of 4, or of $4.0=.8$.

$\frac{3}{80}=\frac{1}{80}$ of 3, 3.0, 3.00, 3.000, or 3.0000, and the last divided by 80 gives 375 ten-thousandths$=.0375$, the last figure of the quotient being of the lowest order to which the numerator is reduced.

NOTES.—1. As it is not always evident to what order the numerator must be reduced, it is generally more convenient to reduce as the division proceeds.

Thus $\frac{18}{84}$.

```
84)18.0(.214285 + or .214285 5/7
   16 8
   ----
    120
     84
    ---
     360
     336
     ---
      240
      168
      ---
       720
       672
       ---
        480
        420
        ---
         60 etc.
```

2. When the quotient is not completed the sign + may be annexed, or the remainder annexed as a common fraction reduced to its lowest terms.

249. RULE.

Annex ciphers to the numerator after the decimal point; divide by the denominator, and point off as many decimal places as there are in the number divided.

NOTE.—Prefix ciphers to the quotient if necessary.

EXAMPLES.

Reduce to decimals—

1. $\frac{5}{8}$, $\frac{1}{6}$, $\frac{7}{25}$, $\frac{3}{125}$, $\frac{60}{400}$.
2. $\frac{3}{2500}$, \$$12\frac{3}{4}$, $25\frac{3}{40}$, $300\frac{7}{400}$, $\frac{256}{8}$.
3. $6.37\frac{3}{4}$, $.07\frac{1}{5}$, $1.4\frac{3}{50}$, $21.0\frac{4}{25}$. *Ans.* 6.3775, etc.
4. $2\frac{9}{16}$, $\frac{3}{75}$, $\frac{250}{42}$, $\frac{64}{125}$, $\frac{80}{1200}$.
5. \$$\frac{4}{25}$, $\frac{2}{50}$, $\frac{6}{200}$, $\frac{8}{125}$, $\frac{125}{875}$.
6. $\frac{13}{40}$, $\frac{7}{400}$, $\frac{23}{150}$, $\frac{4}{1250}$, $\frac{1}{750}$.
7. $\frac{21}{480}$, \$$12\frac{3}{16}$, $25\frac{4}{125}$, $37\frac{13}{50}$.
8. $\frac{5}{800}$, $\frac{41}{300}$, $56\frac{7}{80}$, $19\frac{5}{400}$.
9. $\frac{3}{4}$ of $\frac{5}{14}$, $\frac{4}{7}$ of $\frac{28}{40}$, $\frac{5}{16}$ of $\frac{18}{20}$.
10. \$$47\frac{11}{12}$, $5\frac{50}{48}$, $10\frac{340}{85}$, $17\frac{17}{500}$.

CIRCULATING DECIMALS.

250. 1. When any decimal figure or set of figures is regularly repeated, the number is called a ***Circulating Decimal;*** as, .333, etc., .147147147, etc.

2. The figure or figures repeated are called a ***Repetend.*** Thus in .666, etc., the Repetend is 6; in .135135, etc., the Repetend is 135.

3. Instead of writing the repetend several times, it is generally written only once, and a dot is placed over the single figure, or over the first and last of the set of figures repeated. Thus $.\dot{3}=$ .333, etc. $.\dot{1}5\dot{7}=.157157$, etc.

4. A *Pure Repetend* is a number consisting of the repetend only; as, $.\dot{7}$; $\dot{1}.4\dot{3}$.

5. A *Mixed Repetend* is a number consisting of a repetend and a part before it, called the finite part; as, $.12\dot{6}$; $4.\dot{1}2\dot{7}$.

6. The denominator of a pure repetend consists of as many 9's as there are figures in the repetend. $.\dot{3}=\frac{3}{9}$; $.\dot{2}1\dot{3}=\frac{213}{999}$.

NOTE.—If ciphers precede the repetend, the denominator is the usual number of 9's, with as many ciphers annexed as precede the repetend. Thus $.0\dot{1}\dot{7}=\frac{17}{990}=\frac{17}{99}$ of $\frac{1}{10}$.

7. A mixed repetend is equivalent to the sum of two fractions, and may be reduced as such. Thus $.24\dot{6}=.24+.00\dot{6}=\frac{24}{100}+\frac{6}{900}$.

8. Any common fraction which, in its lowest terms, contains other prime factors than 2 and 5 in the denominator, will produce an indeterminate or circulating decimal.

9. Computations in circulating decimals may be made by extending them to several places, and disregarding small errors, or by reducing them to common fractions.

10. EXAMPLES.

1. Reduce $\frac{5}{12}$, $\frac{7}{15}$, $\frac{8}{13}$, $\frac{9}{14}$.

2. $\frac{42}{19}$, $\frac{50}{90}$, $\frac{110}{99}$, $\frac{440}{999}$, $\frac{10}{9}$.

3. $.\dot{1}\dot{7}$, $.\dot{6}$, $.0\dot{4}$, $\dot{3}$.

4. $.12\dot{7}$, $8.4\dot{2}$, $12.\dot{7}$, $4.\dot{1}\dot{1}$.

DECIMAL CURRENCY.

251. As the various denominations of the money of the United States, called *Federal Money*, have decimal relations, numbers referring to it may be treated as any abstract numbers in the decimal system. (Arts. 411, 412.)

ADDITION OF DECIMALS.

EXAMPLES.

252. 1. Add 6.025, 65.37, 100.0035, and .875.

```
  6.025
 65.37
100.0035
   .875
--------
172.2735 Ans.
```

Explanation.—Since decimals are written upon the same scale as whole numbers, they are added in the same manner, and the tests for the operation are the same. (Arts. 85, 87.)

NOTE.—The decimal points of the several decimals added and of the answer stand in the same column; or, if the numbers be not in columns, the number of decimal places will equal the most places in any of the numbers.

2. Add .37$\frac{3}{4}$, .0256$\frac{1}{2}$, .00015, .5$\frac{1}{3}$, .27$\frac{1}{3}$, and .026.

$$
\begin{array}{lcl}
.37\frac{3}{4} & = & .3775 \\
.0256\frac{1}{2} & = & .02565 \\
.00015 & = & .00015 \\
.5\frac{1}{3} & = & .53333\frac{1}{3} \\
.27\frac{1}{3} & = & .27333\frac{1}{3} \\
.026 & = & .026 \\
\hline
 & & 1.23596\frac{2}{3} \quad \textit{Ans.}
\end{array}
$$

3. What is the sum of 256 thousandths, 3005 millionths, 207 ten-thousandths, 45 hundred-thousandths, 7 hundredths, and 20037 millionths?

4. Add .00675, 4.5680, 3.00007, 2.05, 3.6800$\frac{3}{5}$, .9375, 8.75, 6.4375.

5. What is the sum of 307 millionths, 56$\frac{1}{4}$ ten-thousandths, 68$\frac{3}{4}$ hundredths, 5 hundred-thousandths, 256$\frac{1}{4}$ tenths, 18$\frac{3}{4}$ ten-millionths, and 25 hundredths?

6. Add 375 ten-thousandths, 375 thousandths, 375 hundredths, 375 tenths, and 375 units.

7. A man bought 4 barrels of molasses, containing respectively 30.37$\frac{1}{2}$, 31$\frac{1}{5}$, 33.0756, and 28.6$\frac{5}{8}$ gallons. How many gallons in all?

8. A man bought 5 lots, containing respectively 26.62$\frac{1}{2}$, 220.2007, 56.9$\frac{7}{8}$, 5.8$\frac{3}{16}$, and 150.68$\frac{3}{4}$ acres. How many acres in all?

9. Add 360.00025, 3.75, 567.893, 60,000.637, 200.050006, .0003625, 20.05.

10. Find the sum of 2$\frac{7}{8}$, .625, 6$\frac{9}{40}$, 3.6$\frac{7}{250}$, 26.3125, 5.6, 8$\frac{13}{80}$.

11. Add \$14.80, \$200$\frac{3}{5}$, 62$\frac{1}{2}$ cts., \$1$\frac{1}{8}$, \$40.72$\frac{3}{4}$, \$19.06$\frac{1}{4}$, and \$111.19.

12. What would be the cost of 1 yd. silk @ 95 cts., 1 yd. cassimere @ \$1.75, 1 yd. broadcloth @ \$4.50, 1 yd. doeskin @ \$1.12$\frac{1}{2}$, 1 cravat for \$1.25, 1 pair boots for \$5.20, 1 doz. hose for \$2$\frac{3}{5}$, 1 doz. collars for \$2$\frac{1}{4}$, and 1 doz. handkerchiefs for \$1.40?

13. What is the whole number of square miles occupied by the following cities: New York 22.65 sq. m., Boston 16.25 sq. m., Philadelphia (proper) 12. sq. m., St. Louis 19. sq. m., Chicago 34$\frac{3}{4}$ sq. m., Baltimore 16 sq. m., Cincinnati 48 sq. m., New Orleans 91 sq. m.?

14. 6729.420$\frac{3}{5}$ + 182374$\frac{8}{25}$ + 190.37$\frac{4}{33}$ + 4.002$\frac{1}{75}$ + .00$\frac{7}{16}$ + 82.034.

15. 74$\frac{8}{55}$ + .0001$\frac{5}{15}$ + 180.04$\frac{1}{3}$ + 703.00$\frac{1}{20}$ + MDCCCLXIII + $\frac{8}{65}$.

MULTIPLICATION OF DECIMALS.

253. 1. How much is $\frac{3}{10}\times 2$? $\frac{4}{10}\times 5$?

2. $\frac{7}{100}\times 4$? $\frac{11}{100}\times 6$?

3. $\frac{3}{10}\times\frac{2}{10}$? $\frac{7}{10}\times\frac{3}{10}$?

4. $\frac{12}{100}\times\frac{4}{10}$? $\frac{7}{100}\times\frac{2}{10}$?

5. $\frac{1}{10}\times\frac{1}{10}$? $\frac{1}{100}\times\frac{1}{10}$?

6. $\frac{1}{100}\times\frac{1}{100}$? $\frac{1}{1000}\times\frac{1}{100}$?

7. What is the denominator of the product when tenths are multiplied by units? By tenths? By hundredths? When hundredths are multiplied by hundredths?

8. What is the denominator of the product of any two fractions whose denominators are powers of 10?

254. Multiply .3 by .05.

$$.3\times .05=\frac{3}{10}\times\frac{5}{100}=\frac{15}{1000}=.015.$$

Observe—1st. That this result is in accordance with the General Law respecting the orders of figures in Products. (Art. 102.)

2d. That there *are* as many decimal *places* in the product as there are in both the factors.

255. RULE.

Multiply as in whole numbers, and in the product point off as many decimal places as are in both factors.

NOTE.—If the product does not contain as many figures as the number of decimal places required, prefix decimal ciphers.

EXAMPLES.

1. What is 7 tenths of 201 thousandths?

2. 37.5×4.5; \$$16.37\frac{1}{2}\times .03$.

3. \$$100.15\times .12$; $.0015\times .125$.

4. $70.423\frac{4}{7}\times .0251\frac{2}{3}$. (Art. 224, 15.)

5. $2.10\frac{1}{3}\times .07\frac{2}{7}$; $601.00\frac{1}{11}\times 400.0\frac{2}{9}$.

6. What is the population of Prussia, which comprises 139,499 sq. m., if the average is 169.14 persons to each sq. m.?

7. What is the combined length of 170 bars of iron, each $11.42\frac{1}{8}$ ft. long?

8. What is the value of $.00\frac{1}{15}$ lb. of gold at \$222.942 per lb.?

9. If one dollar in gold be worth 113$\frac{7}{8}$ cts. in currency, what is $2000 in gold worth?

10. In one gill there are 7$\frac{7}{32}$ cubic inches; how many cubic inches in 200 gills?

CONTRACTIONS.

256. To multiply by any power of 10, remove the decimal point as many places towards the right as there are ciphers in the multiplier. (Art. 238, 2.)

NOTE.—If there are not enough figures in the multiplicand, annex integral ciphers.

EXAMPLES.

1. $7.25 \times 100 = 725$.
2. 13.402×10? $\times 1000$? $\times 10000$?
3. $76.0\frac{1}{4} \times 1000$? $.87\frac{1}{2} \times 100$?
4. $\$.12\frac{1}{2} \times 1000$? $\$1.375 \times 100$?
5. $.16\frac{1}{4} \times 2000$?

$$.16\tfrac{1}{4} \times 2000 = .1625 \times 1000 \times 2 = 162.5 \times 2 = 325.$$

6. What is 4000 bu. corn worth @ 51$\frac{1}{4}$ cts.?
7. What is 5000 bu. wheat worth @ 1.18\frac{1}{4}$?
8. What is the value of 5000 bu. corn @ 51$\frac{7}{8}$ cts., and 2000 bu. @ 52$\frac{1}{4}$ cts.?
9. What is the value of 3000 bu. wheat @ 1.15\frac{1}{4}$, and 3000 bu. @ 1.15\frac{3}{8}$?
10. $17.5\frac{5}{32} \times 40000$?

257. 1. To retain only a certain number of decimal places.

Ex. 3269.0434 × .753025, retaining only five decimal places, or the 6th decimal order.

```
3269.0434
  .753025
---------
2288.33038
 163.45217
   9.80713
     6538
     1634
----------
2461.67140
```

Observing the general law of products, we may begin with the left-hand figure of the multiplier, and multiply that figure of such an order in the multiplicand by it as will produce units of the lowest order required. The 6th decimal order being required, and 7 being of the 2d decimal order, 4 of the 5th decimal order must be multiplied by it. The first partial product may be

written in the most convenient place. Then multiplying by 5 of the 3d dec. ord., 3 of the 4th dec. order is the first figure of the multiplicand to be used, and the first figure of this partial product is written directly under the first figure of the first partial product, both being of the same order.

Thus we may continue, using in each case the orders required, dropping one figure of the multiplicand as we multiply by each successive figure of the multiplier, observing to make due allowance for units of the required order arising from the product of neglected figures, adding the nearest number of tens.

The last multiplication begins with 6×5, but as $9 \times 5 = 45$, 4 is added to the product of 6 by 5.

2. This contraction may be more conveniently made by reversing the order of the figures of the multiplier, writing the 2d order under the 4th, the 3d under the 3d, the 4th under the 2d, etc., placing each figure of the multiplier under that figure of the multiplicand into which it is to be multiplied first.

1st. Thus

```
3269.0434
  52 0357.
----------
2288 33038
 163 45217
   9 80713
      6538
      1634
----------
2461.67140
```

2d. Thus 769.4030021 × .0305704 retaining six decimal places.

```
769.403002
 40 75030.
----------
23 082090
   384701
    53858
      308
----------
23.520957
```

EXAMPLES.

1. .706042 × 1.04703, retaining six decimal places.
2. 14.70248 × 90.40031, retaining five decimal places.
3. .70042876 × .54321067, retaining seven decimal places.

NOTE.—All the contractions used in integral multiplication may also be used with decimals.

SUBTRACTION OF DECIMALS.

258. Ex. 1. From 60.025 take 3.0825.

$$\begin{array}{r} 60.0250 \\ 3.0825 \\ \hline 56.9425 \text{ Ans.} \\ \hline 60.0250 \text{ Test.} \end{array}$$

Operation.—Same as in integral subtraction. (Art. 119.)

Tests.—Same as in integral subtraction.

The number of decimal places in the remainder will equal the most in either number.

EXAMPLES.

2. From $.37\frac{1}{2}$ take $.0187\frac{3}{4}$.

$$\begin{array}{r} .37\frac{1}{2} = .375000 \\ .0187\frac{3}{4} = .018775 \\ \hline .356225 \text{ Ans.} \end{array}$$

3. From 4.05 take 2.00075.
4. From 8.1 take $5.37\frac{5}{8}$.
5. From 362 ten-thousandths take 1056 millionths.
6. From 875 thousandths take 62 ten-millionths.
7. From $100.001\frac{1}{3}$ take 93.00075.
8. A man bought $8.75\frac{31}{100}$ yards of linen at one time, and 29.0056 at another. He afterwards sold $25\frac{13}{16}$ yards. How much has he left?
9. From 7 tenths take 7 ten-millionths.
10. From 10001 ten-thousandths take 10001 ten-millionths.
11. $2.0375-.125$? $84.03-75.83759$?
12. $.36-.13-.12-.13$; $.129-.063-.063$.
13. Gain on 2000 lbs. sugar bought @ $11\frac{3}{4}$ cts. and sold @ $12\frac{1}{2}$ cts.?
14. $(43.002-.7856\times100)-2480.125$?
15. $7652.045\frac{1}{3}-(1.-.62\frac{3}{4}\times10)$?

DIVISION OF DECIMALS.

259. 1. How many times can \$.2 be taken from a box containing \$.8?

2. If a man divide .63 of his farm equally among his 3 sons, what part of the farm will each receive?

3. $\frac{9}{10}\div\frac{3}{10}$? $\frac{6}{10}\div\frac{6}{100}$? $\frac{6}{100}\div\frac{6}{10}$?

4. What is obtained by dividing thousandths by thousandths? Tenths by hundredths? Hundredths by tenths?

260. 1. Divide 1.92 by .16.

$$\frac{192}{100} \div \frac{16}{100} = \frac{192}{16} = 12.$$

2. Divide .375 by 12.5.

$$\frac{375}{1000} \div \frac{125}{10} = \frac{375}{\underset{100}{\cancel{1000}}} \times \frac{\cancel{10}}{125} = \frac{375}{12500} = \frac{3}{100} = .03.$$

3. Divide 4.2 by .007.

$$\frac{42}{10} \div \frac{7}{1000} = \frac{42}{\cancel{10}} \times \frac{\overset{100}{\cancel{1000}}}{7} = \frac{4200}{7} = 600.$$

Observation.—1. These results are in accordance with the general law respecting the order of quotient figures. (Art. 133.)

2. When the dividend contains more decimal places than the divisor, the quotient contains a number of decimal places equal to the *excess*.

261. RULE.

Divide as in whole numbers, and point off in the quotient as many decimal places as the dividend has more than the divisor.

NOTES.—1. If at first the divisor contain more decimal places than the dividend, reduce the dividend to the same order as the divisor.

2. In order to begin or continue the division, decimal ciphers may be annexed to the dividend if necessary.

EXAMPLES.

1. 2.06112÷.304.

```
.304)2.06112(6.78
     1 824
       2371
       2128
        2432
        2432
```

Here the order of the first figure of the quotient is (4th − 4th) + 1 = 1st. Or as there are two more decimal places in the dividend than in the divisor, two places are pointed off in the quotient.

2. 13.2÷.033; 6.25÷2.5; 6.25÷.025.
3. .625÷25; 25.6÷.016; 256÷.16.
4. .256÷160; .0025÷50; .001÷100.
5. 4.2÷31¼; \$16.÷\$.25; 3÷1.25.
6. 5÷400; 9÷1500 6.4÷80.

7. $.1 \div .12\frac{1}{8}$; $6\frac{2}{5} \div .08$; $16\frac{2}{3} \div .033\frac{1}{3}$.

8. If 45 bales of goods weigh 6770.9 lbs., what is the weight of one bale?

9. If a horse run $5.12\frac{3}{8}$ miles in $17.6\frac{2}{3}$ minutes, how far does he run in one minute?

10. If a man can build $11.5\frac{3}{4}$ rd. of fence in one day, in how many days can he build $102.16\frac{1}{4}$ rd.?

CONTRACTIONS.

262. To divide by any power of 10, remove the decimal point as many places towards the left as there are ciphers in the divisor. (Art. 238, 1.)

NOTE.—If there are not enough figures in the dividend prefix decimal ciphers.

EXAMPLES.

1. Divide 704.2, 109.125, and 40. by 100.
2. Divide 1107.2, 400.235, 12345. by 1000.
3. Divide 5, 82, 11.5, 428.75 by 1000.
4. Divide 1875, .473, .35, .006 by 10000.
5. $712.8 \div 2000$.

$$712.8 \div 2000 = 712.8 \div 1000 \div 2 = .7128 \div 2 = .3564.$$

6. What is the price of one pencil, if 1000 cost $62.50?
7. If lumber is worth $12.50 per thousand, what is the price per foot?
8. If 10000 bu. wheat be sold for $11462.50, what is the price per bu.?
9. If 3000 lbs. pork be sold for $431.25, what is the price per lb.?
10. What is the gain on 10000 bu. wheat bought @ $1.19\frac{3}{4}$, and sold for $12012.50?

263. When only a limited quotient is required, the operation may be contracted by the following

RULE.

1. *Determine the order of the first figure of the quotient according to the general law for quotient figures.*

2. *Use as many figures of the divisor as there are divisions to be made, or if the divisor contain less, use all of them until the*

number of figures obtained is not less than the number of remaining divisions to be made.

3. *Then at each successive division omit one figure of the divisor, beginning at the lowest order.*

1. Ex. 87.64092503÷12.0432165, retaining six decimal places in the quotient.

```
12.01321|65)87.64092503(7.295377
            84 09251
            --------
             3 54841
             2 40264
             -------
             1 14577
             1 08118
             -------
                6459
                6006
                ----
                 453
                 360
                 ---
                  93
                  84
                  --
                   9
                   8
                   -
                   1
```

By comparing the highest orders of the dividend and divisor, we find that the first figure of the quotient will be of the *first* order, hence six decimal places are required, and seven divisions must be made. Using only the first seven figures of the divisor, we multiply by 7, the first figure of the quotient, making allowance for the units which would come from the product of it and 6, the last omitted figure of the divisor. The 1 of the divisor is next omitted, and the next quotient figure, 2, obtained, etc.

2. 1800.325003462÷42.032, retaining five decimal places in the quotient.

```
42.032)1800.325003462(42.83227
       1681 28
       -------
        119 045
         84 064
        -------
         34 981
         33 625
         ------
          1 356
          1 260
          -----
             96
             84
             --
             12
              9
              -
              3
              2
              -
```

Seven divisions must be made, so all the divisor is used for the first two divisions, then the work is continued as before.

3. .82043506 ÷ 2.03452, giving 6 decimal places.
4. .04352618 ÷ .036245, giving 5 decimal places.
5. .064032112 ÷ .00564028, giving 7 decimal places.
6. 702.45097568 : 4379.58, giving 5 decimal places.
7. 34322.4573 ÷ 8000.009, giving 4 decimal places.
8. 11049.62 ÷ 30.2456, giving 3 decimal places.

264. REVIEW PROBLEMS.

1. What is the value of 52 cattle, of an average weight of 1272 lb. each, @ $5.40 per cwt.?

2. The value of the lumber on the ship "Brilliant," carrying 77,411 ft., and on the ship "Dolphin," at $35. per thousand, was 5859.38\frac{1}{2}$; how many ft. of lumber was on the "Dolphin?"

3. The total export of petroleum from the United States in 1870 was 141,208,155 gallons, and the amount used in the United States in the same year was estimated at one-half this quantity. What would be the value of the entire amount produced, at $11.04 for 46 gallons?

4. The Secretary of the Denver and Rio Grande Railroad reports that the cost of the construction and equipment of the first hundred miles of the road would not exceed $13,500. per mile, as the road is narrow-gauge, or three feet wide, and that this is about $\frac{2}{3}$ of the ordinary cost of a broad-gauge of $4\frac{17}{24}$ ft.; what would be the cost of 750 miles of broad-gauge road at this rate?

5. On a broad-gauge railroad an ordinary freight-car weighs about 20,000 lb., and will carry the same amount of freight; on a narrow-gauge a freight-car weighs about 14,000 lb. and carries 20,000 lb. of freight. What is $\frac{2}{5}$ of $\frac{7}{10}$ of the difference in the weight of 8 loaded cars of the first kind and 16 of the second kind?

6. The Chesapeake and Ohio Railroad Company had completed 227 miles of the road by September, 1871, 95 miles more by October 1st, and the remaining 105 miles were to be completed by September 1, 1872; what would be the cost per mile of the construction and equipment of the entire road, if the entire cost be $\frac{2}{3}$ of $\frac{1}{40}$ of one thousand million dollars?

7. What is the average weight of 192 hogs, whose entire weight is 34176 lb.?

Suggestion.—Divide both dividend and divisor by 4, 12, and 4 in succession, writing only the results. (Art. 147.)

8. Which would be better, to get board at \$7 per week, and on account of "short supplies" pay $\frac{5}{14}$ as much for outside refreshments, or to get satisfactory board at $\frac{5}{56}$ less than $\frac{3}{8}$ more per week? How much would be saved per week?

9. What is the weight of one bar of iron if $7.2\frac{4}{7}$ bars weigh $126.4\frac{7}{18}$ lb.?

10. What is one dollar of currency worth in gold if \$5000. in gold be worth \$5625. in currency?

11. What is the total weight of three pieces of stone weighing exactly $47.1\dot{4}$ lb., $56.01\dot{3}$. lb., and $50.00\frac{2}{3}$ lb., and what is the exact average weight of one piece?

12. What is the shortest piece of wire from which could be cut an exact number of pieces either $\frac{13}{32}$, $\frac{1}{2}$, $\frac{15}{24}$, or $\frac{14}{16}$ of an inch in length?

13. What is the sum of 4 times the unit of the fraction $\frac{2}{11}$ of a mile, and $\frac{1}{7}$ of 14 times the fractional unit of the same fraction?

14. A put \$3000. into business, and B put into another business 2,400,000 times the reciprocal of this amount. Each made $\frac{2}{7}$ of the complement of the smaller amount of capital; what had each then?

15. On the 1st of Sept., 1871, the coin in the United States Treasury amounted to \$90,813,691.06, and the currency to \$7,968,345.66. The total debt, less the cash in the Treasury, was at the same time \$2,274,122,560.38; what was $\frac{2}{3}$ of $\frac{7}{8}$ of the debt?

16. What number increased by $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{3}{10}$ of itself would equal 9760?

17. How many bu. of corn @ 45¢ would pay for 869,500 bu. wheat @ 118¢?

18. If a man buy 3 car loads of wheat @ $117\frac{1}{2}$¢, 2 car loads @ $117\frac{1}{4}$¢, each containing 320 bu., and 10,000 bu. @ $116\frac{1}{2}$¢, and sell 5,000 bu. @ $117\frac{3}{8}$¢, 6,000 bu. @ 118¢, and the remainder @ 117¢, what is his entire gain or loss?

19. If a car load be 20,000 lb., and wheat be estimated at 60 lb. per bu., corn and rye at 56 lb., oats at 32 lb., and barley at 48 lb., how many bushels of grain were received in one day in Chicago in 293 cars of wheat, 284 of corn, 162 of oats, 65 of rye, and 95 of barley?

20. Find the L. C. M. of 7, 9, 15, and 36, and divide the result by $\frac{2}{7}$ of 105.

SECTION XII.

PERCENTAGE.

265. MENTAL PROBLEMS.

Analysis and solution required.

1. What is 2 hundredths of 400 miles?
2. 1 hundredth of \$750?
3. .25 of \$8,000?
4. $.00\frac{1}{2}$ of 90 years?
5. $.00\frac{3}{4}$ of \$24?
6. If a man lost \$50., and this was 25 hundredths of all his money, how much had he at first?
7. If $.12\frac{1}{2}$ of a ton of coal cost \$1.50, what will one ton cost? 3 tons?
8. If 30 hundredths of a farm contain 60 A., how much does the whole farm contain?
9. If the product of two factors be 5.76, and the multiplier be .18, what is the other factor?
10. Of what number is 65 fifteen hundredths?
11. How many hundredths of \$600. is \$8.?
12. 42 yrs. is how many hundredths of 700 yrs.?
13. If the product be 3.6 and the multiplicand 90, what is the multiplier?
14. How many hundredths of \$13. is 52 cts.?
15. How many hundredths of \$84. is 42 cts.?
16. If a man paid \$80. for a horse and gained 20 hundredths of this amount by selling it, how much did he receive for the horse?
17. If a man bought land at \$32. per acre, and sold it so as to gain 30 hundredths of the cost, what was it sold for?
18. If a man weighed 150 lbs. in November, and lost 6 hundredths of his weight during the winter, how much did he weigh in the spring?
19. If in a peach orchard of 600 trees $\frac{14}{100}$ of them die, how many live?
20. From the difference between \$80. and 120 hundredths of \$80, subtract .20 of \$80.

266. 1. ***Percentage*** is a term applied to all *operations* in which *100* is the basis of computation.

2. *Percentage* also means the *product* arising from multiplying by any number of *hundredths* or parts of hundredths.

267. 1. *Per cent.* is a contraction for the Latin phrase *per centum*, which signifies *by the hundred;* hence any number of *hundredths* of a number is so many *per cent.* of it. Thus,

5 *per cent.* of a number is 5 *hundredths* of it.
30 *per cent.* of a number is 30 *hundredths* of it.
$3\frac{1}{2}$ *per cent.* of a number is $3\frac{1}{2}$ *hundredths* of it.
$\frac{1}{2}$ *per cent.* of a number is $\frac{1}{2}$ *hundredth* of it.
125 *per cent.* of a number is 125 *hundredths* of it, and so on.

NOTE.—Instead of the words "per cent.," it is now customary to use the character %: thus, 12 per cent. is written 12%; $2\frac{1}{2}$ per cent., $2\frac{1}{2}$%.

2. The *rate per cent.* is the *number* of hundredths; as in 6 per cent. the *rate* is 6.

3. The per cent. may be expressed decimally by writing it as so many hundredths, and its equivalent common fraction in lowest terms may also be found. Thus,

1	per cent. is written	.01	$=\frac{1}{100}$
7	per cent. is written	.07	$=\frac{7}{100}$
$5\frac{1}{2}$	per cent. is written	$.05\frac{1}{2}$; or .055	$=\frac{11}{200}$
15	per cent. is written	.15	$=\frac{3}{20}$
100	per cent. is written	1.00	
$\frac{1}{2}$	per cent. is written	$.00\frac{1}{2}$; or .005	$=\frac{1}{200}$
$\frac{1}{4}$	per cent. is written	$.00\frac{1}{4}$; or .0025	$=\frac{1}{400}$
$2\frac{1}{3}$	per cent. is written	$.02\frac{1}{3}$	$=\frac{7}{300}$
$\frac{1}{20}$	per cent. is written	$.00\frac{1}{20}$; or .0005	$=\frac{1}{2000}$
25	per cent. is written	.25	$=\frac{1}{4}$
$12\frac{1}{2}$	per cent. is written	.125	$=\frac{1}{8}$, etc.

268. EXAMPLES.

Express decimally, and in lowest terms of equivalent common fractions when convenient.

1. 10%.
2. $12\frac{1}{2}$%.
3. $1\frac{3}{4}$%.
4. $\frac{5}{8}$%.
5. $\frac{1}{10}$%.
6. 2%.

7. 120%.	14. 75%.
8. 250%.	15. $300\frac{1}{4}$%.
9. $1\frac{1}{4}$%.	16. $\frac{4}{9}$%.
10. $\frac{1}{2}$ of 3%.	17. $\frac{7}{16}$%.
11. $\frac{3}{4}$%.	18. $\frac{3}{4}$ of 15%.
12. 500%.	19. $\frac{3}{5}$ of 200%.
13. $16\frac{2}{3}$%.	20. $\frac{7}{4}$%.

269. The *multiplicand* in *Percentage* is called the ***Base.*** *It is that number of which a certain per cent. is taken.*

270. 1. The *sum* of the *base* and *percentage* is called the ***Amount;*** the sum of a unit (1.00) and the per cent. expressed decimally may be called the *Amount per cent.*

2. The *difference* between the *base* and *percentage* is called the ***Difference;*** a unit (1.00), less the per cent. expressed decimally, may be called the *Difference per cent.*

271. All problems in Percentage (Art. 266, 1) refer to two or more of the following five terms:

1. *The Base,* which may be indicated by B.
2. *The per cent.,* which is " %.
3. *The Percentage,* which may be " P.
4. *The Amount,* " " A.
5. *The Difference,* " " D.

NOTE.—*Amount per cent.* and *Difference per cent.* may be indicated by A% and D%.

272. 1. Any two terms in Percentage being given, all the others may be found, the unit (1.00) always being known, and the % being expressed decimally.

2. The following equations are adapted to the solution of *all* simple problems in Percentage. The *first* one is the foundation of all the others, and the relations expressed by it are so simple that they can be easily understood and remembered.

			Base Per ct. Percentage.
I.	1st form.	**B × % = P**	120 × .05 = 6.
	2d form.	B × A% = A	120 × 1.05 = 126 = Amount.
	3d form.	B × D% = D	120 × .95 = 114 = Difference.
II.		B + P = A	120 + 6 = 126
III.		B − P = D	120 − 6 = 114

IV. $\frac{A+D}{2}=B$ $\qquad \frac{126+114}{2}=120$

V. $\frac{A-D}{2}=P$ $\qquad \frac{126-114}{2}=6$

NOTE.—The second and third forms of equation I. are of essentially the same nature as the first form, for the *amount* is equivalent to a certain *per cent.* of the *base*, and the same may be said of the *difference*.

273. TEN PROBLEMS IN PERCENTAGE.

How may the other terms be found from the following:

1. The 1st and 2d? (Art. 271.)
2. The 1st and 3d?
3. The 1st and 4th?
4. The 1st and 5th?
5. The 2d and 3d?
6. The 2d and 4th?
7. The 2d and 5th?
8. The 3d and 4th?
9. The 3d and 5th?
10. The 4th and 5th?

274. Examine the following cases and equations, and translate them, or express in words the relations indicated by them. (Use the example given in Art. 272.)

Case 1st.—*Base* $= P \div \%$; $A - P$; $D+P$; $A \div A\%$; $D \div D\%$; *or* $(A+D) \div 2$.
Case 2d.—*Per cent.* $= P \div B$.
Case 3d.—*Per centage* $= B \times \%$; $A-B$; $D+B$; *or* $(A-D) \div 2$.
Case 4th.—*Amount* $= B+P$.
Case 5th.—*Difference* $= B-P$.

275. APPLICATIONS OF PERCENTAGE.

The five preceding cases cover the whole subject of Percentage in all its numerous and important applications. The importance of fully understanding them cannot be urged too strongly upon one who wishes to become a competent accountant. It is not enough to be able to solve the examples in accordance with the directions of the rules. Rule accountants are always liable to make serious errors. Do I see clearly *why* such a process gives the required result? To this question the student should be able to give an affirmative answer.

There is such a thing as *common sense*, and the use of it in solving practical business problems is a *sine qua non*. The answer of almost any question may be anticipated, at least approximately, previous to its solution. The common-sense student sees from the *conditions* of the question about what answer he may expect. In solving a problem in discount, for example, he knows whether the *present worth* will be nearest \$3, \$30, or \$3000. We have often known "rule students" to hand in the most ridiculous answers to the simplest practical problems.

276. MENTAL PROBLEMS.

Analysis may be required.

1. What is 2% of 16? 4? 850?
2. 8% of 500 miles? 60 mi.? 2000 mi.? 1 mi.?
3. 1000% of \$1000? \$65? \$2? 50 cts.?
4. $\frac{3}{8}$% of \$320? \$80? \$16? \$4000?
5. $\frac{1}{4}$% of \$15.80? \$460? \$5000?
6. $33\frac{1}{3}$% of any number is what part of it?
7. $16\frac{2}{3}$% of 1200 hogs? 90 days? 42 bushels?
8. $66\frac{2}{3}$% of 660 men? 123 sheep? 9 hours?
9. 75% of \$480? \$600? \$1.20? \$1?
10. 4% of \$3? \$19? \$140? \$4.25?
11. $\frac{1}{4}$% of 80? 9600? 2? $\frac{4}{5}$? .06?
12. 120% of \$700? 80 cts.? 50 cts.? 2 cts.?
13. 90% of \$90? \$1.60? 40 cts.? 5 cts.?
14. $\frac{1}{15}$% of \$65? \$3? \$9000? \$105?
15. $\frac{1}{10}$% of \$37.50? $\$5\frac{5}{8}$? \$1923? \$1000?
16. $\frac{4}{10}$% of \$5000? \$120? \$15? \$6300?
17. 4% of 15%? 25%? 1%? 200%?
18. 20% of 2% of \$2500? \$7.50? \$9?
19. 150% of 6% of \$900? \$15? \$1?
20. $1\frac{1}{6}$% of \$300? \$4000? \$2?
21. If 20% per cent. of a man's income be \$240., what is his income?
22. A man received \$75. for some fruit sold on commission for a farmer, and his percentage was \$15.: how much should he send to the farmer?
23. A man's percentage for selling goods was \$25., which was

$62. less than all the money received for them; how much was received?

24. If a horse traveled 48 miles one day, and this was 20% more than he traveled the previous day, how far did he travel the previous day?

25. If a pupil attended school 18 days in March, and this was 10% less than he attended in April, how many days did he attend in April?

26. If the amount of a man's percentage for goods sold and the whole sum received be $60., and the difference between his percentage and the sum received be $40., for how much were the goods sold?

27. What per cent. of $80. is $24?

28. If the amount per cent. be 1.12½, what is the difference per cent.?

29. What is 15% of $60?

30. If a man receive $120. for goods that cost him $80., what is his percentage of gain?

31. If a man send to his employer $320. for goods sold, what was his percentage if the whole amount received for the goods was $360?

32. If the amount of a man's capital and percentage of gain be $440., and the difference between his capital and gain be $200, what is his percentage of gain?

33. If on ten acres 160 bushels of wheat be raised one year, and the percentage gained the next year be 20 bushels, what is the yield the second year?

34. If a ship sail 86 miles the first day, and 5% farther the second day, how far does she sail the second day?

35. What is the difference between $720. and 12½% of the same?

36. If at the beginning of a year a man have $6000., and at the end $7500, what is his per cent. of gain for the year?

37. If a man's debts amount to $1800, and his property to $360 less, what per cent. of his debts can he pay?

38. 72 is 9% of how many times 200?

39. $150 less what per cent. of itself equals 20% of $25.?

40. 120% of what number is 50% of 144?

NOTE.—For a more extended application of Percentage see Part Second.

277. WRITTEN PROBLEMS.

1. What is $6\frac{1}{2}\%$ of $\$72.37\frac{1}{2}$?
2. 32% of $\$1250$?
3. 25% of $12\frac{1}{4}$ hrs.?
4. $1\frac{1}{3}\%$ of 1050 sheep?
5. $33\frac{1}{3}\%$ of 252 cattle?
6. $87\frac{1}{2}\%$ of 1632 ft.? ($\frac{7}{8}$.)
7. 15% of 25% of $\$13.60$?
8. $.796 \div \frac{2}{7} \times 1000 - 8\frac{1}{10}\%$ of the last result?
9. $\frac{14}{15} \div 32\% \times 17 \div 100 + 496\frac{2}{15}$?
10. $76\frac{4}{5} \div (.19\frac{7}{10} - 3\frac{1}{5}\%) \times 99$?
11. $76000 \times (17\% + 23\%) - (\frac{16}{40} \div .\dot{3})$?
12. $(.11 - \frac{3}{50}) \times \frac{2.7}{12} + \frac{1.001}{\frac{1}{10}\%} \times \frac{1}{500}$?

13. A merchant failing was able to pay his creditors but 40 per cent. He owes A \$3500, B \$1200, C \$1134, D \$650. What will each receive?

14. A person at his death leaves an estate worth \$1500; 12 per cent. of which he received from his wife; 20 per cent. from speculation; 30 per cent. from rise of property; 25 per cent. from the estate of an uncle; and the remainder from his father. How much did he receive from each source?

15. A has an income of \$1100 per year; he pays 10 per cent. of it for board; $\frac{1}{2}$ per cent. for washing; 2 per cent. for incidentals; 15 per cent. for clothing; 9 per cent. for other expenses. What does each item cost, and how much has he left?

16. 6 is what per cent. of 25?
17. 12 cts. is what $\%$ of \$3.?

NOTE.—Since only numbers of the same kind can be compared, reduce \$3. to cents, or 12 cts. to decimals of a dollar.

18. What $\%$ of \$40. is \$12?
19. What $\%$ of 120 yds. is 20% of 90 yds.?
20. $2\frac{1}{2}$ dimes is what $\%$ of \$5?
21. 40 men is what $\%$ of 150 men?
22. 150 men is what $\%$ of 40 men?
23. The cent (new coinage) contains 22 parts copper and 3

parts nickle; what per cent. of it is copper and what per cent. nickle?

24. 15 per cent. is what per cent. of 60 per cent.?

25. A person whose annual income is \$450 pays \$125 for board, \$140 for clothing, \$25 for books, and \$30 for sundries; what per cent. of his income is each item, and what per cent. remains?

26. A merchant failing owes \$3500; his property is valued at \$2100. What per cent. of his indebtedness can he pay?

27. A person pays \$13.50 a month for board, which is 30 per cent. of his salary: what is his salary?

$$\frac{\$13.50}{.30}=\$450.\ \textit{Ans.}\quad \text{(Art. 272, 1.)}$$

28. 45 is 10 per cent. of what number?

29. \$3.60 is 15 per cent. of what number?

30. \$5.62½ is 12½ per cent. of what number?

31. Sold cloth for \$3.50 per yard, which was 70 per cent. of its cost; what was the cost of the cloth per yard?

32. A boy spent 60 per cent. of his money for toys, and 25 per cent. for candies, and had 15 cents remaining; how many cents had he at first?

33. The assets of a merchant are \$45000, which is 60 per cent. of his indebtedness; what is his indebtedness?

34. The deaths in a certain city during the year are 980, which is 3½ per cent. of the population; what is the number of inhabitants?

35. A merchant sells 40% of his stock of goods for \$3500.; what is the value of his entire stock?

36. Sold broadcloth at \$5. per yard, and thereby made 25%; what did it cost?

$$\$5\div(1.00+.25)=\$5\div1.25=\$4.\quad \text{(Art. 272, 1.)}$$

NOTE.—Since 25% is gained, \$5, the *amount*, must equal 125% of the cost.

37. A drover lost 12% of a flock of sheep, and then had 2200: how many had he at first?

NOTE.—Since 12% were lost, 2200 must be 88% of the entire flock.

38. 168 is 20 per cent. more than what number?

39. \$63.75 is 15 per cent. less than what?

40. The population of a certain city is 25000, which is 25 per cent. more than it was in 1850; what was the population in 1850?

41. A grocer sells flour as follows:

Extra Family	$5.50 per bbl.
Superfine	4.75 "
Fine	4.25 "

and makes a profit of $12\frac{1}{2}$ per cent.; what was the cost of each brand?

42. A cargo of corn being injured, the owner was obliged to sell the same for $28000, which was at a loss of 30 per cent.; what was the cost of the cargo?

43. The sales of a dry goods firm amount to $90000 per year; $\frac{2}{5}$ of the sales were made at a profit of 25 per cent.; $\frac{3}{10}$ at a profit of 35 per cent.; and the remainder at a profit of 20 per cent.; what was the cost of goods?

44. If a man invest $4500 in a partnership business, and on settlement receive $5760, what is his per cent. of gain?

45. If a young man receive $10000 from his father, and at the end of 3 yrs. has only $5200 left: what per cent. of the whole has he spent each year?

46. What is the difference between $7\frac{1}{2}$% of $2500, and $9\frac{1}{2}$% of $750?

47. A man left $80000 to his two sons, giving to the elder 10% more of the whole than to the younger. In three years the property of each was increased 30%; how much was each one then worth?

48. An insurance agent received $16.50, which was 15% of the amount he collected: how much did he collect?

49. If the number of pupils in a certain school one year was 954, and this was 6% more than the number the previous year, how many were there the previous year?

50. What per cent. of the *amount*, at 4%, is 4% of the *base*?

51. By widening a street 4% it was made 26 yd. wide: how wide was it before?

52. By diminishing the pressure on a steam-boiler 5%, it was then only 64.6 lb.: what was it before?

53. What was the population of Illinois in 1860 if it increased $47\frac{3}{4}$% in ten yr., and was 2,529,410 in 1870?

9

54. What was the population of Kansas in 1870 if it increased 236% in ten yrs., and was 107,206 in 1860?

55. What was the per cent. of increase of population in Nebraska in ten years if the population was 28,841 in 1860, and 116,888 in 1870?

56. What was the population of California in 1850 if in 1870 it was 549,808, having increased $44\frac{7}{10}\%$ from 1860 to 1870, and $310\frac{1}{3}\%$ from 1850 to 1860?

57. What was the population of Massachusetts in 1870 if in 1840 it was 737,700, and increased $34\frac{1}{5}\%$ from 1840 to 1850, $23\frac{4}{5}\%$ from 1850 to 1860, and $18\frac{38}{100}\%$ from 1860 to 1870?

58. If the public debt of the United States amounted to \$2,588,452,213.94 July 1, 1869, and \$2,480,672,427.81 July 1, 1870, what was the per cent. of decrease for one year?

59. If the public debt of Great Britain, March 31, 1870, was £800,681,428 (the Pound Sterling being estimated at \$5.), what per cent. greater was that than the debt of the United States July 1, 1870?

60. Which is the better investment, a house that will rent for \$1200 at 8% on the value, or the same amount in real estate which will be worth \$20,000 in one year? How much better in one year?

SECTION XIII.

DENOMINATE NUMBERS.

NOTE.—*For Tables see Part Third,* Articles

278. A ***Denominate Number*** refers to concrete units (Arts. 8 and 14.)

279. Denominate numbers are of two kinds, *Simple* and *Compound.*

1. A *simple denominate number* refers to units of only one kind or value; as, 7 lbs., 10 A., 19 miles, etc. (Art. 10.)

2. A *compound number* is a denominate number that refers to units of different values but of the same variety or application; as, 2 lb. 10 oz.; 8 hrs. 10 m.; 4 crds. 80 cu. ft. (Art. 11.)

280. MENTAL PROBLEMS.

Reduce—

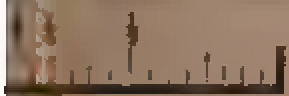
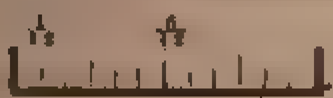
1. 4 ft. 7 in. to inches.
   Repeat the table of Long Measure.
2. 10 rd. 4 yd. to yards. To feet.
3. 4 mi. 3 fur. to rods.
4. 2 m. 4 fur. 20 rd. 2 yd. to feet.
5. 17 leagues to miles. 21 fathoms to feet.
   Repeat the table of Nautical Measure.
6. 6 yd. 1 qr. to inches.
   Repeat the table of Cloth Measure.
7. 364 inches to yards.
8. 273 rods to miles.
9. 2° 16′ to minutes.
   Repeat the table of Circular Measure.
10. 1760″ to degrees.
11. 2 sq. ft. 12 sq. in. to sq. in.
    Repeat the table of Square Measure.
12. 7 A. 3 R. 10 P. to P.
13. 756 links to chains and rods.
    Repeat the table of Surveyors' Measure.
14. 1 A. 2 sq. ch. to P.
15. 4 cu. yd. 12 cu. ft. to cu. ft.
    Repeat the table of Cubic Measure.
16. 482 cu. ft. to cords.
17. 2 gal. 3 qt. 1 pt. to pints.
    Repeat the table of Wine or Liquid Measure.
18. 2 bbl. 2 gal. to quarts.
19. 3 pipes to quarts.
    Repeat the table of Beer Measure.
20. 760 pt. to gallons.
21. 4 w. 2 d. to hours.
    Repeat the table of Time.
22. 9849 secs. to hours.
23. 2 hr. 10 m. to seconds.
24. 960 drams to lbs.
    Repeat the table of Avoirdupois Weight.

25. 2 lb. 8 oz. to drams.
26. 3 T. 3 qr. to pounds.
27. 960 gr. to ounces.
Repeat the table of Troy Weight.
28. 2 lb. 6 oz. 10 pwt. to pwt.
29. 80 scruples to pounds.
Repeat the table of Apothecaries' Weight.
30. 3 lb. 4 dr. to drams.
31. \$7 and 4 dimes to cents.
Repeat the table of Federal Money.
32. 1285 mills to dollars.
33. £2 5s. to pence.
Repeat the table of Sterling Money.
34. 462d. to £, etc.
35. £1 1s. 1d. to farthings.
36. 4 f. 11 centimes to centimes. To francs.
Repeat the table of French Money.
37. 728 centimes to francs.
38. $15\frac{1}{2}$ degrees to quarter degrees.
39. $26\frac{7}{10}$ degrees to tenths of degrees. $\frac{267}{100}$ to degs.
40. How many sheets of paper in 2 quires 7 sheets?
41. How many quires in 6 reams 15 quires?
42. How many reams in 720 sheets?
43. How many miles in 29 fur.? 82 fur.? 153 fur.?
44. How many A. in 800 sq. rd.? 1120 sq. rd.?
45. How many pints in 5 bu.? 2 bu.? 17 qt.?
Repeat the table of Dry Measure.
46. How many cu. in. in 2 cu. ft.? 5 cu. ft.?
47. How many hrs. in 960 sec.? 1440 sec.?
48. How many oz. in 3 lb. 13 oz.? 4 lb. 2 oz.?
49. How many £ in 24s.? 99s.? 480d.?
50. How many minutes in 15 degrees? 70 sec.?

DENOMINATE UNITS.

281. Standard measures of length are called ***Linear Units***, or units of Long Measure. These are the following, having the ratios indicated, and being designated by the abbreviations or symbols given.

Ratios, 8, 40, $5\frac{1}{2}$, 3, 12.
Units—the mile (mi.), furlong (fur.), rod (rd.), yard (yd.), foot (ft.), inch (in.) (Art. 813.)

NOTES.—1 On scales of measure the inch is divided into *binary* parts, that is, halves, quarters, eighths, etc., into *decimal* parts, that is, tenths, hundredths, etc., or into *duodecimal* parts, that is, 12ths, 144ths, etc. On the scale mentioned the inch is called a prime ( ), 12ths are called seconds and the next subdivision thirds (‴), etc. The marks ′, ″, ‴, etc., are called indices. One inch of each scale is here shown, 1 in., binary parts , 1 in. decimal parts ; 1 in., duodecimal parts .

2. In *Cloth Measure* the yd is now divided into binary parts, the old division into quatenary parts of quarters and nails (na. = 2¼ in.) being now seldom used. The following are sometimes used, viz., the Ell Flemish (E. Fl. = 3 qrs.), the Ell English (E. E. = 5 qrs.), and the Ell French (E. Fr. = 6 qrs.)

3. In measuring land boundaries, surveyors use *Gunter's Chain*, 4 rd. in length, and consisting of 100 links (l.), each equal to 7.92 in. In measuring distances merely, engineers use a chain 100 ft. long, consisting of 100 links.

4. Depths at sea are computed in fathoms of 6 ft. each, and 880 fathoms are reckoned as 1 mile. Distances at sea are computed in leagues of 3 knots or geographical miles each, a geographical mile being $\frac{1}{60}$ of a degree on the equator. A "cable length" is about 120 fathoms. (Art. 814.)

282. *Circular Measure* is a species of linear measure applied to the measurement of angles and arcs of circles.

The units and ratios are—

4, 3, 30, 60, 60.
circle (cir.), quadrant (quad.), sign (S.), degree (°), minute (′), second (″), or as the quad. and S. are generally omitted, the ratios would be 360, 60, 60. (Art. 817.)

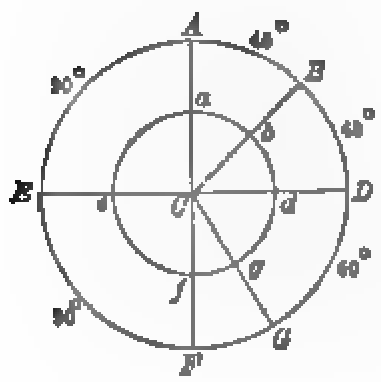
NOTE.—*Circle* here refers to a *circumference.*

1. A degree is one 360th of *any* cir., hence its absolute length is variable, it being equal to $69\frac{1}{6}$ mi. on the equatorial cir. of the earth, but having a less *real* value on every smaller circle.

2. On scales of circular measure the degree, minute, or second is sometimes divided into decimal parts. These are used in surveying, navigation, geography, and astronomy.

3. In the diagram is shown a circle whose diameter, E D, is 1 in., and also one whose diameter, *e d.*, is ½ in. The circumference,

A B D G F E, is divided into the quad. A D, D F, F E, E A, each containing 90° and measuring a ***right angle*** at the point C. Thus the arc E A of 90° measures the right angle E C A, while the angle A C B is measured by an arc of 45°, D C G by an arc of 60° and G C F by an arc of 30°, and the same angles are measured by the corresponding equal arcs of the smaller circle, *a b d g f e*.



4. The degrees on the polar circumference of the earth are called degrees of Latitude (Lat.), and are counted on any meridian from the equator 90° North (N.) and 90° South (S.), hence difference in Lat. may be found by computing the sum when N. and S. Lat. are given, or the difference when both are N. or S.

5. The degrees on the equator, and on every circle of the earth parallel with the equator, are called degrees of Longitude (Long.), and are counted on the equator or any parallel, from any established meridian 180° East (E.), and 180° West (W.), hence difference of Long. may be found by computing the sum when E. and W. Long. are given, or the difference when both are E. or W.

NOTE.—In French Circular Measure the arc of 90° is divided into 100 parts called *grades* (g.), each grade into 100 parts called minutes ('), and each minute into 100 parts called seconds (''). Thus 41g. 17' 29''=41.1729 g. One grade equals $\frac{9}{10}$ of a degree.

283. The term degree (°) is applied also to the unit of measure that indicates temperature or intensity of heat. Relative temperature is measured by the expansion or contraction of mercury, or some other substance. The difference of expansion between the temperature of boiling water and of melting ice is divided into parts called degrees, and the degrees are divided decimally. On the Centigrade scale of Celsius (C.), used mostly in Europe, there are 100° in this space of difference; on the scale of Fahrenheit (F.), used mostly in the United States, there are 180°; on the scale of Reaumur (R.), (the inventor of the mercurial thermometer in 1731), there are 80°. Thus 5° C.=9° F.=4° R. The *zero* of C. and R. is fixed at the point of the temperature of melting ice, and

the *zero* of F. at 32° below this point, 212° F. marking the boiling point. Degrees above 0 are marked +, and those below −.

NOTES.—1. In telegraphy intensity of electrical force, or of resistance to it, is computed in units called *Ohms*.

2. The relative hardness of different substances is expressed in degrees, the diamond being rated at 10° or 13°, and other substances lower. The term degree is also used arbitrarily to express the relative density of any two substances, taking either as a standard.

284. Standard measures of area or surface are called ***Superficial Units***, or units of Square Measure.

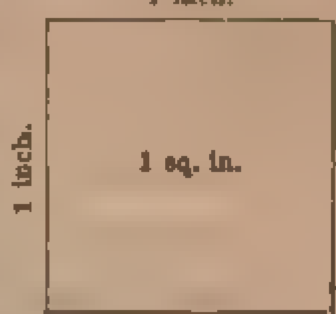
These are—

Ratios,		640,	4,	40,		
The square mile (sq. m.),		acre (A.),	rood (R.),	square rod or pole		
	$30\frac{1}{4}$,			9,		144,
(sq. rd. or P.),	square yard (sq. yd.),			square foot (sq. ft.),		square inch (sq. in.)

(Art. 818.)

A SQUARE INCH.

1 inch.



NOTES.—1. A square inch is the area or surface inclosed in a square each side of which measures one linear inch; a sq. ft. is a square each side of which measures one linear foot or 12 inches.

2. The area of any rectangular figure is equal to the product of the number of units in its two dimensions, the length and breadth. Thus the diagram

A B C D, which is 8 units in length and 5 units in breadth, is found to contain (5 × 8) = 40 *square units*; the linear unit in this case being *a b* = $\frac{1}{10}$ inch, and the square unit *a b c d* = $\frac{1}{100}$ sq. inch, and there being 5 rows, each containing .8 sq. in.

3. The sq. ft. is divided duodecimally into primes, seconds, etc., the prime (′) being $\frac{1}{12}$ of a sq. ft.

4. In Surveyor's Measure the units used are—

Ratios,	36,	640,	10,		16,	625,
The Township (Tp.),	section (Sec.),	A.,	square chain (sq. ch.),		P.,	square link (sq. l.).

In the survey of government lands, townships are bounded by parallels and meridians, and each section is divided into quarter sections of 160 acres each. (Art. 819.)

5. Stone dressing, and glazing, are generally computed in sq. ft.; painting, calcimining, etc., by squares of 100 sq. ft. each; bricklaying, when the wall is 12 inches thick, by the hundred square ft., the sq. yd., or the thousand brick, a common brick being 8 in. long, 4 in. wide, and 2 in. thick, 27 of which make a solid foot.

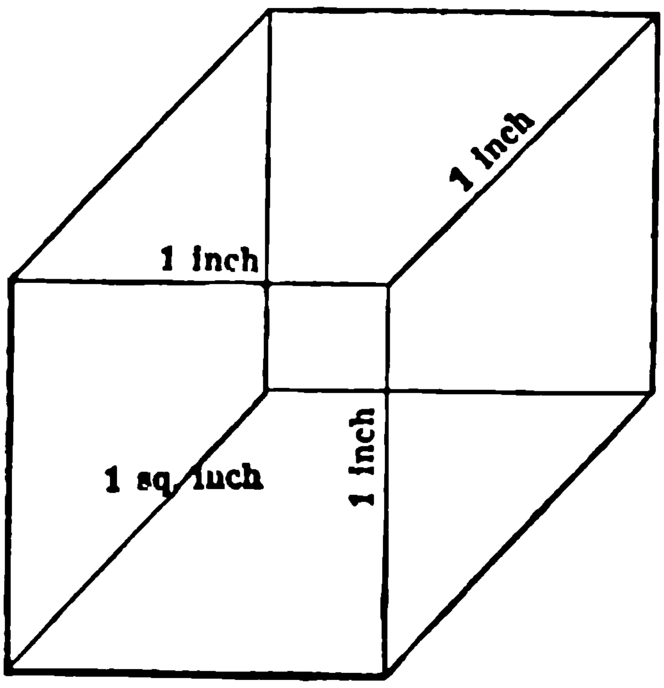
285. Standard measures of volume are called ***Solid*** or ***Cubic Units,*** units of Cubic Measure or of Capacity.

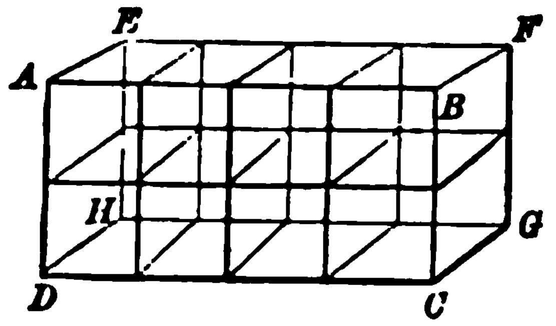
The primary cubic units are—

Ratios, 27, 1728,

The solid or cubic yard (cu. yd.), cubic foot (cu. ft.), cubic inch (cu. in.) (Art. 820.)

A CUBIC INCH.





NOTES.—1. A solid inch, or cubic inch, is the volume of a cube each edge of which measures one inch, and the area of each side or face being one square inch; a cubic foot is a cube each edge of which measures 1 ft.

2. The volume of any rectangular solid is equal to the product of the number of units in the three dimensions. Thus in the diagram a solid is represented 4 units (each ¼ in.) long, 2 units high, 1 unit wide, containing ($4 \times 2 \times 1$)=8 cubic units, there being 2 rows containing 4 cubic units each; and if there were 3 layers of 2 rows each, the volume would be 24 cubic units; if the breadth, height, and length were each 4, then the contents would be equal to $4 \times 4 \times 4$, or 4 cubed = 64 cubic units. A solid whose three dimensions are equal is called a *cube.*

3. The cu. ft. is divided duodecimally, the prime or ′ containing 144 cu. in., the ″ 12 cu. in., and the ‴ 1 cu. in.

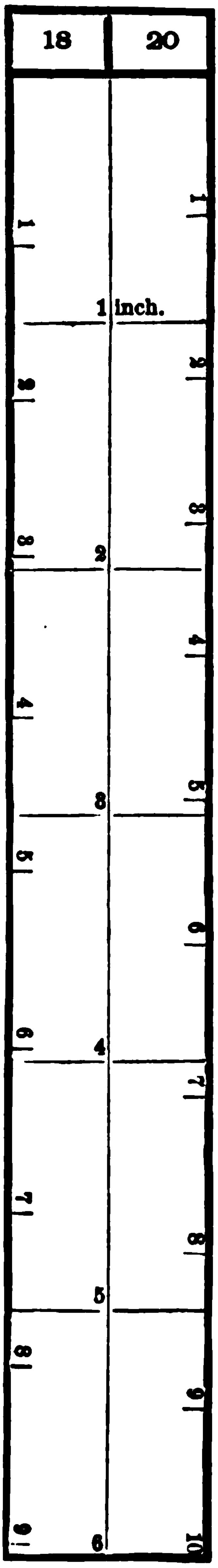
4. In measuring *wood,* the units used are—

Ratios, 8, 16,

The Cord (C. or crd.), cord foot (crd. ft.), cu. ft.; the cord thus containing 128 cu. ft. The usual dimensions of a cord of wood are 8 ft. long, 4 ft. wide, and 4 ft. high. ($8 \times 4 \times 4$=128.) A cord of *stone* contains 100 cu. ft.

5. In *Lumber Measure* the unit employed is *one foot of board measure,* which is 1 ft. long, 1 ft. wide, and 1 inch thick; that is, just a cubic prime, or $\frac{1}{12}$ of a cu. ft. The instrument generally used is the *Board Rule,* frequently made in the form of a hexagonal staff. At one end of this, where the scale begins, are marked the numbers, 14, 16; 18, 20; 22 and 24, indicating the number of ft. in length of different kinds of common lumber, and the scale for each length consists of the ordinary scale of inches, and near this a scale showing the number of ft. board measure, corresponding to the length and width of the board when only one inch thick, the number of

inches in thickness being used as an abstract multiplier in other cases. The width being measured with the scale for the given length, the number of ft. of lumber may be read at once or readily computed. It may be observed that 12 inches is divided into as many parts on each scale as there are feet in the length of the board. A portion of the scale is here shown.



Thus if an inch board 18 ft. long be 5 in. wide, it contains a little more than 7½ ft.; one 20 ft. long and 5½ in. wide, contains about 9 ft. If a 20 ft. timber be 3½ in. wide and 4 in. thick, it would contain about (6 × 4)=24 ft. of lumber.

Similar scales are found on the carpenter's square, showing the ft. of lumber in half-inch boards, or in inch boards of half the length (of from 16 to 30 feet) indicated.

For finding contents of timber of varying dimensions, see MENSURATION.

6. Heavy timber is sometimes estimated by the *ton* of *round timber* at 40 cu. ft., or *ton* of *hewn timber* at 50 cu. ft.

7. Quantities of stone and masonry are estimated by the *perch* of 24¾ cu. ft., the ordinary dimensions being 16½ ft. long, 1 ft. high, and 1½ ft. thick. Generally 27 brick make a cu. ft. In such estimates for walls of buildings the outside dimensions are taken.

286. The *units* of *capacity* for ***Liquid Measure*** (L. M.), (called also Wine Measure,) used in measuring nearly all liquids, are—

Ratios, 4, 2, 4.

The gallon (gal.), quart (qt.), pint (pt.), and gill (gi.) (Art. 823.)

NOTES.—1. The following measures are also used to some extent:

Ratios, 2, 2, 2.

The tun, pipe, or butt, hogshead (hhd.), and barrel (bbl.), but the bbl. varies from 31½ gal. to 36 gal., and the hogshead from 50 gal. to 100 or more. "Barrel bulk" is a space of 5 cu. ft.

2. The standard gallon of Liquid or Wine Measure contains 231 cu. in. The gallon of Beer Measure, containing 282 cu. in. is now seldom used here. Ale is sold in some places by weight at 8 lbs. per gallon.

3. In mixing liquid medicines, apothecaries use—(Art. 824.)

Ratios, 8, 16, 8, 60.
The gallon (Cong.), pint (O.), fluid ounce (f. ℥), fluid drachm (f. ʒ), and minim (♏), and one minim equals one *drop*. The gal. equals the gal. L. M.

287. The units of capacity for ***Dry Measure,*** used for all grains, fruit, etc., are—

Ratios, 4, 8, 2.
The bushel (bu.), peck (pk)., quart (qt.), and pint (pt.) (A "heaped bushel" contains about 5 pk.)

NOTES.—1. The standard bushel measure is $18\frac{1}{2}$ inches in diameter and 8 inches deep.

2. In buying and selling grain, and some other articles, they are computed at a certain number of pounds per bushel, or barrel, as follows:

ARTICLES.	Lbs. per bushel in most places.	Lbs. per bu. in particular States.
Apples, dried	28	25 Indiana and Illinois.
Barley	48	46 Iowa.
Beans, Castor	46	
Beans, white	60	
Bran	20	
Buckwheat	....	52 Ill., Ia., Ky.; 50 Ind., Pa.; 48 Mass.; 42 Mich.; 40 Wis.
Clover seed	60	
Corn on ear	70	68 Indiana.
Corn shelled	56	
Flax seed	56	
Grass seed, timothy	45	46 Wisconsin.
Grass seed, red top or blue.	14	
Oats	32	33 Iowa; 30 Pa., Maine.
Onions	57	52 Massachusetts.
Peaches, dried	28	33 Iowa, Ill., Ind.
Peas	60	
Potatoes, common	60	
Potatoes, sweet	55	
Rye	56	
Salt	50	56 Kentucky.
Turnips	55	
Wheat	60	
	Lbs. per bbl.	
Beef, pork, or fish	200	220 of fish in Maryland.
Flour	196	
Salt	280	At New York salt works.
Soap	256	

NOTE.—For rates of estimating gross weight of freight on R. R.'s see Part Third.

288. Standard measures of *gravity* are called ***Units of Weight.***

The units of ***Avoirdupois Weight,*** used for weighing nearly all ordinary articles, are—

Ratios, 20, 100, 16, $437\frac{1}{2}$.
The ton (T.), hundred weight (cwt.), pound (lb.), ounce (oz.), grain (gr.). (Art. 826.)

NOTE.—Until recently the ounce was divided into 16 drams (dr.); the cwt. formerly contained 112 lbs., and the T 2240 lbs. The latter is still used at the United States custom-house and in some coal mines. The cwt. is divided into quarters of 25 lb.

289. Troy Weight is used for weighing precious metals, gems, etc., and in scientific experiments. Its units are—

Ratios, 12, 20, 24.
The pound (lb.), ounce (oz.), pennyweight (pwt.), and grain (gr.)

NOTES.—1. The Troy pound is the standard of all other weights, it is the weight of 22.7944 cu. in. of pure water at its greatest density. The grains mentioned in Avoirdupois Weight are Troy grains. (Art 827.)

2. In mixing medicines not liquid, apothecaries use Troy grains, and also the peculiar units—

8, 3, 20.
The ounce (℥ or oz.), dram (ʒ or dr.), scruple (℈ or sc.), and grain (gr.)
Medicines are bought and sold by Avoirdupois Weight. (Art. 828.)

3. In weighing diamonds the *carat* of about 3.2 gr. is used. The term *carat* also denotes one 24th part of pure gold; thus gold that contains 4 parts of alloy is said to be 20 carats fine. The *pearl* grain, equal to ¼ Troy grain, is also used in weighing diamonds and pearls.

290. Standard measures of duration are called ***Units of Time.*** These are—

Ratios, 10, 10, 12, 4, 7, 24,
The century, decade, year (yr.), month (mo.), week (w.), day (da.),
60, 60.
hour (hr.), minute (m.), and second (sec.). (Art. 821.)

NOTES.—1 The ratio of the mo. to the w., 4, is not accurate, as the month varies from 28 to 31 days. Among business men the mo. is reckoned at 30 days, and the *year* at 52 w., or 365 da.

2. The exact length of a *Tropical* or *Solar Year* is determined by one entire revolution of the earth in its orbit, and indicated by the sun's occupying twice in succession the same apparent place in the heavens. It is equal to 365 da. 5 h. 48 m. 49.7 sec. The *Common* or *Civil Year* is reckoned at 365 days. Every fourth year, except the centuries not multiples of 400 (as 1800), is reckoned at 366 days, and is called a *leap year*.

3. In 1752, in order to correct an accumulated error in the Julian calendar, the British government called the 3d of September the 14th, thus dropping 11 days. This *corrected* mode of reckoning is called New Style (N. S.), and the former method the Old Style (O. S.) The O. S. is still used in Russia, and the error now amounts to 12 days, so that N. S. dates are 12 days in advance of O. S. dates. Old and new style dates are combined thus, Mch. $\frac{1}{13}$.

4. The centuries and years counted from the beginning of the Christian era forward are designated as A. D., and those counted backward from the same time are designated as B. C.

291. 1. ***Longitude*** and ***Time*** are so related, that in certain cases one may be computed from the other. (Art. 822.)

2. As the earth revolves eastward on its axis once in about 24 hrs., every place must pass through one-24th of 360°, that is, 15° in one hr., hence 1° in 4 m., and 1′ in 4 sec. of time. If then the difference in the Long. of two places be known, their difference in time may be computed, and *vice versa* the difference in Long. may be computed. Thus London is about 77° E. of Washington; hence the difference in time between the two places must be $\frac{77}{15}$ hr. = 5 h. 8 min., and since 1° makes a difference of 4 m., 2° would afford 8 m. Now if the time at Washington were 2 h. 20 m. P. M., the time at London would be about 5 h. 8 m. later, or 7 h. 28 m. P. M.

15°)77°
5 and 2° Rem.
$2 \times 4 = 8.$

292. Standard measures of *value* are called ***Units of Money*** or ***Currency.*** The American units are the dollar ($), and cent (ct. or ¢). (Art. 829.)

Ratio, 100.

NOTES.—1. In 1837 the standard weight of the United States silver dollar was fixed at 412.5 grains, $\frac{9}{10}$ of which is required to be of pure silver. In 1849 the standard weight of the gold dollar was fixed at 25.8 grains, $\frac{9}{10}$ of which is required to be of pure gold. Thus 371.25 grains of pure silver equal in value 23.22 grains of pure gold.

2. The units of English currency are—

Ratios, 20, 12, 4.

The pound sterling (£), shilling (s.), penny (d.), and farthing (qr.) (Art. 847.)

The equivalent of the £ is 113 grains of pure gold, and it is generally rated at $4.84, but this value is variable.

3. The term shilling is frequently used in the United States in stating the price of articles, and it indicates old divisions or equivalents of parts of the dollar. Its value varies in different States as follows: In the New England States, and in Indiana, Illinois, Missouri, Mississippi, Texas, Virginia,

Kentucky, and Tennessee, 1s.=16⅔ cts., and $1.=6s.; in New York, Ohio, Michigan, and North Carolina, 1s.=12½ cts., and $1.=8s.; in Pennsylvania, New Jersey, Delaware, and Maryland, 1s.=13⅓ cts., and $1.=7½s.; in Georgia and South Carolina, 1s.=21⅔ cts., and $1.=4⅔s. These rates are liable to variations by custom; as, in Illinois, the shilling is rated frequently at 12½ cts.

4. The units of French currency are—(Art. 845.)

Ratio, 100.

The franc (f.) and the centime, the franc being commonly rated at $.186 in the United States, though the real mint value is $.1936.

293. WRITTEN PROBLEMS.

NOTE.—The same general principles are applied in the reduction, transformation, combination, and comparison of compound and of simple numbers.

To reduce a denominate number to a simple denominate number of lower denominations. (Art. 74, 3.)

EXAMPLES.

1. Reduce 5 lb. 6 oz. 10 pwt. 18 gr. of silver to grains.

lb.	oz.	dwt.	gr.	
5	6	10	18	*Ans.* 31938 gr.

$$\begin{array}{rl} 12 & \\ \hline 66 & \text{oz.} = (5\times12)+6. \\ 20 & \\ \hline 1330 & \text{pwt.} = (66\times20)+10. \\ 24 & \\ \hline 31938 & \text{gr.} = (1330\times24)+18. \end{array}$$

2. How many seconds in 10 hours?

Ans. 36000 s.

$$\begin{array}{rl} 10 & \text{h.} \\ 60 & \\ \hline 600 & \text{m.} \\ 60 & \\ \hline 36000 & \text{s.} \end{array}$$

NOTE.—For convenience in reducing, the ratios of the scale are used as *multipliers*. These, however, are really the logical multiplicands, and the

number of units to be reduced the logical multipliers. Thus 10 h. × 60 cannot equal 600 minutes, but 60 m. × 10 = 600 m., and since the numerical results are the same in the two cases, the more convenient method may be used, provided the pupil understands *why*. Both factors can *not* be denominate.

3. Reduce $\frac{7}{9}$ lb. butter to drams.

$$16 \text{ oz.} \times \tfrac{7}{9} = \tfrac{112}{9} \text{ oz.}; \ 16 \text{ dr.} \times \tfrac{112}{9} = \tfrac{1792}{9} \text{ dr.} = 199\tfrac{1}{9} \text{ dr.}$$

This is equivalent to $\frac{7}{9} \times 16 \times 16 = \frac{7 \times 16 \times 16}{9} = \frac{1792}{9} = 199\frac{1}{9}$.

4. Reduce $\frac{7}{320}$ yd. to inches.

$$\tfrac{7}{320} \text{ of } 3 \text{ ft.} = \tfrac{21}{320} \text{ of } 12 \text{ in.} = \tfrac{63}{80} \text{ in.}$$

This is equivalent to $\frac{7 \times 3 \times \overset{3}{\cancel{12}}}{\underset{80}{\cancel{320}}} = \frac{63}{80}$; or $\frac{7 \times \overset{9}{\cancel{36}}}{\underset{80}{\cancel{320}}} = \frac{63}{80}$.

NOTE.—When a simple number is to be reduced to another simple number of a lower denomination, it is better to multiply at once by the ratio of the superior to the inferior unit.

5. Reduce $12\frac{1}{2}$ bu. to pints.
6. In $\frac{7}{8}$ of an acre how many perches?
7. Reduce 12 h. 20 m. to seconds.
8. Reduce $\frac{3}{4}$ hhd. of wine to pints.
9. Reduce .375 T. to pounds (Avoirdupois).
10. In .7 of a bushel how many pints?
11. In 8.75 yd. how many nails?
12. Reduce $2\frac{2}{3}$ days to minutes.
13. Reduce $5\frac{3}{4}$ cords to solid feet.
14. In .45 of a rod how many inches?
15. Reduce 12 cubic feet to cubic inches.
16. Reduce 13.5 hhd. of beer to quarts.
17. Reduce 5 lb. 6 oz. 12 pwt. of gold to pwt.
18. Reduce 5.24 lb. of calomel to ounces.
19. Reduce 7 lb. (Troy weight) to grains.
20. Reduce .65 of a yard to quarters.
21. In .24 of a ream of paper how many sheets?
22. In $\frac{2}{5}$ of a barrel of flour how many pounds?
23. Reduce 7 lb. $8\frac{2}{3}$ oz. of butter to drams.
24. In $\frac{3}{500}$ lb. of brass how many ounces?

294. To reduce a fraction to integers of lower denominations.

EXAMPLES.

1. Reduce $\frac{3}{5}$ da. to integers.

$$24 \text{ h.} \times \tfrac{3}{5} = \tfrac{72}{5} \text{ h.} = 14\tfrac{2}{5} \text{ h.}$$
$$60 \text{ m.} \times \tfrac{2}{5} = \tfrac{120}{5} \text{ m.} = 24 \text{ m.}$$
$$\text{hence } \tfrac{3}{5} \text{ da.} = 14 \text{ h. } 24 \text{ m.}$$

NOTE.—In such a case, only the proper fraction is reduced, as many integers as possible being expressed in superior units.

2. Reduce .85 da. to integers.

$$\left.\begin{array}{rrl} & 24 & \text{h.} \\ & .85 & \\ \hline & 120 & \\ & 192 & \\ \hline & 20.40 & \text{h.} \\ \text{and} & 60 & \text{m.} \\ & .4 & \\ \hline & 24.0 & \text{m.} \end{array}\right\} \text{equivalent to} \left\{\begin{array}{r} .85 \\ 24 \\ \hline 3\,40 \\ 17\,0 \\ \hline 20.40 \\ 60 \\ \hline 24.00 \end{array}\right.$$

Hence .85 da. = 20 h. 24 m.

3. Reduce .375 hhd. to integers.
4. Reduce .9 lb. Troy to integers.
5. Reduce $\frac{5}{8}$ rod to integers.
6. Reduce .5625 cwt. to integers.
7. Reduce $30\frac{7}{16}$ hhds. to integers.
8. Reduce $\frac{5}{9}$ mile to integers.
9. Reduce 250.35 lbs. Troy to integers.
10. Reduce .8 mile to integers.
11. Reduce £$\frac{2}{3}$ to integers.
12. Reduce .45 peck to integers.
13. What is the value of $\frac{2}{5}$ week?
14. What is the value of .75 bu.?
15. What is the value of $\frac{9}{16}$ day?

295. To reduce a simple denominate number to a denominate number of higher denominations.

EXAMPLES.

1. Reduce 15969 gr. to pounds.

```
24)15969 gr.(665
   144
   ---
    156            20)665 pwt.
    144              -----
    ---            12)33 oz. 5 pwt.
     129             --
     120              2 lb. 9 oz.
     ---
       9 gr.
```

Hence 15969 gr.=2 lb. 9 oz. 5 pwt. 9 gr.

2. Reduce $\frac{7}{9}$ inch to the fraction of a yard.

$$\tfrac{7}{9}\text{ in.}\div 12=\tfrac{7}{108};\ \tfrac{7}{108}\text{ ft.}\div 3=\tfrac{7}{324}.$$

Hence $\frac{7}{9}$ in.$=\frac{7}{324}$ yd.

This is equivalent to $(\frac{7}{9}\div 12)\div 3=\dfrac{7}{9\times 12\times 3}=\dfrac{7}{9\times 36}=\dfrac{7}{324}$.

NOTE.—When a simple number is to be reduced to another simple number of a higher denomination, it is better to divide at once by the proper ratio.

3. Reduce .48 of a nail to the decimal of a yard. *Ans.* .03 yd.

```
4).48 n.  )
  ----    |
4).12 qr. } =16).48
  ----    |      ---
  .03 yd. )      .03
```

4. Reduce 25.6 dr. to the decimal of a pound. *Ans.* .1 lb.

```
16)25.6 dr. )
   ----     |
 16)1.6 oz. } =256)25.6
    ---     |       ----
     .1 lb. )        .1
```

5. Reduce 414 gal. wine to hhd.
6. Reduce 2461 pwt. to pounds.
7. Reduce 1357 pts. to bushels.
8. Reduce 98 furlongs to miles.
9. Reduce 307200 perches to square miles.
10. Reduce 4032 gills to hhd. of wine.
11. Reduce $\frac{3}{10}$ gal. to the fraction of a hhd.
12. Reduce $\frac{7}{15}$ hours to the fraction of a day.

13. Reduce $6\frac{2}{3}$ pt. to the fraction of a bu.
14. Reduce 645 in. to yd.
15. Reduce 2176 cu. ft. to cords.
16. Reduce $1152\frac{3}{4}$ qt. to hhd.
17. Reduce 623 nails to yards.
18. Reduce 23.04 drams to lbs.
19. Reduce 184.8 hours to weeks.
20. How many acres in a street 5 rods wide and $2\frac{1}{2}$ miles long.

296. To find what part one denominate number is of another, to indicate their ratio.

EXAMPLES.

1. 2 ft. 6 in. is what part of a yard? *Ans.* $\frac{5}{6}$.

2 ft. 6 in.$=$30 in., and 1 yd.$-$36 in.; 30 in.$\div$36 in.$=\frac{5}{6}$.

Suggestion.—Reduce denominate numbers to the same denomination, then compare.

2. What part of a week is 5 d. 10 h.? *Ans.* $\frac{65}{84}$.
3. What part of 2 acres is 3 R. 25 p.?
4. What decimal part of 5 hours is 40 minutes?
5. What decimal part of 5 gals. is 3 qts. 1 pt.?
6. What part of \$5 is $87\frac{1}{2}$ cents?
7. What decimal part of a gallon is 3 pints?
8. What part of .45 lb. Troy is .45 oz.?
9. $\frac{3}{4}$ oz. is what part of $\frac{2}{3}$ lb. Avoirdupois?
10. 4 quires of paper is what decimal part of a ream?
11. What part of a mile is 6 fur. 16 rds.?
12. What decimal part of a pound is 10 oz. 4 pwts.?
13. What decimal part of a bushel is 3 pks. 4 qts.?
14. What part of a week is 3 d. 17 h. 36 m.?

297. For reducing denominate numbers of one variety to those of another variety, the following common equivalents are given:

1. Equivalents of volume—

1 gal. Liquid or Wine Measure	=	231	cu. in.
1 gal. Beer Measure	=	282	"
1 gal. Dry Measure	=	268.8	"
1 bushel	=	2150.42	"

Method.—Reduce to cubic inches, then to the denomination required.

2. Equivalents of weight—

1 lb. Avoirdupois	=7000	grains Troy.
1 lb. Apothecary or Troy	=5760	"
1 oz. Avoirdupois	= 437.5	"
1 oz. Apothecary or Troy	= 480	"
1 carat, Assayer's weight	= 240	"
1 carat, diamond weight	= 3.2	"
1 cu. ft. pure water	=1000	oz. Avoirdupois.

Method.—Reduce to Troy grains, then to the denomination required.

EXAMPLES.

1. Reduce 2 hhd. 20 gal. L. M. to B. M.

2. Reduce 7 bu. to qts. L. M.

3. Does a man gain or lose in quantity if he buys a barrel of ale by B. M. and sells it in qts. by Liquid Measure?

4. Reduce 4 lb. 10 oz. Av. to Apoth. drams.

5. Reduce 16 lb. 9 oz. Troy to Av. weight.

6. How many prescriptions requiring 5 Apoth. ounces each, could a druggist fill from 3 lb. 6$\frac{6}{7}$ oz. tincture of rhubarb bought by Av. weight?

7. What is the weight of 2 gal. water?

8. To how many Troy ounces is the combined weight of the following diamonds equal, viz.: The Regent 136 carats, the Koh-inoor 186 carats, the Orloff 195 carats?

9. What is the weight of water that would just fill a bushel measure?

10. What is the weight of a cubic inch of gold in Troy ounces, gold being 19.26 times as heavy as water?

ADDITION OF COMPOUND NUMBERS.

298. MENTAL PROBLEMS.

1. What is the sum of 3 ft. 2 in. and 5 ft. 4 in.?

2. 2 lb. 4 oz. + 8 lb. 3 oz.

3. £4 10s. + 15s. 5d.

4. 9 w. 5 da. + 7 w. 8 da. + 2 w. 3 da.
5. 4 yd. 2 ft. 8 in. + 20 yd. 2 ft. 10 in.
6. 11 A. 3 R. + 9 A. 2 R.
7. 5 bu. + 2 bu. 3 pk. + 3 pk. 5 qt.
8. $\frac{2}{3}$ yd. + 5 ft. + 18 in.
9. $\frac{1}{5}$ bu. + $\frac{3}{4}$ pk.

NOTE.—Observe the general principle concerning all numbers that are to be combined.

299. WRITTEN PROBLEMS.

(*See* SIMPLE ADDITION, Art. 85.)

1. Add together 5 lbs. 6 oz. 13 pwts. 22 grs.; 12 lbs. 9 oz. 18 pwts.; 7 oz. 19 pwts. 21 grs.; 24 lbs. 11 oz. 18 grs.

lbs.	oz.	pwt.	grs.	
5	6	13	22	
12	9	18		
	7	19	21	
24	11		18	
43	11	12	13	*Ans.*

Explanation.—Having written numbers of the same denomination in the same column, add, reducing as far as possible units of a lower denomination to those of a higher. In this example the sum of the grains is 61 grs. = 2 pwts. 13 grs. Write 13 grs., and add the 2 pwts. to the column of pwts., and proceed as before.

2. A man purchased 4 loads of corn: the first contained 25 bu. 3 pks. 7 qts. 1 pt.; the second, 30 bu. 2 qts.; the third, 37 bu. 1 pk.; the fourth, 29 bu. 1 pk. 7 qts. 1 pt. How much did he buy?

3. Find the sum of 5 gals. 3 qts. 1 pt.; 10 gals. 1 pt. 1 gill; 25 gals. 1 pt.; 19 gals. 1 qt. 1 gill; and 30 gals. 1 pt. 3 gills.

4. A man has 4 farms. The first contains 110 A. 3 R. 25 P.; the second, 95 A. 1 R. 20 P.; the third, 205 A. 0 R. 15 P.; and the fourth, 90 A. 3 R. 35 P. How many acres in all?

5. I purchase of one merchant 19 yds. 3 qrs. of cloth; of a second, 25 yds. 3 qrs. 2 na.; of a third, 17 yds. 3 na. How many yards do I buy?

6. Add 17 mi. 3 fur. 25 rd. 2 yd. 2 ft. 9$\frac{1}{2}$ in.; 14 m. 6 fur. 1 yd. 1 ft. 7 in.; 5 mi. 18 rd. 1 ft. 6$\frac{3}{4}$ in.; 25 mi. 7 fur. 32 rd. 3 yd. 2 ft. 5$\frac{1}{4}$ in.; 6 fur. 5 yd. 11$\frac{1}{2}$ in.

7. Add 14 w. 5 da. 13 h. 46 m.; 7 w. 6 da. 20 h. 16 m. 50 s.; 7 w. 4 da. 21 h. 18 sec.; 4 da. 29 m. 12 s.; 18 w. 7 h. 40 m. 40 s.

8. Add $16\frac{4}{7}°$; 22° 42′ 52″; $\frac{1}{4}$ sign; 3 s. 25° 45″; 11 s. 58′.40″.

9. 8 m. 6 fur. 8 ch. 2 p. 20 l.+20 m. 7 fur. 9 ch. 3 p. 19 l.+18 m. 5 fur. 8 ch. 20 l.

10. 41 cords, 120 cu. ft. 1400 cu. in.+70 C. 78 cu. ft. 1640 cu. in.+127 cu. ft. 1256 cu. in.+57 C. 1122 cu. in.

11. 71 deg. 56 m. 5 fur. 28 rd. 12 ft. 10 in.+16 deg. 26 m. 4 fur. 15 rd. 14 ft. 11 in.+52 deg. 25 m. 7 fur. 32 rd. 15 ft. 8 in.+23 deg. 47 m. 6 fur. 35 rd. 11 ft. 5 in.+47 deg. 23 m. 5 fur. 16 rd. 13 ft. 10 in.

12. 824 A. 2 R. 28 P. 220 sq. ft. 100 sq. in.+715 A. 30 P. 250 sq. ft.+3 R. 25 P. 118 sq. ft. 98 sq. in.+63 A. 1 R. 37 P. 132 sq. in.

13. $\frac{18}{24}$ bu.+$\frac{76}{24}$ pk.+$\frac{82}{30}$ qt.+$\frac{140}{28}$ pt.

14. 14 yd. 3 qr. 1 na.+17 yd. 2 qr. 3 na.+$\frac{76}{56}$ yd.

15. 7 m. 260 rd. 3 yd. 2 ft.+$\frac{80}{82}$ m.

16. $\frac{124}{30}$ A.+9 A. 120 rd.+10 A. $\frac{52}{13}$ rd.

17. 5 T. 18 cwt. 65 lb. 13 oz. 200 gr.+$\frac{13}{9}$ T.

18. 32 m.+4.28 fur.+320.6 rd.+1 m. 7 fur.

19. 123 sq. yd. 5 sq. ft. $112\frac{1}{8}$ sq. in.+89 sq. yd. 7 sq. ft. 100.375 sq. in.+97 sq. yd. 6 sq. ft. 97.5 sq. in.

20. 5.25 da.+$\frac{29}{35}$ h. 40 m.+2 da. 14 hr. $\frac{18}{40}$ m.

MULTIPLICATION OF COMPOUND NUMBERS.

300. MENTAL PROBLEMS.

1. What is the amount of 2 lb. 3 oz.+2 lb. 3 oz.+2 lb. 3 oz.?
2. How much cloth in 4 pieces, each containing 21 yd. 3 qr.?
3. How much grain in 10 bags, each containing 4 bu. 3 pk.?
4. How many inches in 5 pieces of wire, each 1 ft. 2 in. long?
5. How many A. in 12 lots of 5 A. 40 sq. rd. each?
6. How much water can flow through a pipe in 5 m. at 5 gal. $3\frac{1}{2}$ qt. per minute?
7. What is 6% of £4 9s.?
8. How much wood in 4 piles of 2 crd. 6 crd. ft. each?
9. How many sheets of paper in 5 packages of 1 ream, 2 quires, 16 sheets each?
10. What is the weight of 12 rings at 15 pwt. 8 gr. each?

301. WRITTEN PROBLEMS.

(*See* SIMPLE MULTIPLICATION, Art. 103.)

Ex. 1. Multiply 5 fur. 35 rd. 16 ft. 9 in. by 5.

m.	fur	rd.	ft.	in.	
	5	35	16	9	
				5	
3	5	20	$\frac{1}{2}$	9	
			$\frac{1}{2}$	-6	
3	5	20	1	3	*Ans.*

2. What is the distance round a square field, each side of which is 35 rd. 5 yd. 2 ft. 8 in. in length?

3. What is the weight of 5 watch chains, each containing 1 oz. 7 pwt. 13 gr. of gold?

4. Bought 7 loads of corn, each containing 29 bu. 3 pk. 7 qt. 1 pt.; how much corn did I buy?

5. Bought 11 pieces of broadcloth, each containing 34 yd. 1 qr. 3 na.; how many yards did I buy?

6. How much wine in 7 casks, each containing 75 gal. 3 qt. 1 pt.?

7. Multiply 17 rd. 11 ft. $9\frac{2}{3}$ in. by 156.

8. If a man can mow 2 A. 1 P. 200 sq. ft. in one day, how much can he mow in 10 days?

9. How many bu. wheat from 14 A. if the average yield be 16 bu. 19 qt. for each acre?

10. How much iron in thirteen loads of 5 cwt. 27 lbs. 10 oz. each?

11. What is the weight of 1000 silver dollars? (Art. 292, 1.)

12. If a man own 25 lots of .874 A. each, how much land has he in all?

13. If a ship sail 1° of Lat. in 6 hr. 17 m. how long would it take for it to sail from 12° S. Lat. to 10° 30′ N. Lat.?

14. How much carpeting would be required for 2 rooms of 25 yd. 3 qr. each, and 3 rooms of 18 yd. 2 qr. each?

15. If a party of men construct 1 m. 170 rd. 13 ft. of railroad in one day, how much could they construct in six days?

SUBTRACTION OF COMPOUND NUMBERS.

302. MENTAL PROBLEMS.

1. If from a bbl. of oil containing 28 gal. 3 qt. a man draw out 10 gal. 2 qt., how much will remain?

2. How much more land in a lot of 6 A. 3 R. 30 P. than in one of 5 A. 3 R. 20 P.?

3. If a man arrive in New York with £200 8s. 11d., and in one week spend £2 9s. 2d., how much is left?

4. If a druggist buy 1 lb. of camphor gum, and then sell 7 ʒ to each of 9 persons, how much would remain?

5. What is the difference between 20 yr. and 16 yr. 2 mo. 15 da.?

6. How much greater than an angle of 40° 10″ is an angle of 45° 30′?

7. If a man stored 18 bu. 7 qt. of grain in a bin, and some one stole 3 bu. 1 pk., how much was left?

8. If a man has 16 gal. of milk and sells 2 qt. 1 pt. to each of 20 families, how much remains?

9. If a man be 32 m. from home, and on one day walk 12 m. 3 fur., and the next day 10 m. 6 fur. toward home, how far away will he then be?

10. If a sounding-line be 100 fathoms long, and all but 26 fathoms 4 ft. be let out, what is the depth of the water?

303. WRITTEN PROBLEMS.

(*See* SIMPLE SUBTRACTION, Art. 119.)

Ex. 1. From 12 lb. 6 oz. take 7 lb. 9 oz. 13 pwt. 22 gr.

11	17	19	24	Minuend changed in form.
lb.	oz.	pwt.	gr.	
12	6			Minuend.
7	9	13	22	Subtrahend.
4	8	6	2	Remainder.

Ex. 2. From 3 m. 7 fur. 30 rd. take 5 fur. 38 rd. 10 ft. 9 in.

3	6	69	$15\frac{1}{2}$	12	Minuend changed in form.
m.	fur.	rd.	ft.	in.	
3	7	30		0	
	5	38	10	9	
3	1	31	$5\frac{1}{2}$	3	
			$\frac{1}{2}=$	6	
3	1	31	5	9	*Ans.*

3. From 1 m. take 4 fur. 3 rd. 4 yd. 2 ft. 6 in.
4. From 2 T. 4 cwt. take 17 cwt. 2 qr. 8 lb.
5. How long from June 12, 1855, to April 3, 1859?

	mo.	da.	
1859	4	3	
1855	6	12	
3	9	21	*Ans.*

6. How long from the signing of the declaration of Independence, July 4, 1776, to the battle of New Orleans, January 8, 1815?

7. From the battle of Lexington, April 18, 1775, to the battle of Montebello, May 5, 1859?

8. How long from the battle of Saratoga, Sept. 7, 1777, to Perry's victory, Sept. 10, 1813?

9. If from a tract of land of 740 A. there be sold to each of two men 120 A. 3 R. 15 P., and to each of two others 50 A. 21 P., how much will remain?

10. If a man travel directly south 2 m. 7 fur. 2 rd. 11 ft., then north 3 m. 10 rd. 20 ft., then south 1 m. 3 fur. 12 rd., how far will he be from the place of starting?

11. What is the difference in Lat. of Washington 38° 52′ 20″ N., and San Francisco 37° 46′ N.? (Art. 282, 4.)

12. What is the difference in Lat. of Paris 48° 50′ 72″ N., and Rio Janeiro 22° 54′ S.?

13. What is the difference in Lat. of Chicago, Ill., 41° 54′ N., and Rome, Italy, 41° 53′ 52″ N.?

14. What is the Lat. of Valparaiso, S., if the Lat. of New York be 40° 42′ 43″ N., and the difference in Lat. be 73° 46′ 43″?

15. What is the Lat. of Edinburgh, N., if the Lat. of Paris be 48° 50′ 12″ N., and the difference in Lat. be 7° 6′ 48″?

DIVISION OF COMPOUND NUMBERS.

304. MENTAL PROBLEMS.

1. How many rings of 18 pwt. each can be made from 4 oz. 10 pwt. of gold?

2. If 2 lb. 10 oz. of candy be divided equally among 5 children, how much would each receive?

3. What is $\frac{1}{3}$ of 1 m. 3 fur. 10 rd.?

4. What is $\frac{2}{3}$ of 13 C. 7 crd. ft.?

5. How many spoons of 3 oz. each could be made from 2 lb. 8 oz. of silver?

6. What is $\frac{1}{7}$ of 9 d. 1 hr. 42 m.?

7. How many rulers of 14 in. each could be made from a piece of rosewood of the proper width and 3 ft. 7 in. long?

8. If a man weave 52 yd. 2 ft. 10 in. of carpet in 10 days, how much can he weave in one day?

9. How many soles 8 in. long and 3 in. wide could be cut from a piece of leather 6 ft. long and 3 ft. 9 in. wide?

10. If 8 horses eat 3 bu. of oats in one day, how much would 8 horses eat?

305. WRITTEN PROBLEMS.

(*See* SIMPLE DIVISION, Art. 134.)

Ex. 1. Divide 1 m. 3 fur. 28 rd. 5 yd. 2 ft. 8 in. by 5.

	m.	fur.	rd.	yd.	ft.	in.
5)	1	3	28	5	2	8
		2	13	4	1	$5\frac{1}{5}$

2. A man divided 1578 acres of land equally between 7 children. What was the share of each?

3. A piece of cloth containing 36 yd. 3 qr. will make 5 suits of clothes. How much cloth in each suit?

4. Seven men purchased 8 cwt. 3 qr. 20 lb. of sugar. What was the share of each?

5. Four men agreed to share equally 3 sacks of coffee, each containing 2 cwt. 1 qr. 15 lb. What was the share of each?

6. How many rotations will a wheel make in passing over $.6\dot{6}$ of a mile if it be 11 ft. 8 in. in circumference?

7. How many curb-stones of 4 ft. 6 in. each will it take for one side of a block 40 rd. long?

8. How many baskets of 1 bu. 5 qt. each will hold 24 bu. 1 pk. 1 qt. of peas?

9. How many gold dollars can be made from 7 lb. 5 oz. 40 gr. pure gold, and how much will remain? (Art. 292, 1.)

10. What is $\frac{1}{30}$ of a solar year? (Art. 290, 2.)

11. Divide $\frac{7}{9}$ m. by $\frac{5}{8}$ rd.
12. Divide 48 cu. ft. by $7\frac{7}{12}$ cu. ft.
13. Divide £.374 by .98s.
14. Divide $\frac{2}{3}$ oz. (gold) by .38 lb.
15. Divide 6 in. by 40 rd.

306. LONGITUDE AND TIME. (Art. 291.)

1. What is the difference of time in Boston, 71° 3′ 58″ W. Long., and New York, 74° 3″ W. Long.

2. What is the difference of time in Boston, and Paris, 2° 20′ $22\frac{1}{2}$″ E. Long.?

3. When it is noon at New York it is 11 h. 6 m. 2 s. A. M. at Chicago; what is the Long. of Chicago?

4. When it is noon at New York it is 4 h. 30 m. 42 s. P. M. at Dublin; what is the Long. of Dublin?

5. When it is noon at Washington, 77° 15′ W. Long., what time is it at Sidney, Australia, 152° 20′ E. Long.?

6. What is the difference of time between New Orleans, 90° W. Long., and Calcutta, 88° 19′ 2″ E. Long.?

7. San Francisco is in 122° 23′ W. Long., and the difference of time between that place and Rome, Italy, is 8 h. 59 m. 39 s.; what is the Long. of Rome, E.?

8. What time is it at Portland, Me., when it is 3 A. M. at Astoria, Oregon, the Long. of the former place being 70° 15′ W., and of the latter 123° 48′ W.?

9. What time is it at Cape Horn when it is 4 h. 30 m. P. M. at the Cape of Good Hope, the Long. of the former place being 68° W., and of the latter place 18° 29′ E.?

10. When it is midnight at Washington, what time is it at Sitka, Alaska, the Long. of Washington being 77° 15′ W., and of Sitka 135° 18′ W.?

11. When it is noon at Greenwich, Eng., find the respective time at each of the following places, viz.:

Pekin 116° 27′ E., Jerusalem 35° 13′ E., London 5′ 48″ W., Paris 2° 20′ $22\frac{1}{2}$″ E., Berlin 13° 30′ E., New York 74° 3″ W., Chicago, Ill., 87° 38′ W., St. Augustine, Fla., 81° 35′ W., and Hamburg, 9° 58′ 33″ E.

THE METRIC SYSTEM.

307. The ***Metric System*** is the French system of denominate numbers, having for its fundamental unit the ***Meter***, and having 10 as the constant ratio in its scale of notation.

308. 1. The ***Meter*** (Fr. Metre, Greek *metron*, a measure) *is the unit of length.* It is equal to one ten-millionth of a quadrant of the polar circumference of the earth, as estimated by French astronomers.

NOTES.—1. The Meter was adopted as the standard of measure by France in 1799, and later by several other countries of Europe. The use of the Metric System in government and legal computations was authorized by the United States Congress in 1866. It is also used in most of the arts and sciences. The Meter is equal to 39.37 inches. The equivalent values given for the different units of this system are those adopted by the United States in 1866.

2. The value of all other units of this system are derived from the meter or referred to it.

2. The ***Are*** (Latin, *area*), is the *superficial unit* for the measurement of land. It is a square each side of which is 10 meters. It is equal to 119.6 sq. yds. Surfaces of small extent are computed in *square meters.*

3. The ***Stere*** (Greek, *stereos*, solid), is the *unit of volume* for the measurement of solids. It is a cube each edge of which is 1 meter. It is equal to 35.3174 cu. ft. Volume is also computed in *cubic meters* and *liters.*

4. The ***Liter*** (Fr. Litre, Greek, *litra*, a certain coin), is the unit of volume for measuring the *capacity* of vessels, computing the quantity of liquids, etc. It is a cube each edge of which is $\frac{1}{10}$ meter; hence *cubic decimeters* are equal to *liters.* The liter is equal to 61.02803 cu. in., .908 qt. dry measure, or 1.0567 qt. wine measure. A *kiloliter* constitutes the *Stere.* The *hectoliter* is used for grain.

5. The ***Gram*** (Fr. Gramme, Greek, *gramma*, a certain small weight) is the *unit of weight.* It is the weight of a cube of pure water, at its greatest density, each edge of which is $\frac{1}{100}$ of a meter. It is equal to 15.432 grains. A *liter* of water weighs 1 kilogram, or 1000 grams.

309. The names of the superior orders in this system are derived mostly from the Greek numeral adjectives, and the names of the inferior orders from the Latin numeral adjectives, the Greek ***deka*** signifying 10, ***hekaton*** 100, ***kilioi*** 1000, *myrioi* 10000, and the Latin *decimus* signifying 10th, *centesimus* 100th, and *millesimus* 1000th. Thus the following is the scale of notation for the meter, and we may observe that the methods of all operations with metric numbers are the same as for any others in a decimal system of notation.

Myriameter.	Kilometer.	Hectometer.	Decameter.	*Meter.*		Decimeter.	Centimeter.	Millimeter.
1	1	1	1	1	.	1	1	1

NOTES.—1. The names of the different orders of other units are derived in a similar manner, as may be seen in the Tables, Part Third. The second order of the *Are* is not used.

2. The *kilogram* (kilo) is the ordinary commercial unit of weight.

3. In the United States Post Office Department one-half oz. is reckoned equal to 15 grams.

4. The new nickel United States *fice cent* coin weighs 5 grams, and is 2 centimeters or $\frac{1}{50}$ of a meter in diameter; the silver *franc* also weighs 5 grams.

310. 1. The following approximate equivalents may be used for ordinary reduction to common denominations when great accuracy is not required:

1	millimeter	=	about	$\frac{1}{25}$ inch.
1	decimeter	=	"	4 in.
1	meter	=	"	1.1 yd.
5	meters	=	"	1 rd.
1	decameter	=	"	.5 chain.
1	kilometer	=	"	.625 mile or 200 rd.
1	centiare	=	"	1.2 sq. yd.
1	hectare	=	"	2.5 acres.
1	liter	=	"	.375 gal. or $1\frac{1}{18}$ qt.
1	hectoliter	=	"	$2\frac{5}{6}$ bu. or $\frac{5}{6}$ bbl.
1	kilogram	=	"	2.2 lb. Avoir. or 32 Troy ounces.
$1\frac{1}{60}$	tonneaux	=	"	2240 lb.
1	tonneaux	=	"	2200 lb.

2. Metric numbers may be reduced to ordinary denominations more accurately by using the following multipliers or their reciprocal divisors, and ordinary denominations may be reduced to metric units by the reverse process:

To reduce	multiply by	or divide by
Meters to inches,	39.3685	.0254
Kilometers to miles,	.62135	1.6094
Sq. meters to sq. ft.,	10.763	.09291
Ares to sq. rd.,	3.953	.2529
Hectares to acres,	2.4709	.4047
Liters to qt.,	1.05656	.9465
Hectoliters to bu.,	2.837	.3524
Liters to cu. in.,	61.012	.01639
Steres to cu. yd.,	1.3078	.7646
Grams to grains,	15.432	.9648
Kilograms to lb. Av.,	2.206	.4536
Tonneaux to T. of 2000 lb.,	1.103	.9066

(For more extended tables see the American Annual Cyclopædia for 1866, from which the above tables of reduction are taken,)

311. MENTAL PROBLEMS.

1. How many meters in a decameter? A kilometer? In 500 decimeters? 80 centimeters?
2. How many centimeters in 92 meters? In 2.75 meters? In 42 decameters?
3. How many inches in 1000 meters? How many kilometers? How many centimeters?
4. How many ares in 4 hectares? In 785 centiares? How many sq. rd. in 10 ares?
5. How many liters in a hectoliter? A decaliter? In 700 centiliters?
6. How many grams in 400 decigrams? In 750 centigrams? In .2 kilogram?
7. What is the amount of 30 decagrams and 15 grams?
8. What is the difference between 95 myriameters and 250 kilometers?

9. If a man have 7 francs and 150 centimes, and spend 500 centimes, how much will he have left?

10. How many kilograms would 1000 francs weigh? (Art. 309, 4.)

312. WRITTEN PROBLEMS.

1. How many kilometers in 7280 meters, 985 decameters, 200500 centimeters, and 42 myriameters?

2. How many acres in 549 ares, 67250 centiares, and 48 hectares?

3. How many lb. in 25 boxes of goods, each weighing 76 kilograms and 450 grams?

4. How many sq. meters on a floor 2 decameters long and 18 meters wide?

5. How long would it take a ship to go 1800 miles if it went 150 kilometers in one day?

6. How many bushels in 325 hectoliters of wheat?

7. How many liters in 63 gal. of wine?

8. How many kilograms in a bbl. of flour?

9. How many cords in 7520 steres of wood

10. How many five cent pieces of 5 grains each could be coined from 25 lb. avoir. of nickel, and how many milligrams would remain?

DUODECIMALS.

313. ***Duodecimals*** (Art. 281, 1) are added and subtracted like other compound numbers.

314. In determining the relative value of the terms of the product in multiplication of duodecimals, every duodecimal may be regarded as a common fraction, having some power of 12 for a denominator. Thus $5' = \frac{5}{12}$ of a foot. $5'' = \frac{5}{12 \times 12} = \frac{5}{144}$. Hence $5' \times 5'' = \frac{5}{12} \times \frac{5}{12 \times 12} = \frac{25}{12 \times 12 \times 12}$, or $25'''$

In a similar manner it may be shown that the index of the product of any two orders of duodecimals will be equal to the sum of the indices of the two factors.

315. EXAMPLES.

1. How many sq. ft. in a board 9 ft. 5 in. long and 2 ft. 8 in. wide?

9 ft.	5′	$5' \times 8' = 40'' = 3'\ 4'' = \frac{5}{12} \times \frac{8}{12}$
2 ft.	8′	$9 \text{ ft.} \times 8' = 72' = 6 \text{ ft.} = 9 \times \frac{8}{12}$
6	3′ 4″	
18	10′	$5' \times 2 \text{ ft.} = 10' = \frac{5}{12} \times 2$
25 sq. ft.	1′ 4″	$9 \text{ ft.} \times 2 \text{ ft.} = 18$ sq. ft. (Art. 284, 2.)

NOTE.—The indices of the multiplier are retained for convenience; it is really to be considered as an abstract number.

2. Multiply 12 ft. 8′ by 4 ft. 10′.
3. Multiply 4 ft. 6′ 4″ by 8 ft. 8′.
4. Multiply 10 ft. 6′ 6″ by 4′ 8″.
5. How many square feet in a board 12 ft. 9 in. long and 1 ft. 1′ 4″ wide?
6. How many cubic inches in a block 2 ft. 9′ long, 1 ft. 3′ wide, and 2 ft. 4′ high?
7. Required the solid contents of a block 4 ft. 4′ long, 2 ft. 3′ wide, and 10′ high?
8. How many square feet in 60 boards, each board being 5 ft. 4′ long, and 1 ft. 2′ wide?
9. How many feet of lumber will it take for the floors of 8 rooms, each 15 ft. 8′ 6″ long, and 12 ft. 7′ 2″ wide?
10. How many sq. ft. in the walls and ceiling of a room that is 20 ft. 9′ long, 18 ft. 5′ 4″ wide, and 12 ft. 6′ high, making no allowance for windows or doors?

NOTE.—When the multiplier is a simple number, proceed as in multiplication of compound numbers.

316. In Division of Duodecimals the same general principle is observed as in multiplication, and the dividend may be considered as a product.

Thus, divide 10 sq. ft. 2′ 10″ by 5 ft. 7′ (placing the divisor at the right of the dividend for convenience).

10 sq. ft. 2′ 10″	5 ft. 7′ divisor,	
5 7′	1 ft. 10′ quotient,	
4 7′ 10″		
4 7′ 10″		

7

The first term of the quotient is readily found. In determining the second term, the 4 ft. may be considered as 48′, which, combined with 7′, makes 55′, and this is equal to 5 ft. × 11′; but since some allowance must be made for the product of 7′ × 11′, we see that only 10′ can be placed in the quotient, and multiplying 5 ft. 7′ by 10′ we get 4 ft. 7′ 10″.

EXAMPLES.

1. Divide 87 sq. ft. 6′ 2″ 4‴ by 6 ft. 5′.
2. If the volume of a room be 9604 cu. ft. 3′ 6″, and it be 30 ft. 4′ long, and 25 ft. 6′ wide, how high is it?
3. If it cost $12.75 to plaster the wall of a room each side of which is 15 ft. 8′, at 20 cts. per sq. yd., what is the height of the room?
4. How many yards of carpet 2 ft. 10′ wide will be required for a room that is 20 ft. 6′ long, and 15 ft. 6′ wide?
5. Divide 2 sq. ft. 7′ 8″ by 10′ 11″.
6. Divide 1870 sq. ft. 10′ by 27 ft. 2′.
7. Divide 786 cu. ft. by 20 sq. ft. 8′ 8″.
8. From the sum of 29 sq. ft. 11′, and 14 sq. ft. 9′ 10″, take the product of 5 ft. 7′ and 3 ft. 10″.
9. To 15 times 8 ft. 5′ 6″ add the quotient of 89 sq. ft. 11′ 8″ divided by 15 ft. 10′.
10. Divide 18 cu. ft. 7′ by 20 sq. ft. 2′.

NOTE.—When the divisor is a simple number, proceed as in division of compound numbers.

317. MISCELLANEOUS PROBLEMS.

Ex. 1. What will .65 of a ream of paper cost at 20 cents a quire?

2. What will $\frac{4}{5}$ of a ream of paper cost at $\frac{3}{4}$ of a cent per sheet?

3. What will $\frac{3}{5}$ of a barrel of beef cost at $6\frac{1}{4}$ cents a pound?

4. What must be the height of a wood-bed that is 12 feet long and $3\frac{1}{2}$ feet wide to hold just one cord?

5. What will it cost to excavate a cellar $18\frac{1}{2}$ feet long, $15\frac{1}{4}$ feet wide, and 9 feet deep, at 20 cents per cubic yard?

6. How many cords of wood in a pile 40 feet long, $7\frac{1}{2}$ feet high, and 4 feet wide?

7. What will .75 of a hhd. of wine cost at 75 cents a pint?

8. Bought 12 barrels of flour at $6.50 per barrel, and sold the same at retail at 4 cents a pound. How much did I gain?

9. The cabin of the steamer Bostona is 165 feet long and 18 feet wide. What will it cost to carpet the same with Brussels carpeting $\frac{3}{8}$ of a yard wide at 80 cents per lineal yard?

10. At 25 cents a sq. yd., what will it cost to plaster the ceiling of a room $18\frac{1}{2}$ feet long and 16 feet wide?

11. At 20 cents a sq. yd., what will it cost to plaster both sides of a partition wall 52 feet long and $13\frac{1}{2}$ feet high, and another wall 149 feet long and 11 feet high?

12. A gentleman's garden 200 feet long and 180 feet wide is enclosed by a tight board fence $5\frac{1}{2}$ feet high. What will it cost to paint the fence at 10 cts. per sq. yd.?

13. How many bricks, each being 8 in. long and 4 in. wide, will it take to surround the above garden with a walk 6 feet in width? What will be the cost of the bricks at $4 per thousand?

14. A miller ground 5000 bushels of wheat, taking from each bushel 4 quarts of wheat as toll. How many bushels of wheat does he grind for his customers, and what does he receive for the work, wheat being worth $87\frac{1}{2}$ cents a bushel?

15. What will be the cost of 25 boards, each being 15 ft. long and 10 in. wide, at $30 per thousand?

16. What cost 9 cwt. 1 qr. 18 lbs. 12 oz. at $6.40 per cwt.?

17. What will 10 lbs. 8 oz. 8 pwts. of gold cost at $300 per pound?

18. What will $3\frac{3}{4}$ hhds. of molasses cost at 10 cents per quart?

19. What will be the cost of papering the walls of a room 40 feet long, 30 feet wide, and 9 feet high, at 30 cents a bolt, each bolt being 9 yards long and 18 inches wide?

20. A farmer sold 30 bu. 2 pks. 1 qt. $1\frac{5}{8}$ pts. of clover seed at $3.60 per bushel. How much did he receive?

21. How many bushels of coal will a boat 100 feet long, 42 feet wide, and 4 feet deep contain, a bushel of coal being $1\frac{3}{8}$ of a cubic foot?

22. If there are 6 yds. 3 qrs. 2 na. in one suit of clothes, how many yards will clothe an army of 128,000 men?

SECTION XIV.

RATIO AND PROPORTION.

RATIO.

318. The comparison of numbers is the foundation of all arithmetical problems and of their solution.

In order to compute any equivalent, sum, difference, product or power, divisor or factor, one or more *comparisons* must be made.

From a single comparison of two terms we may find either their Arithmetical Ratio (difference) or their Ratio (quotient).

Two terms that are compared are called a *couplet;* the first is called the *antecedent,* the second is called the *consequent.*

319. In Arithmetical Ratio the consequent is always the smaller term, and the value of the ratio equals the antecedent minus the consequent.

320. MENTAL PROBLEMS.

1. What is the arithmetical ratio of 27 and 19? 400 and 240? 60 and 15? 81 and 21? 70 and 90? 120 and 80?

2. What is the consequent if the antecedent be 45 and the difference 17? 80 and 70? 120 and 45? 79 and 19? 111 and 43?

3. What is the antecedent if the consequent be 19 and the difference 20? 600 and 200? 99 and 100? 230 and 88? 112 and 79? 490 and 157?

321. 1. In ***Ratio*** either term may be used as the standard of comparison, and the value of the ratio is the quotient arising from measuring or dividing by the term used as the standard. In this book the consequent is regarded as the standard.

NOTE.—The consequent is regarded as the divisor or standard by Greenleaf, Walton, White, Stoddard, and others, while the antecedent is regarded as the divisor by Davies, Felter, Robinson, Smith, and others. The former is known as the English method, and the latter as the French method

2. The value of every ratio is really determined by taking 1 as a standard of measure for each of the terms. Thus 18 : 6 equals

$\frac{18}{6}:1=3:1$; that is, 18 contains 3 times as many units as 6, or 18 equals 3 units of 6 each. $7:14=\frac{7}{14}:1=\frac{1}{2}:1$.

3. The colon, used as the sign of ratio, is thought by some to be a part of the sign of division, the horizontal line of the sign $\div$ being omitted. This seems probable if the consequent be used as the divisor, or as the denominator, when the ratio is expressed in the form of a fraction.

4. When the value of a ratio is one, it is called a ratio of *equality;* when it is greater than *one* it is called an *increasing* ratio, or a ratio of greater inequality; when it is less than one it is called a *decreasing* ratio, or a ratio of lesser inequality.

5. Only numbers of the same kind can be compared, and the value of the ratio is always abstract, and yet concrete numbers of different kinds may be compared by regarding them both as abstract numbers, and the value of a ratio may be applied to some concrete unit.

6. If the ratio equals the antecedent divided by the consequent, the antecedent must equal the consequent multiplied by the ratio, and the consequent must equal the antecedent divided by the ratio.

322. Ratios are of two kinds—

1. A *Simple Ratio* is the ratio of any two integral or fractional numbers; as, 18 : 16; 4 lb. 8 oz. : 1 lb. 2 oz.; $\frac{4}{5}:\frac{2}{3}$; .45 : .9, etc.

2. A *Compound Ratio* is the product of two or more simple ratios, and may be expressed in three ways, viz.: $(5:4)\times(3:2)\times(3\div4)$; $\frac{5}{4}\times\frac{3}{2}\times\frac{3}{4}$; or,

$$\begin{array}{c}5:4\\3:2\\3:4\end{array}$$

NOTE.—Numbers used as the terms of a compound ratio can only be regarded as abstract. Sometimes concrete numbers are improperly used thus for convenience. Thus we may ask the ratio of a box 20 inches long, 6 in. wide, 4 in. deep, to one 10 inches long, 5 in. wide, 3 in. deep, and, for convenience, express it by

$$\begin{array}{r}20:10\\6:\ \ 5\\4:\ \ 3\end{array}$$

comparing length with length, etc., but properly the cu. in. in the first should be compared with the cu. in. in the second, giving only the *simple* ratio, 480 : 150.

3. The *reciprocal* of a ratio (Art. 183) is sometimes called an *inverse* ratio. Thus the reciprocal of 12 : 18 is 18 : 12, and the latter is called the inverse ratio of 12 to 18.

REDUCTION OF RATIOS.

323. 1. A simple ratio may be reduced to its lowest terms by dividing both terms by their G. C. D. Thus 18 : 42 becomes 3 : 7, and $\frac{12}{20}$: $\frac{4}{5}$ becomes 3 : 4.

2. A compound ratio may be reduced to a simple ratio by taking the product of all the antecedents for a single antecedent, and the product of all the consequents for a single consequent. Thus

$$\left.\begin{matrix} 4:7 \\ 10:3 \\ 9:5 \end{matrix}\right\} \text{becomes } 360:105.$$

324. MENTAL PROBLEMS.

1. 4 : 7? 3 : 6? 45 : 15? 2 : 6? 15 : 45? 84 : 12?
2. $\frac{2}{3}$: $\frac{3}{4}$? $2\frac{1}{2}$: $\frac{5}{8}$? $\frac{2}{5}$: $\frac{7}{10}$? 1.5 : 45? 72 : .9?
3. 3 : 6? 10 : 2.5? 975 : 100? 1.6 : 8? .49 : 7?
4. $13 : $91? $4 : $.25? $2.25 : $.75? $1. : $.01?
5. 10 ft. : 2 ft. 6 in.? 2 m. : 80 rd.?

   14 oz. : 1 lb. 12 oz.? $\left.\begin{matrix} 3:7 \\ 8:2 \end{matrix}\right\}$? $\left.\begin{matrix} \frac{2}{3}:\frac{1}{2} \\ \frac{1}{3}:\frac{2}{3} \end{matrix}\right\}$?

Other similar problems may be given.

325. WRITTEN PROBLEMS.

1. $4960 : $24.80? 920 cu. ft. : 1440 cu. in.?
2. Pop. of New York, 926,341 : pop. of Boston, 250,526?
3. Pop. Philadelphia, 674,022 : pop. San Francisco, 149,482?
4. Pop. Chicago, 298,983 : pop. San Francisco?
5. Pop. New Orleans in 1870, 191,322 : pop. in 1860, 168,675? What per cent. increase?
6. Pop. St. Louis, 310,864 : pop. Washington, 109,204?
7. Pop. Kansas City in 1870, 32,260 : pop. in 1860, 4,418? What per cent. increase?
8. R. R.'s in N. Y. in 1870, 3,928 m. : 2,682 m. in 1860?
9. R. R.'s in Ill. in 1870, 4,823 m. : R. R.'s in Ill. in 1860, 2,790 m.?
10. Miles R. R. in Ill. 1870 : miles R. R. in N. Y. 1870?

11. *Number* of inhabitants in N. Y., 4,364,411 : the ***number*** of miles R. R. in N. Y. in 1870 ?

12. Number of pop. Penn., 3,515,993 : number of miles R. R. in Penn. in 1870, 4,656 ?

13. Number of pop. Ill., 2,529,638 : number of miles R. R. in Ill. in 1870, 4,823 ?

14. Number of miles R. R. in N. Y. in 1870 : number of sq. m. in N. Y., 47,000 ?

15. Number of miles R. R. in Ill. in 1870 : area of Ill., 55,410 sq. m. ?

16. If the antecedent be $7980, and the ratio 41.5, what is the consequent ?

17. If the consequent be 118 yd., and the ratio $\frac{1}{3}$, what is the antecedent ?

18. If the ratio be $\frac{4}{15}$, and the antecedent 172, what is the consequent ?

19. If the ratio be 70, and the consequent $4.75, what is the antecedent ?

20. If the antecedent be 400, and the ratio be equal to the consequent, what is the ratio ?

PROPORTION.

326. 1. In all comparisons of numbers a unit is the real standard. Thus $5+7=12$; that is, 12 contains the same number of units as 5 *and* 7; $7\times5=35$; that is, 35 contains the same number of units as five 7's; $7-5=2$; that is, 7 contains 2 more units than 5; $7\div5=\frac{7}{5}$; that is, 7 contains seven-fifths as many units as 5.

2. An *equation of ratios*, that is, an expression for an equality of ratios, is called a ***Proportion***. Thus $12:4=18:6$, is a proportion. Proportion is generally indicated by two colons, : :, which are regarded by some as the extremities of the lines which make the sign of equality. Thus $24:6::40:10$, and it is read, 24 is to 6 as 40 to 10, and the four terms of the two couplets are said to be in *Geometrical Proportion.*

3. The antecedents of the couplets are the antecedents of the proportion, and the consequents of the couplets are the consequents of the proportion.

4. The first and last terms are called the *extremes*, and the middle terms are called the *means*.

5. Any four numbers that can be formed into a proportion are called *proportionals*; as, 84, 12, 77, and 11, because 84 : 12 :: 77 : 11. Any three numbers that can be formed into a proportion by repeating one of them for the two means, are also called proportionals; as, 4, 32, and 256, because 4 : 32 :: 32 : 256. The number used for the middle terms is called the *mean proportional* of the other two, thus 32 is the *mean* of 4 and 256.

6. Both ratios of a proportion must be of the same kind, that is, both *increasing*, or both *decreasing*, otherwise they cannot be equal. Hence, in every proportion, if the first term be *less* than the second, the third term must be *less* than the fourth, and if the first term be *greater* than the second, the third term must be *greater* than the fourth.

327. 1. In an *Arithmetical Proportion*, if a number equal to each consequent be added to both sides of the equation, the form of it will then indicate a method of finding any one term when the others are given. Thus in $17-5=28-16$, if 5 and 16 be added to both members of the equation, it becomes $17+5-5+16=28+16-16+5$, or since $5-5=16-16=0$, these terms may be omitted, and we have $17+16=28+5$.

2. As this may be shown for any arithmetical proportion, we conclude that *the sum of the extremes equals the sum of the means* of an arithmetical proportion, the terms being regarded as abstract.

3. How might either extreme be found? $18-15=27-(d)$? $(a)-16=40-25$?

4. How might either mean be found? $49-(b)=52-20$? $60-40=(c)-70$?

5. If the means were equal, how might they be found? $19-(b)=(b)-11$?

6. If the extremes be equal, $(a)-17=24-(a)$?

328. EXAMPLES.

1. A man had at first \$10,000; he lost \$4,000, and then had as much as his partner, who had \$12,000 at first; how much did his partner lose?

2. One city lost 500 inhabitants in one year, and then had as

many as another which had at first 22,000 and lost 250; how many had the first before losing any?

3. At first A had \$25,000; he lost as much money in one year as B put into business, and B had at the end of the year \$7,500, having lost \$2,500; how much did A lose?

4. A put in as much as B lost. In one year A lost \$3,500. B put in \$12,500, and at the end of the year both had an equal amount. What did A put in?

5. Make three correct substitutions in each of the blanks in the following: ( ) $-$ \$1800 $=$ \$1800 $-$ ( ).

329. 1. In any *Geometrical Proportion*, if one of the terms be missing, it may be found in accordance with the general relations of terms in division. Thus if in the proportion $42 : 14 :: 60 : (20)$, the last term be missing, we know that $42 \div 14 = 60$ divided by the required term; that is, the *quotient* 3 and the *dividend* 60 are known, and the divisor is required, hence the divisor must be $60 \div 3 = 20$. The complete operation may be expressed by the equation $60 \div \frac{42}{14} = 20$, or (Art. 185) $\frac{60 \times 14}{42} = 20$. Now observe that 60×14 is a *dividend*, 42 a *divisor*, and 20 a *quotient* in this last equation, hence $60 \times 14 = 42 \times 20$; that is, the product of the means in this proportion equals the product of the extremes.

2. The same fact may be shown by multiplying both members of the equation, $42 \div 14 = 60 \div 20$, by factors equal to the consequents. Thus $42 \div 14 \times 14 \times 20 = 60 \div 20 \times 20 \times 14$, or better, $42 \times 20 \times (14 \div 14) = 60 \times 14 \times (20 \div 20)$; then as $14 \div 14 = 1$, and $20 \div 20 = 1$, these terms may be eliminated or omitted, and $42 \times 20 = 60 \times 14$; or, by cancellation, $\frac{42 \times \cancel{14} \times 20}{\cancel{14}} = \frac{60 \times \cancel{20} \times 14}{\cancel{20}}$.

NOTE.—1. It will be seen that these relations are only numerical, and exist between the terms of a proportion considered only as abstract numbers, and this agrees with the principle that the value of a *ratio* is really *abstract*. Thus 54 men : 27 men :: \$10. : 5.; but it cannot be true that the product of 54 men and \$5. equals the product of 27 men and \$10., and yet $54 \times 5 = 27 \times 10$.

2. Whenever in the solution of problems by proportion, the first term is used as a *divisor* of the product of the two means, it is *not* used as a term of a *ratio*, but as a term of a *proportion*. *It* is one of two *factors*, whose product is equal to that of two other known factors, viz., the 2d and 3d terms of the proportion, which themselves are not treated *together* as terms of

their respective *ratios, for as such,* they are not related and cannot be combined.

3. As the above may be shown for *any proportion,* we conclude that *the product of the means of any proportion, equals the product of the extremes, the terms being considered as abstract.*

4. Any missing term of the four may be regarded as one of two factors whose product is known; therefore,

1st. *If the product of the two means of a proportion be divided by either extreme, the quotient will be the other extreme.*

2d. *If the product of the two extremes of a proportion be divided by either mean, the quotient will be the other mean.*

5. EXAMPLES.

Find the missing term in each of the following proportions:

1. 15 : 20 :: 90 : (   ).
2. (   ) : 16 :: 90 : 20.
3. 45 : 90 :: (   ) : 28.
4. 27 : (   ) :: 108 : 12.
5. $\frac{1}{3}$: $\frac{5}{4}$ :: $\frac{3}{4}$: (   ).
6. $2\frac{1}{2}$: (   ) :: $\frac{3}{8}$: 4.
7. $\frac{2}{3}$: $\frac{3}{4}$ :: (   ) : $\frac{5}{6}$.
8. 2.5 : 62.5 :: 15 : (   ).
9. 3.6 : 7.2 :: (   ) : 9.4.
10. $2\frac{1}{3}$: $7\frac{1}{2}$ :: $\frac{1}{4}$: (   ).
11. $\frac{1}{3}$: $\frac{1}{12}$ :: (   ) : $\frac{1}{6}$.
12. $\frac{1}{3}$: $\frac{1}{3}$ :: $\frac{1}{4}$: (   ).

330. 1. A ***Simple Proportion*** is an equality of two simple ratios; as, 17 : 51 :: 90 : 270.

2. A ***Compound Proportion*** is an equality of a compound ratio and a simple ratio; as, $\begin{array}{r}12 : 6\\ 48 : 16\end{array}$ :: 90 : 15; or an equality of two compound ratios; as, $\left.\begin{array}{r}25 : 5\\ 80 : 20\end{array}\right\} :: \left\{\begin{array}{l}36 : 18\\ 70 : 7.\end{array}\right.$

3. Besides the relations of *magnitude* between the terms of ratios in a proportion, as determined by comparison, there exist also natural logical relations, which may be called relations of *cause* and *effect*, and relations of *condition* and *the conditioned.*

4. A *cause* is either *efficient,* that is, *producing* an effect, or *satisfactory,* that is, *sufficiently accounting for* an effect.

5. A *condition* is anything that *limits* or *qualifies* a cause or an effect.

NOTE.—Any problem whose terms are related as above described, may be readily solved by a proportion if the required term be *simple*

6. An ***Inverse Proportion*** is one in which as a *cause increases* the *effect decreases ;* or one in which the same condition that *decreases an effect* with a given cause, renders a *greater cause* necessary to produce a given effect. (Art. 331, Ex. 1.)

7. The terms of the two members of a proportion may consist of the following elements, there being eight different cases possible; four simple and four compound:

			(Ratio of) (Ratio of)
8. 1*st case.*—Simple.			*Two causes* :: *two effects.* Conditions being equal or 0.
2*d*	"	"	*Two causes* :: *two conditions.* Effects being equal.
3*d*	"	"	*Two conditions* :: *two effects.* Causes being equal.
4*th*	"	"	*Two conditions of causes* :: *two conditions of effects.* Equal causes and equal effects.
5*th*	"	Compound.	*Conditioned causes* :: *two effects.*
6*th*	"	"	*Two causes* :: *conditioned effects.*
7*th*	"	"	*Conditioned causes* :: *conditioned effects.*
8*th*	"	"	*Several conditions of causes* :: *several conditions of effects.*

The last case having equal causes and equal effects.

NOTES.—1. The two terms of a *ratio* should be of the same kind, but when the terms of a *proportion* are considered as abstract numbers they may be transposed or otherwise changed in any manner not inconsistent with Art. 329, 3. Thus the proportion $a : b :: c : d$, may be changed as follows:

1. $c : d :: a : b$. 2. $b : a :: d : c$.
3. $a : c :: b : d$. 4. $c : a :: d : b$.

2. The couplets in a compound proportion may be changed from one member to the other by inverting them. Thus,

$$\left.\begin{matrix} a : b \\ c : d \end{matrix}\right\} :: \left\{\begin{matrix} e : k \\ l : m \\ s : t \end{matrix}\right. \quad \begin{matrix} \text{may become} \\ " \\ " \\ \\ \end{matrix} \quad \begin{matrix} a : b \\ c : d \\ k : e \\ m : l :: s : t. \end{matrix}$$

3. Equal multiples or equal factors of the terms of a couplet may be used instead of the original terms.

9. ILLUSTRATIONS.

1st Case.—If 10 men earn \$32. in a week, how much will 15 men earn in a week, the men being of equal average ability, and working the same number of hours per day?

1st cause : 2d cause :: 1st effect : 2d effect.

That is, 10 : 15 :: 32 : (48).

2d Case.—If 18 men cut 8 crd. wood in 3 hr., in how many hours would 24 men cut the same amount?

NOTES.—1. The greater the effect produced in a unit of time, the less the number of units of time conditional to a certain effect produced by a greater cause. 18 : 24 :: (2¼) : 3, or 24 : 18 :: 3 : (2¼).

2. *Time* may be regarded as a condition of either the cause or the effect. (Compare the illustration of Case 2d with the illustration of Case 5th.)

3d Case.—If 4 houses afford \$5800. rent in 5 yr., how much would they afford in 7 yr. at the same rate?

5 : 7 :: 5800 : (8120).

4th Case.—If a man can trade 84 gal. of spirits 80½ proof, for 3 bbl. of sugar of grade 7, of what grade can 3 bbls. of sugar be procured for 84 gal. of spirits 92 proof?

80.5 : 92 :: 7 : (8).

5th Case.—If 5 men earn \$12.50 in 10 hr., how much can 8 men of the same ability earn in 9 hr.?

$$\begin{array}{l} 5 : 8 \\ 10 : 9 :: 12.50 : (18). \end{array}$$

6th Case.—If 12 boys can pick 3 cases of berries, each containing 8 small boxes, in 20 minutes, how many such boys could pick 6 cases, each containing 12 small boxes, in the same time?

$$\begin{array}{lcl} 3 : 6 & & 3 : 6 \\ 12 : (36) :: 8 : 12, & \text{or} & 8 : 12 :: 12 : (36). \end{array}$$

NOTES.—1. The operation of solving the proportion may be shortened by cancellation. (Art. 152.)

$$\overset{3}{\cancel{12}} : (36) :: \begin{array}{c} \cancel{3} : \cancel{6}^{2} \\ \underset{4}{\cancel{8}} : 12 \end{array}, \text{ or } \frac{\overset{2}{\cancel{6}} \times \overset{3}{\cancel{12}} \times 12}{\cancel{3} \times \underset{4}{\cancel{8}}} = 36 \quad \textit{Ans.}$$

2. Observe that when a *mean* is required the *extremes* are the factors of a dividend, but when an *extreme* is required the *means* are the factors of a dividend.

7th Case.—If, in a certain time, 4 engines of 10 horse-power, having a speed of 700, can saw 140 cords of wood of 60 degrees hardness, making 4 cuts, how many cords of wood of 70 degrees hardness can be sawed by 7 engines of 12 horse-power, having a speed of 800, and making 3 cuts, in the same time?

$$\left.\begin{array}{r}4:7\\10:12\\700:800\end{array}\right\} :: \left\{\begin{array}{l}140:(384)\\60:70\\4:3\end{array}\right. \quad\Bigg|\quad \text{or}\ \begin{array}{l}4:7\\10:12\\700:800\\70:60\ \text{(Art. 330, 8, Note 2.)}\\3:4\quad ::140:(384),\end{array}$$

$$\text{or}\ \frac{\cancel{7}\times\overset{\cancel{3}}{\cancel{12}}\times\overset{8}{\cancel{800}}\times\overset{6}{\cancel{60}}\times 4\times\overset{2}{\cancel{140}}}{\cancel{4}\times\cancel{10}\times\underset{\cancel{100}}{\cancel{700}}\times\cancel{70}\times\cancel{3}}=384\ \textit{Ans.}$$

8th Case.—If 10 men in 6 days of 10 hrs. each, dig 2 cellars, each 40 ft. long, 27 ft. wide, 4 ft. deep, in ground of 70 degrees hardness, of what width can 2 cellars, each 40 ft. long and 5 ft. deep, be dug by 10 men in 10 days of 8 hr. each, in ground of 80 degrees hardness?

$$\left.\begin{array}{r}6:10\\10:\ 8\end{array}\right\} :: \left\{\begin{array}{l}4:5\\27:(25\frac{1}{5})\\70:80\end{array}\right. \quad \text{or}\ \frac{\cancel{10}\times\cancel{8}\times\overset{2}{\cancel{4}}\times\overset{9}{\cancel{27}}\times\overset{7}{\cancel{70}}}{\underset{\cancel{2}}{\cancel{6}}\times\cancel{10}\times 5\times\underset{\cancel{8}}{\cancel{80}}}=\frac{126}{5}=25\tfrac{1}{5}.$$

SIMPLE PROPORTION.

331. 1. A ***Simple Proportion*** is an equality of two simple ratios.

2. The method of finding the fourth term of a simple proportion, the other three being given, or of solving problems by means of a simple proportion, is sometimes called the *Rule of Three.*

3. In stating a problem in simple proportion, the first and second terms must be of the same denomination; also the third and the fourth.

NOTE.—It is generally more convenient to regard the term required as the fourth term.

Ex. 1. If 5 men can do a piece of work in 18 days, how many men can do it in 10 days?

STATEMENT.

10 days : 18 days :: 5 men : *Ans.*

$$\begin{array}{r} 5 \\ \hline 10)90 \\ \hline 9 \end{array}$$ men, 4th term, or *Ans.*

Explanation.—The answer (or fourth term) is to be in men, therefore 5 men is the third term. If 5 men can do a piece of work in 18 days, it will require more men to do the same work in 10 days (less time). Hence the second ratio is *increasing*, and the first must be increasing, or 18 days must be made the second term. Hence, 10 days : 18 :: 5 men : *Ans.* or 9 men. Or, arranging the terms according to their logical relations (Art. 330, 3 and 8, case 2d),

5 : (9) :: 10 : 18.

4. RULE.

Place the number of the same denomination as the answer sought for the third term. If the answer is to be GREATER *than the third term, place the greater of the other two numbers for the second term, and the less for the first; if the answer is to be less than the third term, place the* LESS *of the two numbers for the second term, and the greater for the first, or arrange the terms according to their logical relations.*

Then divide the product of the means by the extreme known, or divide the product of the extremes by the mean known; the quotient will be the term required.

EXAMPLES.

2. If 5 peaches cost as much as 7 apples, how many apples can you buy for 35 peaches?

3. What will 450 feet of lumber cost at $17 per thousand?

4. If 150 cows cost $1800, how many cows can be bought for $132?

5. If 5 men can mow 8 acres of grass in one day, how many men can mow 32 acres in the same time?

6. If a horse travels 15 miles in 1 h. 40 m., how far, at this rate, can it travel in 12 hours?

7. If a 5 cent loaf of bread weigh 4 ounces when flour is $4 per barrel, what should be the weight of a loaf when flour is $7.50 per barrel?

8. If 5 yards of cloth cost $17, how many yards can be bought for $102?

9. A man received $45 for 30 days' work, how much should he receive for 25 days' work?

10. If 12 oz. of pepper cost 20 cents, what will 7 lbs. of pepper cost?

11. A merchant failing can pay but 70 cents on each dollar of his indebtedness. He owes A $1690, B $2000, and C $1100; what will each receive?

12. A merchant failing owes A $900, B $1200, C $1400, and D $1500. His property is valued at $2800; what will each creditor receive?

13. A man paid $7.50 for berries at the rate of $1.20 for 30 qt.; how many did he buy?

14. When 41 T. of coal can be bought for $348.50, how much can be bought for $900?

15. If the expense of a family of 3 persons be $86.57 for 4½ weeks, what would it be for 183 days?

16. If 9 men sell $42000 worth of goods in 16 days, how many men could sell the same amount in 48 days, at the same rate?

17. How long will it take a man to pay a debt of $7560 if he pay $320 in seven months?

18. If a man trade 720 bu. oats for 700 bu. corn, how much corn could be procured for 1000 bu. oats?

19. If 7 schooners carry 1,575,000 ft. lumber, how much would 19 carry?

20. How many bushels of wheat @ $1.06¾ will pay for 5000 bu. corn @ $.45½?

21. If butter be worth 18 cts. per lb., and 42 lb. sugar be exchanged for 30 lb. butter, what is the price of the sugar per pound?

22. If a locomotive run 85 miles in 4¼ hr., in what time could it run 222 miles at the same rate?

23. If a post 12 ft. high casts a shadow of 7 ft. in length at noon, how high is a steeple that casts a shadow of 85 ft. at the same time?

24. If the shares of two partners in the profits of their business one month be $275 and $350, what would be their shares when

the entire profits were $550 if divided in the same ratio as before?

25. If $1\frac{3}{4}$ hhd. of molasses cost $40., what would $3\frac{1}{4}$ hhd. cost?

26. If $3000 in gold be worth $3363.75 in currency, what amount of gold can be purchased for $3500 in currency?

27. If 85% of the circulation of a certain bank be $225,000, what is the amount of its entire circulation?

28. Construct and solve three different problems from the following statement: In $11\frac{1}{2}$ days 25 men earn $690., and in 18 days 31 men earn $1339.20.

COMPOUND PROPORTION.

332. 1. A ***Compound Proportion*** is an equality of two compound ratios, or of a compound ratio and a simple one.

2. In solving problems in Compound Proportion, sometimes called the Double Rule of Three, it is more convenient to make the second ratio always *simple*. The first ratio may be reduced to a simple ratio by multiplying the antecedents together for a new antecedent, and the consequents together for a new consequent. Hence, every compound proportion may be reduced to a simple one.

3. The third term of a compound proportion may be of the same denomination as the answer sought, and each of the simple ratios that compose the compound ratios must be of like denominations.

Ex. 1. If 5 men can mow 20 acres of grass in 3 days by working 8 hours each day, how many men will it take to mow 80 acres of grass in 4 days, working 6 hours each day? *Ans.* 20 men.

STATEMENT.

$$\left.\begin{array}{r} 20\text{ A.} : 80\text{ A.} \\ 4\text{ days} : 3\text{ days} \\ 6\text{ hours} : 8\text{ hours} \end{array}\right\} :: 5\text{ men} : Ans. \quad \text{Or, } \frac{80 \times 3 \times 8 \times 5}{20 \times 4 \times 6} = 20.$$

Explanation.—The answer required being in men, place 5 men for the third term. If it take 5 men to mow 20 acres, it will require *more* men to mow 80 acres in the same time; hence, 80 acres must be made the second term of the first simple ratio of the compound ratio. If it take 5 men when they work 3 days, it will require *less* men when they work 4 days; hence 3 days is the sec-

ond term of the second simple ratio. If it take 5 men when they work 8 hours per day, it will require *more* men when they work but 6 hours per day; hence, 8 hours is the second term of the third simple ratio. Reducing the compound ratio to a simple one, we have $20 \times 4 \times 6 : 80 \times 3 \times 8 :: 5 :$ *Ans.*, from which we find the fourth term to be 20.

4. *By Cancellation.*—Instead of stating a problem in compound proportion in the above form, it is more convenient to arrange the third and second terms in one column, the first terms in another column, and cancel the factors common to the two. The correctness of the process is evident from the fact that the product of the third and second terms constitutes a *dividend*, and the product of the first terms a *divisor*. The quotient is the fourth term.

$$\begin{array}{r|l} & 5 \\ \cancel{20} & \cancel{80}\ \cancel{4} \\ \cancel{4} & \cancel{3} \\ \cancel{2}\ \cancel{6} & \cancel{8}\ 4 \end{array} \qquad \text{Or, } \frac{5 \times \overset{\cancel{4}}{\cancel{80}} \times \cancel{3} \times \overset{4}{\cancel{8}}}{\cancel{20} \times \cancel{4} \times \underset{\cancel{2}}{\cancel{6}}} = 20 \textit{ Ans.}$$

$$5 \times 4 = 20 \textit{ Ans.}$$

5. *Logical Statement.* Art. 330, 8. *5th Case.*

$$\begin{array}{l} \cancel{5} : (20) \\ \cancel{3} : \cancel{4} \\ \cancel{8} : \cancel{6} :: \cancel{20} : \cancel{80} \\ \cancel{4} : \cancel{2} \qquad \cancel{4}\ \ 20 \end{array}$$

As the required term is a mean, we may divide the product of the extremes, *all* of whose terms are known by the product of the known means.

$$\frac{5 \times 3 \times 8 \times 80}{4 \times 6 \times 20} = 20.$$

6. RULE.

Arrange the terms of each of the simple ratios of the compound ratio as in SIMPLE PROPORTION. (Art. 331, 4.)

Then, if an extreme term be required, it will be equal to the quotient of the product of the means divided by the product of the known extremes, and if a mean term be required, it will be equal to the product of the extremes divided by the product of the known means.

NOTES.—1. In determining which number of each ratio is to be the second term, reason from what is *given* to what is proposed or required.

2. *Cancel* factors common to an extreme and a mean term.

EXAMPLES.

2. If \$900 produce \$50 in 9 months, what sum will produce \$450 in 5 months?

3. If it cost \$25 to lay a sidewalk 10 feet wide and 90 feet long, what will it cost to make a walk 6 feet wide and $\frac{1}{2}$ of a mile long?

4. If 16 men can excavate a cellar 90 feet long, 40 feet wide, and 10 feet deep in 15 days of 8 hours each, in how many days of 9 hours each can 3 men excavate a cellar 60 feet long, 36 feet wide, and 8 feet deep?

5. If 30 men, by working 8 hours a day, can in 12 days dig a ditch 40 rods long, 12 feet wide, and 4 feet deep, how many men, by working 12 hours a day for 9 days, can dig a ditch 300 rods long, 9 feet wide, and 6 feet deep?

6. If 12 men will build 48 rd. of brick sidewalk 14 ft. wide in $13\frac{1}{2}$ days of 9 hr. each day, how many rods of sidewalk 12 ft. wide can 9 men build in 16 days, working 12 hr. each day?

7. If a man can travel 810 miles in 30 days, traveling 6 hr. each day, how many miles per hour must he travel to go 300 miles in $7\frac{1}{2}$ days, traveling 9 hr. each day?

8. If in 3.25 days of 10 hr. each, 7 men can construct an embankment 3 rd. long, 16.5 ft. wide, and 8 ft. high, how long an embankment, 15 ft. wide, and 8.5 ft. high, can 9 men construct in 4.5 days of 9.5 hr. each?

9. If 62 men in a fort have provisions enough to last 27 days by allowing each man 30 ounces per day, how many more lb. of provisions will be required to furnish 75 men for 40 days, allowing each man 38 ounces per day?

10. If 25% of the value of 12 lots be \$5400, what is 18% of the value of 14 lots of the same grade?

11. If in 12 days of 10 hr. each, 14 men can cut 54 crd. of wood, how much can 17 men cut at the same rate in 18 days, working 8 hours each day?

12. If 90 persons, working 91 days of 10 hr. each, can manufacture 21451 lb. of yarn, how much more could be made by employing 150 persons for 80 days, for $10\frac{1}{4}$ hr. per day?

13. If a block of granite 8 ft. long, $5\frac{1}{2}$ ft. wide, $2\frac{1}{4}$ ft. thick, weigh 16,335 lb., what would be the weight of a granite column 20 ft. high and $2\frac{1}{4}$ ft. square?

14. If a timber of yellow pine 40 ft. long, $1\frac{1}{2}$ ft. square, of 100 degrees hardness, weigh 3780 lb., what is the weight of a timber of white pine, 32 ft. long, $1\frac{1}{4}$ ft. square, and of $71\frac{3}{7}$ degrees hardness?

15. If 140 cu. ft. of bituminous coal of 93 degrees hardness, weigh 7000 lb., how many cu. ft. in 2000 lb. of anthracite coal of 100 degrees hardness?

PRACTICE.

333. 1. Many problems commonly occurring in commercial transactions may be readily solved by exercising a little tact, especially when the prices or quantities are aliquot parts of some power or multiple of 10, or when they can be separated into *aliquant* parts, convenient for mental computation.

2. *Aliquots* are equal parts of a whole; *aliquants* are any parts equal or unequal, thus, 5 and 5 are aliquots of ten, 7 and 3 are aliquants of 10.

3. By the use of reciprocals, ratios, aliquots, and aliquants, rapid and accurate computations may be made.

4. This method is called ***Practice,*** and is illustrated in the following examples.

NOTE.—See table of aliquots, Art. 156, and let the pupil make a table of aliquots of 1 yr. and 1 mo., and use any contractions learned.

EXAMPLES.

1. Required the cost of 24 yds. of muslin at $12\frac{1}{2}$ cts. a yd.

Solution.—At $1.00 a yd. it is worth $24.00.
At $12\frac{1}{2}$ cts. a yd. it is worth only $\frac{1}{8}$ of $24.00, which is $3.00, *Ans.*

2. Find the cost of 56 yds. at $37\frac{1}{2}$ cts. a yd.

Solution.—At $1.00 a yd. the cost=$56.00.
At 25 cts. a yd. the cost=$\frac{1}{4}$ of $56.00=$14.00.
At $12\frac{1}{2}$ cts. a yd. the cost=$\frac{1}{2}$ of $14.00=$7.00.
The sum of the last two results=$21.00. *Ans.*

3. Required the cost of 56 bbls. of flour at $\$6.87\frac{1}{2}$ a bbl.

100 bbl.	cost	\$687.50
50	"	343.75
6	"	41.25
56	"	\$385.00

NOTE.—See table of aliquot parts.

4. Required the cost of 75 gals. of wine at $\$3.93\frac{3}{4}$ a gal.

100 gal.	cost	\$393.75
25	"	$98.43\frac{3}{4}$
75	"	$\$295.31\frac{1}{4}$

5. Find the cost of 25 bu. at $\$1.16\frac{2}{3}$ a bu.

$16\frac{2}{3}$ cts.=	$\frac{1}{6}$	\$25.00	=cost @ \$1.00,	
		$4.16\frac{2}{3}$=	"	$.16\frac{2}{3}$
		\$29.166=	"	$\$1.16\frac{2}{3}$

or

100 bu.	cost	$\$116.66\frac{2}{3}$
25	"	$29.16\frac{2}{3}$

6. Required the cost of 75 yds. at $43\frac{3}{4}$ cts. a yard.

100 yd.	cost	\$43.75
25	"	$10.93\frac{3}{4}$
75	"	$\$32.81\frac{1}{4}$

7. Required the cost of 45 bu. at $56\frac{1}{4}$ cts. a bu.

50 cts.=	$\frac{1}{2}$	\$45.00	=cost @ \$1.00,	
$6\frac{1}{4}$ " =	$\frac{1}{8}$	22.50 =	"	.50
		2.8125=	"	$.06\frac{1}{4}$
		\$25.3125=	"	$.56\frac{1}{4}$

or

50 bu.	for	$\$28.12\frac{1}{2}$
5	"	$2.81\frac{1}{4}$
45	"	$\$25.31\frac{1}{4}$

8. Find the cost of 9762 bu. at 25 cts. a bu.

25 cts.=	$\frac{1}{4}$	\$9762.	=cost @ \$1.00 a bu.	
		\$2440.50=	"	.25 "

9. Required the cost of 7 yds. 3 qrs. at 75 cts. a yard.

2 qrs.=	$\frac{1}{2}$	\$0.75	=cost of 1 yd.,	
		7		
		\$5.25 =	"	7 yds.
1 "	$\frac{1}{2}$	.375 =	"	2 qrs.
		.1875=	"	1 qr.
		\$5.8125=	"	7 yds. 3 qrs.

or

@ \$1.	cost=\$7.75
.25 cts. "	= $1.93\frac{3}{4}$
@ .75 cts.	$=\$5.81\frac{1}{4}$

10. Required the cost of 256 bu. of corn at $18\frac{3}{4}$ cts. a bu

$12\frac{1}{2}$ cts. =	$\frac{1}{8}$	\$256.00 = cost @ \$1.00		or 200 bu. cost \$3	
$6\frac{1}{4}$ "	$\frac{1}{2}$	32.00 = "	$.12\frac{1}{2}$	50 "	
		16.00 = "	$.06\frac{1}{4}$	5 "	
		\$48.00 = "	$.18\frac{3}{4}$	1 "	
				256 "	\$4

11. Find the cost of 15 lbs. 15 oz. of butter at 25 cts. a l

16 lb.	cost	\$4.00
1 oz.	"	.0156
15 lb. 15 oz. cost \$3.9844		

12. Required the cost of 9 lb. 7 oz. of cheese at $12\frac{1}{2}$ cts. a

4 oz. =	$\frac{1}{4}$	\$0.125	= the cost of 1 lb.			
		9				
		1.125	=	"	"	9 lbs.
2 " =	$\frac{1}{2}$	3125	=	"	"	4 oz.
1 " =	$\frac{1}{2}$	15625	=	"	"	2 "
		78125	=	"	"	1 "
		\$1.1796875	=	"	"	9 lbs. 7 oz., or,

10 lb.	cost	\$1.25
8 oz.	"	.0625
1 "	"	.0078
9 "	"	.0703
9 lb. 7 oz. cost \$1.1797		

13. Required the cost of 5 cwt. 3 qrs. 10 lbs. of sugar at a cwt.

600 lb.	cost	\$57.00
10 lb.	"	.95
5 lb.	"	$.47\frac{1}{2}$
15 lb.	"	$1.42\frac{1}{2}$
585 lb.	"	\$55.575

14. Find the cost of 15 gals. 3 qts. 1 pt. of molasses at 68 a gal.

10 gal.	cost	\$6.875
6 "	"	4.125
16 "	"	11.00
1 pt.	"	.0859
15 gal. 3 qt. 1 pt. cost \$10.9141		

15. Required the cost of 875 bu. at $1.06\frac{1}{4}$ a bu.

$6\frac{1}{4}$ cts. = $\frac{1}{16}$	\$875.00 = cost @	\$1.00	or 1000 bu.	cost	\$1062.50
	54.69 =	"	$.06\frac{1}{4}$	125	"	$132.81\frac{1}{4}$
	\$929.69 =	"	$\$1.06\frac{1}{4}$	875	"	$\$929.68\frac{3}{4}$

16. If a man walk 24 m. 7 fur. 25 rds. in one day, how far can he walk in 5 d. 11 h. 50 m., if he walk 12 hr. per day?

Remark.—This example may be solved in the same manner as the preceding; the only difference is, the multiplicand (24 m. 7 fur. 25 rds.) is a compound number.

17. What will be the cost of 3 qrs. 2 na. at \$4.50 a yard?

18. Required the cost of 13 cwt. 3 qrs. 20 lbs. of cheese at $\$9.12\frac{1}{2}$ a cwt.?

19. Find the cost of a ham, weighing 15 lbs. 3 oz. at 13 cts. a lb.

20. What will be the cost of 17 A. 1 R. 15 P. of land at \$25.25 per acre?

21. Find the cost of 19 yds. at $\$4.37\frac{1}{2}$ a yd.

22. What are 156 bu. 3 pks. 7 qts. 1 pt. of wheat worth at $93\frac{3}{4}$ cts. a bu.?

23. Find the cost of $87\frac{1}{2}$ yds. at $87\frac{1}{2}$ cts. a yard.

24. If a man walk 27 m. 5 fur. 15 rds. in one day, how far can he walk in 15 d. 10 h. 45 m., if he walk 12 hr. per day?

25. If a man earn 6 lb. 15 oz. 15 dr. of cheese in one day, how much can he earn in 7 d. 7 h., working 10 hr. per day?

26. A man can plow 2 A. 1 R. 25 P. in a day; how much can he plow in $5\frac{3}{4}$ days?

27. Find the cost of 6 T. 5 cwt. 3 qrs. 20 lbs. of hay at $\$16.62\frac{1}{2}$ a T.

28. Required the cost of 10 loads of coal, each containing $15\frac{1}{2}$ bu. at $12\frac{1}{2}$ cts. a bu.

29. What will be the cost of making 29 m. 7 fur. 35 rds. of road at \$975.75 a mile?

30. Required the cost of 10 cords 75 ft. of wood at $\$2.87\frac{1}{2}$ a cord?

31. Required the cost of 55 bbls. of flour at $\$6.68\frac{3}{4}$ a bbl.?

SECTION XV.

ANALYSIS.

334. Arithmetical ***Analysis*** may be defined as the art of—

1st. Logically deducing from the terms of a problem the conditions and relations required in its solution, which are not fully stated.

2d. So combining principles and operations as best to apply them to the peculiar relations and conditions of each problem.

NOTE.—Many problems to which this may be applied are generally more readily solved by Algebra.

EXAMPLES.

1. If A can do a piece of work in 6 days, B in 8 days, and C in 10 days, how long would it take them all? How long would it take each two?

2. If A and B earn \$17. in one week, A and C \$16., B and C \$15., how much does each man earn?

3. If the sum of two numbers be 39, and one be $\frac{1}{6}$ more than the other, what are the two numbers?

NOTE.—Six parts may represent one number; then $1+\frac{1}{6}=1\frac{1}{6}=\frac{7}{6}$ or 7 parts for the other, making 13 parts in both.

4. A man sold a horse and harness for \$160., and $\frac{1}{10}$ of the price of the horse was equal to $\frac{1}{6}$ the price of the harness; what was the price of each?

NOTE.—16 parts in the \$160.

5. Divide 72 into parts that shall have the ratios of 4, 3, 2.

NOTE.—Any number may be divided into proportional parts by making the sum of the proportional numbers given, the first term, each proportional number a second term, and the number to be divided the third term of proportion. Thus, $4+3+2=9$, then,

$$\left.\begin{array}{l} 9 : 4 :: 72 : (\quad) \\ 9 : 3 :: 72 : (\quad) \\ 9 : 2 :: 72 : (\quad) \end{array}\right\} \text{will give the parts required.}$$

6. An estate is to be divided as follows among 4 children (see Robinson's Algebra): The first is to have $200. more than $\frac{1}{4}$, the second $340. more than $\frac{1}{5}$, the third $300. more than $\frac{1}{6}$, and the fourth $400. more than $\frac{1}{8}$; what is the value of the estate, and what will each receive?

NOTE.—The difference between the whole and the sum of the several fractional parts mentioned, must equal the excess of money over the same parts.

7. How should an estate of $120,000. be divided between three heirs so that their portions should be as 7, 8, and 9? (Ex. 5, note.)

8. A man, after spending $500 more than $\frac{1}{4}$ of his salary, had $100. more than $\frac{1}{3}$ left; what was his salary?

9. A has $\frac{1}{2}$ as much money as B, or $\frac{1}{3}$ as much as C, and the three have $96.; how much has each?

10. When are the hour and minute hands of a watch together the second time after noon?

NOTE.—In 12 hrs. they will be together 11 times; hence they will be together the second time in $\frac{2}{11}$ of 12 hrs.

11. If 2 bu. of corn and 3 bu. of barley cost $3.06; and 1 bu. corn and 2 bu. barley $1.88, what is the price of each?

NOTE.—If 1 bu. corn + 2 bu. barley cost $1.88, double these,

or 2 bu. corn + 4 bu. barley, would cost	$3.76;
now if we subtract the cost of 2 bu. corn + 3 bu. barley . .	3.06
the difference will be the cost of 1 bu. barley	= $.70

12. A man who agreed to work 10 hrs. @ 60 cts., and to forfeit 30 cts. for every hour of the ten he did not work, received $3.30 on settlement; how many hours did he work?

NOTE.—Compare what he received with the full price for 10 hrs., observing that when idle he loses both his wages and his forfeit.

13. A crew of 70 persons have just provisions enough to last them 14 days, allowing each 20 oz. a day, and they receive 15 more persons without provisions; how much can be allowed each one in order that the provisions may last 20 days?

14. A can paint a house in 15 days, working 8 hrs. a day; B can do the same work in 10 days, working 10 hrs. a day; how long would it take both working 7 hrs. a day?

15. Smith was 34 yr. old when John was born; now John lacks only 4 yr. of being half as old as Smith. What is the age of each?

ALLIGATION OR COMPOUNDING.

335. In various kinds of business, it is sometimes convenier or necessary to mix articles of different values or qualities, thu forming a compound whose value or quality differs from that (its ingredients. This process is called ***Alligation*** (*L. ad*, t and *ligatus*, bound); a name suggested by the method of solvin some of its problems by joining or *binding* together the terms.

The various problems in Alligation may be divided into tw classes, commonly called *Alligation Medial* and *Alligation Alte nate.* These are solved by *Analysis.*

ALLIGATION MEDIAL.

336. Alligation Medial teaches the method of findin the average value or quality of a mixture, the quality and quan tity of whose ingredients are known.

This is a simple application of the principle of averages. (Ar 187.)

EXAMPLES.

1. A farmer mixed together 50 bushels of oats, at 40 cents pe bushel; 30 bushels of barley, at 50 cents per bushel; and 25 bush els of corn, at 60 cents per bushel. What was a bushel of th mixture worth?

OPERATION.

Cts.		Cts.
40×50	=	2000
50×30	=	1500
60×25	=	1500
105	)	5000
		$47\frac{13}{21}$

Explanation.—Since the value of 50 bushel of oats, at 40 cents a bushel, is 2000 cents; o 30 bushels of barley, at 50 cents a bushel, 150 cents; and 25 bushels of corn, at 60 cents a bushel, 1500 cents; the value of the mixture i 2000 cents + 1500 cents + 1500 cents = 500 cents. But the mixture contains 50 bushels + 30 bushels + 25 bushels = 105 bushels. Hence the value of 1 bushel of the mixture is $\frac{1}{105}$ of 5000 cents = $47\frac{1}{2}$ cents.

2. A goldsmith melted together 12 oz. of gold, 20 carats fine 6 oz., 18 carats fine; and 10 oz., 16 carats fine. What was th quality of the mixture? *Ans.* $18\frac{1}{7}$ carats.

OPERATION.

Carats.	Carats.
20×12	240
$18 \times 6 =$	108
$16 \times 10 =$	160
28	$)508$
	$18\frac{1}{7}$

Explanation.—Since a carat of gold is the *twenty-fourth* part of the mass regarded as a *unit* (here an oz.), 12 oz. of gold, each oz. containing 20 carats of pure gold, contain 12 times 20 carats $=$ 240 carats; 6 oz. of gold, each containing 18 carats, contain 108 carats; 10 oz. of gold, each containing 16 carats, contain 160 carats. Hence, 12 oz. $+$ 6 oz. $+$ 10 oz. $=$ 28 oz. of mixture contain 240 carats $+$ 108 carats $+$ 160 carats $=$ 508 carats, and 1 oz. of the mixture must, therefore, contain $\frac{1}{28}$ of 508 carats $= 18\frac{1}{7}$ carats.

NOTE.—Regarding a carat as a unit of measure of the pure gold in a given mass is not essential to the explanation of the above solution. For, suppose the comparative qualities of the above varieties of gold, represented respectively by the numbers 20, 18, and 16. Now, as these numbers represent the *comparative qualities* of the three varieties of gold, it is clear they must contain a *common unit of quality*. The number 20 denotes that the quality of the first variety contains this common unit of quality 20 times; and, hence, 20 is the measure of its quality. But the effect of 12 oz. in determining the *quality of a mixture* is 12 times as great as the effect of 1 oz.; hence, 12×20, or 240, is the *effective quality* of 12 oz. of gold, if the quality of 1 oz. is 20.

RULE.

Multiply the value or quality of each article by the number of articles, and divide the sum of the products by the sum of the articles. The quotient will be the average value or quality of the mixture.

3. A grocer mixed 15 lbs. of coffee, at 18 cents a pound; 35 lbs., at 16 cents a pound; and 40 lbs., at 14 cents a pound. What is a pound of the mixture worth?

4. A grocer mixed 25 gallons of wine, at 90 cents a gallon; 40 gallons of brandy, at 75 cents a gallon; and 10 gallons of water without price. What is a gallon of the mixture worth?

5. What is the price per lb. of a mixture of teas @ \$1.20, \$1.32, \$1.50, and \$1.55, if 25 lbs. of each kind be taken?

6. What is the average price of a compound of sugars, consisting of 35 lb. @ $8\frac{3}{4}$¢, 25 lb. @ $9\frac{1}{8}$¢, 30 lb. @ $10\frac{3}{8}$¢, and 45 lb. @ 11¢?

7. What is the gain or loss in buying 100 lbs. sugar @ $12\frac{1}{2}$¢,

75 lb. @ $12\frac{3}{4}$¢, 80 lb. @ $13\frac{3}{4}$, and selling all at an average price of $12\frac{7}{8}$¢?

8. A mixture of 90 lbs. of coffee is made from coffees worth 16, 17, 18, and 19 cts., in the proportion of 3, 2, 1, $1\frac{1}{2}$; how much of each kind is to be taken, and what is the average price?

9. If 18 gal. spirits, 20 under proof, that is of grade 80, be mixed with 24 gal., 10 under proof, and after drawing out 14 gal. there be added 10 gal. 10 over proof, what is a gal. of the last compound worth, proof-spirits being worth $1.00 per gal.?

10. If 10 gal. water be mixed with 90 gal. milk, worth 8 cts. per qt., and after selling 10 gal. of this, 10 gal. more of water be added, what is the last compound worth per qt.?

ALLIGATION ALTERNATE.

337. 1. ***Alligation Alternate*** is the process of finding what quantities of certain ingredients or elements may be compounded to form a mixture of a required grade or value.

2. This is used in mixing different grades of sugar, coffee, liquors, or other articles of commerce that are compounded to meet the various demands of the market for special grades.

338. The most common case of Alligation Alternate is that of compounding with certain quantities of certain grades, sufficient of one or more other grades to form a grade required.

Thus a grocer may have four grades of Rio coffee, say 20 lb. @ 16¢, 25 lb. @ $17\frac{1}{2}$¢, 30 lb. @ $18\frac{3}{4}$¢, and a larger quantity @ $19\frac{1}{4}$¢, of which he wishes to make a compound, using all of the first three kinds mentioned, and sufficient of the fourth kind to make the mixture worth $18\frac{1}{2}$¢ per lb.

One lb. of the first kind, compared with the average price, would show a gain of $2\frac{1}{2}$¢ ($18\frac{1}{2}-16$); hence, if sold @ $18\frac{1}{2}$¢,

On 20 lb. of the grade	@ 16¢,	the gain would be . .	50¢
On 25 lb. "	@ $17\frac{1}{2}$¢,	" . .	25¢
On 30 lb. "	@ $18\frac{3}{4}$¢,	the loss would be . .	$7\frac{1}{2}$¢

Then the gains, $(50+25)-$loss ($7\frac{1}{2}$), shows a balance of $67\frac{1}{2}$¢ gain, which is to be adjusted by taking a sufficient quantity @ $19\frac{1}{4}$. On 1 lb. @ $19\frac{1}{4}$, the loss is $\frac{3}{4}$¢, then $67\frac{1}{2}\div\frac{3}{4}=90$, will give the quan-

tity required to balance a gain of 67½¢; hence 90 lb. of the last kind must be taken.

NOTE.—Observe that the price of any unit of the compound must be the average price of the elements, that the loss on one or more elements must be balanced by the gain on one or more other elements, and that the entire process is one of simple comparisons and special adjustment to obtain a certain average.

339. RULE.

Compute the balance of gain or loss on the definite quantities taken; then cancel this balance by taking a sufficient quantity of one or more of the same or other elements furnished.

NOTE.—It is generally convenient to arrange the numbers showing the different grades, gains, etc., in columns, and to mark gain +, and loss −.

EXAMPLES.

1. A grocer wishes to mix 100 lb. of coffee @ 12 cts., and 90 lb. @ 10¢, with coffee @ 9¢, to make a grade worth 11¢. How much of each kind must he take?

OPERATION.

Av. @	El's @	Gain or loss per lb.	Lbs.	Total gain or loss.	Balance.	Total value.
	12	1 −	100	100 −		12.00
11	10	1 +	90	90 +		9.00
					10 −	
	9	2 +	5		10 +	.45

Proof.—If 195 lb. are worth . . $21.45
1 lb. is worth 11

Since, in taking 100 @ 12, and 90 @ 10, there is a balance of 10 loss, as many parts @ 9 must be taken as there are 2's (the gain on 1 @ 9) in 10; that is, (100−90) ÷ 2 = 5, which is the number to be entered in the column showing how many parts or units of each element must be taken. The headings of the several columns indicate their use. The proof is by Alligation Medial.

2. A farmer wishes to mix 60 bushels of corn, at 60 cents a bushel, with rye, at 75 cents; barley, at 50 cents; and oats at 45 cents. What quantity of rye, barley, and oats must be taken that the mixture may be worth 65 cents a bushel?

	60	5+	60	300+
65	75	10−	30	300−
	50	15+		
	45	20+		

NOTE.—Observe that the loss on 30 @ 75 will balance the gain on 60 @ 60, and that if elements @ 50 or 45 be included, they must be balanced by taking more @ 75; the loss on 3 @ 75 balancing the gain on 2 @ 50, and the loss on 2 @ 75 balancing the gain on 1 @ 45. Hence, take 60 @ 60, 35 @ 75, 2 @ 50, and 1 @ 45.

3. In what proportions may a grocer mix 100 lb. of coffee @ 12¢ with coffee at 15, 10, and 8 cents, to make a grade worth 11 cts. per lb.?

4. How much water is required to reduce 70 gal. proof spirits to thirty under proof; that is, grade 100 to grade 70?

5. How much water is required to reduce forty gal. 40 over proof to proof?

6. How much gold, 24 carats fine, must be mixed with 10 oz. 21 carats fine, and 10 oz. 20 carats fine, to make a quality of gold 22 carats fine?

340. To form an indefinite or a definite amount of a compound from several elements, all of which are to be used in due proportions.

Thus, a compound of sugars at 7, 9, 12, and 13 cts., may be required to be worth 11 cts. per lb.

On every lb. @ 7¢, sold @ 11¢, there will be a gain of 4¢, hence on $\frac{1}{4}$ lb. there will be a gain of 1¢. Thus we may show that in each case the reciprocal of the difference between the average price or grade and the price or grade of the element, indicates the amount of the element on which there would be a unit of gain or loss, and it is only necessary to so adjust and modify these as to balance the gains and losses, to solve any problem. For convenience, columns may be formed.

Av. @	El's. @	Lb.		Lb.	Value.
	7	$\frac{1}{4}$+	$\div\frac{1}{4}$	1	7
	9	$\frac{1}{2}$+	$\div\frac{1}{2}$	1	9
11	12	1−	$\div\frac{1}{2}$	2	24
	13	$\frac{1}{2}$−	$\div\frac{1}{4}$	2	26
			Proof	6	66
					11

In this case, as there are two gains and two losses, no adjustment is necessary, and the reciprocals entered (in the column of lb.) indicate the proportional quantities required. As it is more convenient, however, to take integral lb., equi-multiples or equi-aliquots of these primary num-

bers may be taken, and we may consider particular elements as balancing one another, and treat them as thus combined, indicating the combinations by linking.

Generally the simplest derivative proportionals may be obtained by dividing balancing or combined elements by the greatest common divisor of the reciprocals entered. Thus the 1st and 4th above may be divided by $\frac{1}{4}$, and the 2d and third by $\frac{1}{2}$, giving a new column of proportional parts. The proof is, of course, the same as in Art. 339. The three steps are—1st. Find reciprocals; 2d. Adjust; 3d. Reduce, if necessary.

341. RULE.

When the number of elements of higher grade than the average is equal to the number of elements of lower grade than the average, take the reciprocal of the difference between the value of each element and the average value: this will indicate the quantity of each element required for the compound.

When there is not an equal number of higher and lower grades, such elements may be repeated in the compound as will secure an equal adjustment of gains and losses.

NOTES.—1. For convenience, the elements and reciprocals may be arranged in columns, those reciprocals affording gain being marked +, and loss −, and balancing results being united by curved lines or links.

2. Primary balancing results, divided by their G. C. D., will give the integral number of proportional parts that may be used.

3. Equi-multiples, or equi-aliquots, of proportional parts, or of elements and the average, may be taken, if more convenient in the operation.

342. EXAMPLES.

1. Combine grades of 20, 18, 24, 22, and 23 to form a grade of 21.

OPERATION.

Av.	El's.	Reciprocal parts.	Adjustment of parts. No. gains and losses.	Adjustment of parts. Primary results.	G. C. D's.	Derived proportional parts.	Value.
	20	1+	2+	2	$\div\frac{1}{2}$	4	80
	18	$\frac{1}{3}$+	1+	$\frac{1}{3}$	$\div\frac{1}{3}$	1	18
21	24	$\frac{1}{3}$−	1−	$\frac{1}{3}$	$\div\frac{1}{3}$	1	24
	22	1−	1−	1	$\div\frac{1}{2}$	2	44
	23	$\frac{1}{2}$−	1−	$\frac{1}{2}$	$\div\frac{1}{2}$	1	23
						9	) 189
						Proof	21

Notes.—1. Observe in this example that *one* of the elements affording gain must be taken *twice* to balance *two* elements of loss.

$$21\left\{\begin{array}{l} 20 \\ 18 \\ 24 \\ 22 \\ 23 \end{array}\right. \begin{array}{r|c|c|c|} 1+2= & 3 & & 3 \\ & 3 & \div 3 & 1 \\ & 3 & \div 3 & 1 \\ & 1 & & 1 \\ & 1 & & 1 \end{array}$$

2. Another method is frequently used, called the *method by linking*. Thus, each element above the average grade is linked with one below the average, and the differences are written opposite the element with which each one is linked. It will be observed in this example, that the gain on 3 @ 20 balances the loss on 1 @ 22 *and* 1 @ 23, etc. The primary results may be reduced as in the other method.

2. Mix sugars at 7, 10, and 12 cts., to form a grade worth 9 cts. per lb.

3. Mix teas at 50, 60, 70, and 90 cts. to form a grade worth 80 cts.

Note.—Each of these numbers may be divided by 10 before comparing.

4. How can a farmer mix oats at 40 cts., corn at 50 cts., rye at 70 cts., and wheat at 90 cts., to form a compound worth 60 cts.?

5. A merchant has teas worth 60, 75, 80, and 100 cents per lb. How much of each kind must he take to make a mixture worth 85 cents per lb.?

6. A wine-merchant wishes to mix wine worth \$1.20 and \$1.40 per gallon with water. How much of each kind must he use to make a mixture worth \$1.00 per gallon?

7. A goldsmith wishes to combine gold 22 carats fine; 19 carats fine; 18 carats fine; and 17 carats fine. In what proportion must they be united that the compound may be 20 carats fine?

8. A wine-merchant wishes to fill a cask containing 36 gallons with a mixture of wines worth \$1.00, \$1.20, \$1.50, and \$1.60 per gallon. How many gallons of each kind must he take that the mixture may be worth \$1.40 per gallon?

Note.—For proportional parts in any amount see Art. 334. Ex. 5.

9. A trader wishes to fill 10 casks, each containing 28 gallons, with a mixture of brandy, rum, and water. If the brandy is worth 80 cents a gallon, and the rum 95 cents, how many gallons of each must be taken that the mixture may be worth 75 cents?

10. What is the least number of pounds of each element in a mixture of teas worth 98 cts. per lb., formed of grades @ 85, 105, 95, and 110 cts.?

11. Mix sugars at $14\frac{1}{8}$, $13\frac{1}{8}$, $12\frac{7}{8}$, and $12\frac{3}{4}$ cts., to form a grade at $13\frac{1}{2}$ cts.?

12. Mix coffees at 16, $17\frac{1}{4}$, $18\frac{3}{4}$, and $19\frac{1}{4}$ cts. to form a grade worth $18\frac{1}{2}$ cts.?

13. Mix liquors at 80, 90, 115, 130, and 145 cts. per gal., to form a grade worth $1.00 per gal.

14. Make up 1200 lb. sugar worth $12\frac{7}{8}$ cts., from grades worth 14, $13\frac{1}{4}$, 13, $12\frac{1}{2}$, and 12 cts.

15. A man sold 13 fowls for $8.45, receiving the following prices, viz.: For a turkey $1., for a goose 75 cts., for a duck 60 cts., and for a chicken 50 cts. How many of each did he sell?

SECTION XVI.

INVOLUTION.

343. MENTAL PROBLEMS.

1. What is 2×2? $2 \times 2 \times 2$? $2 \times 2 \times 2 \times 2$?

2. What product is obtained by using 3 three times as a factor?

3. (3×3) used twice as a factor? How many factors of 3 each would give the same product?

4. What is the third power of 4, or the product of three 4's?

5. What is the quotient of the 4th power of 2 divided by the 2d power? What power of 2 is this quotient?

6. What power of 3 is obtained by dividing the 2d power by 3?

7. What is the quotient of the first power of any number divided by the number itself?

NOTE.—The results of successive divisions of any power by the number of which it is a power may be shown as follows:

$$64 \div 4 = 16 = 4^3 \div 4 = 4^2$$
$$16 \div 4 = 4 = 4^2 \div 4 = 4^1$$
$$4 \div 4 = 1 = 4^1 \div 4 = 4^0$$
$$1 \div 4 = \frac{1}{4} = \frac{1}{4^1}$$
$$\frac{1}{4} \div 4 = \frac{1}{4 \times 4} = \frac{1}{4^2}$$
$$\frac{1}{4 \times 4} \div 4 = \frac{1}{4 \times 4 \times 4} = \frac{1}{4^3}$$

In a similar manner it may be shown that the 0 power of any number is an expression equivalent to 1, and that the reciprocals of powers are 1 divided by the powers. It is evident that there is really no *zero* power of any number, for nothing but 1 can be produced by using 1 as a factor, and much less can any product result from using 1 *no times* as a factor.

344. 1. The process of obtaining any power of a number is called ***Involution.***

2. The *first power* of a number is the number itself. This is also called the *root* of all the powers.

3. Any power higher than the first is the product obtained by using the number several times as a factor.

345. The name of any power is derived from the number of factors, equal to the first power, of which it is the product. Thus the product of four 5's is the 4th power of 5; the product of six 10's is the 6th power of 10; or $5 \times 5 \times 5 \times 5 = 5^4 = 625$, etc.

346. 1. The *second* power of any number is also called the ***square*** of the number, because the product of two equal dimensions of any area gives the area of a square. (Art. 284.)

2. The *third* power of a number is also called its ***cube,*** because the product of three equal dimensions of any volume gives the contents or volume of a cube. (Art. 285.)

347. The *degree* of a power is indicated by an ***index or exponent,*** which is a smaller figure placed at the right, and a little above the root or first power. Thus,

2^4 indicates the fourth power of 2.
6^{10} indicates the tenth power of 6.
$(\frac{3}{4})^2$ indicates the square of $\frac{3}{4}$.
$(2+3)^3$ indicates the cube of $(2+3)$.
$(5^2)^6$ indicates the sixth power of the square of 5.

NOTE.—The parenthesis is used in connection with the index in cases of

common fractions, any combination of several terms, or any modified expression of a single term.

348. 1. *The square of any number consists of twice as many figures as the root, or one less.*

2. *The cube of any number consists of three times as many figures as the root, or one or two less.*

NOTE.—Let the pupil observe these facts respecting all the squares and cubes he is required to find, and let him *also* observe how many figures in the 4th, 5th, 6th, and other powers as compared with the number of figures in the root.

349. 1. *Linear units* in the decimal system *have a ratio of 10.*

Thus 42526 linear feet are equivalent to—

4	units, each	10000	ft. long.
+2	"	1000	"
+5	"	100	"
+2	"	10	"
+6	"	1	"

2. The *squares* of decimal units *have a ratio of 100.*

Thus 42526 sq. ft. are equivalent to—

4	units, each	100	ft. square, hence	each	containing	10000 sq. ft.
+25	"	10	"	"	"	100 "
+26	"	1	"	"	"	1 "

3. The *cubes* of decimal units *have a ratio of 1000.*

Thus 42526 cu. ft. are equivalent to—

42	units, each	10	ft. cubed,	hence containing	1000 cu. ft.
+526	"	1	"	"	1 "

and 21,316,457 cu. ft. are equivalent to—

21	units, each	100	ft. cubed, hence	each	containing	1000000 cu. ft.
+316	"	10	"	"	"	1000 "
+457	"	1	"	"	"	1 "

350. Any combination of factors may be separated into any convenient number of composite factors. Thus,

$$3 \times 2 \times 5 \times 6 \times 9 \times 7 \times 2 \times 4 = (3 \times 2 \times 5) \times (6 \times 9) \times (7 \times 2 \times 4),$$
$$\text{or } (3 \times 2) \times (5 \times 6) \times (9 \times 7) \times (2 \times 4),$$

or any other form of the combination involving the use of these terms as factors only.

So $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = (3 \times 3 \times 3) \times (3 \times 3 \times 3) \times 3 = 3^3 \times 3^3 \times 3 = 3^7$, and

$5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 =$
$(5 \times 5 \times 5) \times (5 \times 5 \times 5) \times (5 \times 5 \times 5) = 5^3 \times 5^3 \times 5^3 = 5^9$.

As the same may be shown for any number, we conclude that—

1st. *The product of any two or more powers of a number* equals the power whose index is the *sum* of the indices of the factors. Thus $4^2 \times 4^3 = 4^5$. $25^3 \times 25^2 \times 25^4 = 25^9$.

2d. *The product arising from using any power of a number several times as a factor* equals the power whose index is equal to the index of the factor multiplied by the number of times it is used as a factor. Thus $4^2 \times 4^2 \times 4^2 = (4^2)^3 = 4^6$. $172^3 \times 172^3 \times 172^3 \times 172^3 = (172^3)^4 = 172^{12}$.

$(49^2)^3 \times (49^2)^3 \times (49^2)^3 = (49^6)^3 = 49^{18}$.

351. To find a power of any number—

1. Find the third power of 516.

$$516 \times 516 \times 516 = 137388096.$$

2. Find the 12th power of 1.12.

OPERATION.

$$1.12 \times 1.12 \times 1.12 \times 1.12^3 \times 1.12^6 = 1.12^{12} = 3.8959738+.$$

RULE.

To find either the 2d or 3d power, use the first power as many times as a factor as there are units in the index of the power required. For powers above the third, find the product of any two or more powers of the number, the sum of whose indices equals the index of the required power.

NOTE.—The second part of this rule is equivalent to an extended application of the first part. (Art. 350.)

352. The following table of squares and cubes should be learned *perfectly* by the pupil, as a knowledge of these is most frequently required in finding roots:

No.	Square.	Cube.	No.	Square.	Cube.
1	1	1	7	49	343
2	4	8	8	64	512
3	9	27	9	81	729
4	16	64	10	100	1000
5	25	125	11	121	1331
6	36	216	12	144	1728

NOTE.—The squares might be continued much farther to advantage, and higher powers of the numbers, from 1 to 9, might be learned.

353. EXAMPLES.

1. 125^2.
2. 64^2.
3. 4.5^3.
4. 56^2.
5. $(\frac{4}{5})^2$.
6. 9^3.
7. 99^3.
8. 999^2. (Art. 112.)
9. 9^5.
10. 99^3.
11. 999^3.
12. 100^5.
13. $(6\frac{1}{2})^2$.
14. $(9\frac{1}{2})^2$.
15. 1.06^4.
16. 1.10^4.
17. 2.12^{10}. $(2.12^2 \times 2.12^3)^2$.
18. $(\frac{11}{17})^2$.
19. $(5 \times 7 \times 5)^2$.
20. 125^3. (Art. 182.) $\frac{1000}{8}$.
21. 972^2.
22. $.6^{10}$.
23. 1.125^2.
24. $(3\frac{1}{2})^4$.
25. $(\frac{7}{8} \text{ of } .2)^4$.
26. $(8\frac{2}{3})^3$.

NOTES.—1. In squaring a mixed number, when the fraction is $\frac{1}{2}$, the square of the fraction may be annexed to the product of the whole number by a factor greater by 1. Thus $(8\frac{1}{2} \times 8\frac{1}{2}) = (8 \times 9) + (\frac{1}{2} \times \frac{1}{2}) = 72\frac{1}{4}$.

2. When there would be more than six or seven decimal places in a power, it is generally sufficient to drop the rest as in contraction of multiplication of decimals. (Art. 257.)

INVOLUTION BY ANALYSIS.

354. If any number be separated by analysis into two parts whose units have a ratio of 10, and be involved in this form to any power, the form of the partial products will show the relation of the parts of the root to the complete power. Thus $27 = 20 + 7$; $125 = 120 + 5$; $340 = 300 + 40$; $16500 = 16000 + 500$, the ratio between the two orders in the analysis of each number being 10.

Now take any one of these and involve it, keeping the parts separate, and compare the results with those obtained by the ordinary process. Thus take $340 = 300 + 40$, and multiply each part of the multiplicand by each part of the multiplier, at first simply indicating the composition of the partial products, and afterwards combining them. Use A, B, C, etc., to refer to the several parts in the operation.

$$
\begin{array}{r}
300+40 = \\
\underline{300+40 =} \\
(300\times 40)+40^2 = A \\
\underline{300^2+(300\times 40)\qquad = B} \\
300^2+2(300\times 40)+40^2 = C \\
\underline{300+40 =} \\
(300^2\times 40)+2(300\times 40^2)+40^3 = D \\
\underline{300^3+2(300^2\times 40)+(300\times 40^2)\qquad = E} \\
300^3+3(300^2\times 40)+3(300\times 40^2)+40^3 = F
\end{array}
$$

$$
\begin{array}{lr}
300+40= & 340 \\
300+40= & \underline{340} \\
A= \quad 12000+1600= & 13600 \\
B=90000+12000 \quad = & \underline{1020} \\
C=90000+24000+1600= & 115600 \\
300+40= & \underline{340} \\
D= \quad 3600000+960000+64000= & 4624000 \\
E= \quad 27000000+7200000+480000 \quad = & \underline{346800} \\
F=27000000+10800000+1440000+64000= & 39304000
\end{array}
$$

Thus $340^2=(300+40)^2=300^2+2(300\times 40)+40^2$, or letting t represent the 1st part and u the 2d; $t^2+2tu+u^2=(t+u)^2$. Thus also $340^3=(300+40)^3=300^3+3\ (300^2\times 40)+3\ (300\times 40^2)+40^3$, or letting t represent the 1st part and u the second; $t^3+3t^2u+3tu^2+u^3=(t+u)^3$.

355. As *any* number may be analyzed and involved in a similar manner, we conclude that—

1st. *The square of any number* is equivalent to the square of the number separated into two parts, which is equal to *the square of the first part, plus twice the product of the first part by the second part, plus the square of the second part.*

2d. *The cube of any number* is equivalent to the cube of the number separated into two parts, which is equal to *the cube of the first part, plus three times the square of the first part into the second part, plus three times the first into the square of the second, plus the cube of the second part.*

NOTES.—1. These same relations may be shown with the geometrical figures of the square and the cube.

2. A knowledge of these relations is of utility in mental involution, and of great importance in evolution.

356. EXAMPLES.

NOTE.—Only partial products and their sum should be *written*.

1. $128^2 = (120+8)^2 = 120^2 + 2\ (120 \times 8) + 8^2 = 14400 + 1920 + 64 = 16384$

2. 1190^3.

OPERATION.

$$\begin{aligned} 1100^3 &= 1331000000 \\ 3\,(1100^2 \times 90) &= 326700000 \\ 3\,(1100 \times 90^2) &= 26730000 \\ 90^3 &= \underline{729000} \\ 1190^3 &= 1685159000 \end{aligned}$$

3. $$64^3 = \left\{\begin{aligned} &216000 = t^3 \\ &+\ 43200 = 3\,t^2 u \\ &+\ 2880 = 3\,t u^2 \\ &+\ \underline{64} = u^3 \end{aligned}\right.$$
$$262144 = t^3 + 3\,t^2 u + 3\,t u^2 + u^3 = (t+u)^3$$

4. 1290^3.
5. 85^3.
6. 78^3.
7. 47^3.
8. 39^3.
9. 5.6^3.
10. 98^3.
11. 1170^3.

EVOLUTION.

357. MENTAL PROBLEMS.

1. Of what number is 81 the square?
2. Of what is 64 the cube?
3. Of what is 125 the cube?
4. Of what is 1.44 the square?
5. Of what is 625 the cube?
6. Of what is 1728 the cube?
7. Of what is 81 the cube?
8. Of what is 1331 the cube?
9. How much does 92 exceed a perfect cube? Of what is a greater part the cube?

10. What is the greatest square in 160?
11. What cube in 999? Of what?
12. What square in 99? Of what?
13. What cube in 700? Of what?
14. What cube in 540? Of what?
15. What cube in 472? Of what?

358. 1. The process of finding one of several equal factors of a product is called *extracting the root of a power*, or *Evolution*. It is simply a process of factoring. (Art. 164.)

2. The names of different roots correspond with the names of the powers. Thus one of *two* equal factors, or the *root* of *a square*, is called *the square root;* one of *three* equal factors, *the cube root;* one of *four* equal factors, *the fourth root,* etc.

359. The root required is generally indicated in either of two ways—

1st. The radical sign, $\sqrt{}$, is placed before the number, with a smaller index before it, except the index of the square root, which is generally omitted. Thus the square root of 25 is indicated by $\sqrt{25}$, the cube root of 27 by $\sqrt[3]{27}$, etc.

2d. Roots are also indicated by using a fractional index at the right of the number. Thus $\sqrt{144}=144^{\frac{1}{2}}$; $\sqrt[3]{1000}=1000^{\frac{1}{3}}$, and these are read as *powers*, thus, "1000 to the one-third power."

Fractional indices arise from factoring powers. Thus if 7^4 be separated into two factors it becomes $7^2 \times 7^2$, and as $2=\frac{4}{2}$, this may be written $7^{\frac{4}{2}} \times 7^{\frac{4}{2}}$. In a similar manner $7=7^{\frac{1}{2}} \times 7^{\frac{1}{2}}$.

Hence a *root of any power* may be indicated by a fraction expressing *the ratio of the index of the power to the index of the root*. Thus $\sqrt[3]{18^3}=18^{\frac{3}{3}}=18^1=18$; $\sqrt{20^4}=20^{\frac{4}{2}}=20^2=400$, and $\sqrt{81^1}=81^{\frac{1}{2}}=9$. $\sqrt[3]{(\frac{2}{5})^4}=(\frac{2}{5})^{\frac{4}{3}}$.

360. A number may be a perfect or an imperfect power of a required root. Thus, 25 is a perfect square but not a perfect cube. The *exact* root of an imperfect power cannot be found, and such a number is called *a surd*. Prime numbers are imperfect powers of all roots except themselves, which are the first roots of their first powers. Thus $\sqrt{8}$, $\sqrt[3]{16}$, $\sqrt{2}$, $\sqrt[3]{36}$, are surds. Surds are also sometimes called *irrational* quantities, and perfect powers, as $\sqrt[3]{8}$, $\sqrt{16}$, $\sqrt{36}$, are sometimes called *rational* quantities.

361. 1. ***Logarithms*** (log.) are indices of powers. Every number may be regarded as some power of 10, which is called the base, and logarithms are the indices of these powers of 10, most of them being proper or improper fractions. Thus the index or logarithm of 1000 is 3, that is 1000 is the 3d power of 10; the log. of 251.19 is about 2.4 or $\frac{24}{10}$, that is 251.19 is the 10th root of the 24th power of 10; $251.19=10^{\frac{24}{10}}=10^{2.4}$

2. Knowing the logs. of two or more numbers, the log. of their product may be found simply by adding the logs. of the numbers (Art. 350, 1); the log. of their quotient by finding the difference of the logs. The log. of any power of a number may be found by multiplying the log. of the number by the index of the power required (Art. 350, 2), and the log. of any root of a number by dividing the log. of the number by the index of the root required. Thus, $10^2\times10^3=10^5$; $10^5\div10^3=10^2$; $(10^2)^3=10^6$; $(10^4)^{\frac{1}{2}}=10^2$.

3. For the use of logarithms extensive tables are required. Only the fractional part of the index, expressed decimally, is given in the table, the integral part being known to be one less than the number of integral figures in the number. A single example may illustrate their utility.

If the 20th power of 1.08 be required, we find in the table that

the log. of 1.08 is	.0334238
and multiplying this by 20	20
we obtain	0.6684760

as the log. of the power required, and the tables show this to be the log. of the number 4.6609671+, hence $1.08^{20}=4.6609671+=10^{.6684760}$.

4. Conversely, if the 20th root of 4.6609671+ were required, its log. divided by 20 would give the log. of the root required, and this would be the log. of 1.08.

SQUARE ROOT.

362. The ***Square Root*** of a number is one of two equal factors of that number. (Art. 358, 2.) Thus $\sqrt{16}=4$; $\sqrt{\frac{4}{9}}=\frac{2}{3}$; $\sqrt{.25}=.5$.

NOTE.—A distinction should be made between the root of a number and

an equivalent of that root. Thus $\sqrt{\frac{64}{4}}=\frac{8}{2}$, of which 4 is the equivalent, but 4 is not the square root of $\frac{64}{4}$.

363. 1. A method of finding the square root may be discovered by an examination of square numbers, observing the manner in which they are formed, and the relation which the orders of the square bear to those of the root.

2. $1^2=1$
$10^2=100$
$100^2=10000$
$1000^2=1000000$

Observe that the square of a unit of each higher order occupies two more places than the square of a unit of the next lower order. (Art. 349, 2.)

3. The first nine numbers are—

1, 2, 3, 4, 5, 6, 7, 8, 9,

and their squares

1, 4, 9, 16, 25, 36, 49, 64, 81.

From this it is seen that the square of any number composed of one order of figures, cannot contain more than two orders.

Conversely, the square root of any number composed of one or *two* orders is composed of but *one* order.

4. It will further be seen that the numbers in the second line above are the only perfect squares found below 100, and that the square root of any number between any two of these consecutive perfect squares is between the two corresponding roots above. Thus, 75 is not a perfect square, and its approximate square root is between 8 and 9.

5. The first nine numbers of *tens* are—

10, 20, 30, 40, 50, 60, 70, 80, 90,

and their squares,

100, 400, 900, 1600, 2500, 3600, 4900, 6400, 8100.

From which it is seen that the square of *tens* gives no order below *hundreds* or above *thousands.*

6. In the same manner it may be shown *that the square of any number must contain at least twice as many orders, less one, as the number squared.* (Art. 348, 1.)

NOTE.—If the left-hand figure of the number to be squared be more than *three,* the square will contain *just twice as many orders as the root.* Thus, the square of 456 contains six orders.

7. Hence, if a number whose square root is required, be separated into periods of two figures each, the number of periods will indicate the number of figures in the root. Thus the square root of 726148, or of 45692, consists of three figures.

NOTE.—If decimal periods be annexed to any number, the root will contain 1 decimal figure for every two decimal places used.

364. Observe further, that *the square of any number separated into tens and units is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.* (Art. 355, 1.)

365. These two principles concerning the number of orders in any square, and the relation of the elements of the root to the square, determine the methods of extracting the square root.

Ex. What is the square root of 4225 ?

OPERATION.

```
                 4225(65
6 × 2 = 12       36
  120+5=125)625
             625
```

Explanation.—Since 4225 is composed of four orders, its root will be composed of but two ; and since the square of units is composed of units and tens, and the square of tens, of hundreds and thousands, we separate the number into periods of two figures each, by placing a dot over units and another over hundreds.

Now 42 must contain the square of the ten's figure of the root. The greatest perfect square in 42 is 36, the square root of which is 6. Hence 6 is the ten's figure of the root. Subtracting the square of the ten's figure of the root from 42 hundreds, we have 6 hundreds for a remainder, to which, if the 25 units be added, we shall have 625, which is composed of *twice the product of the tens of the root by the units (to be found) plus the square of the units.*

Now the product of tens by units gives no order below tens, hence 62 tens must contain *twice the product of the tens by the units.* It may contain more, since the square of *units* may give *tens.*

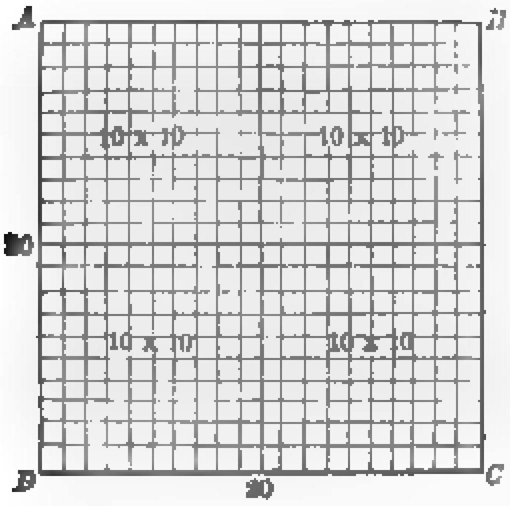
If 62 tens be divided by 2 × 6 tens, or 12 tens, the quotient, 5, will be the *unit figure of the root.* By placing 5, the unit figure, at the right of 12 tens, and multiplying the result, 125, by 5, the product will be twice the tens by the units, plus the square of the units.

366. The process of this evolution may also be illustrated by the use of diagrams.

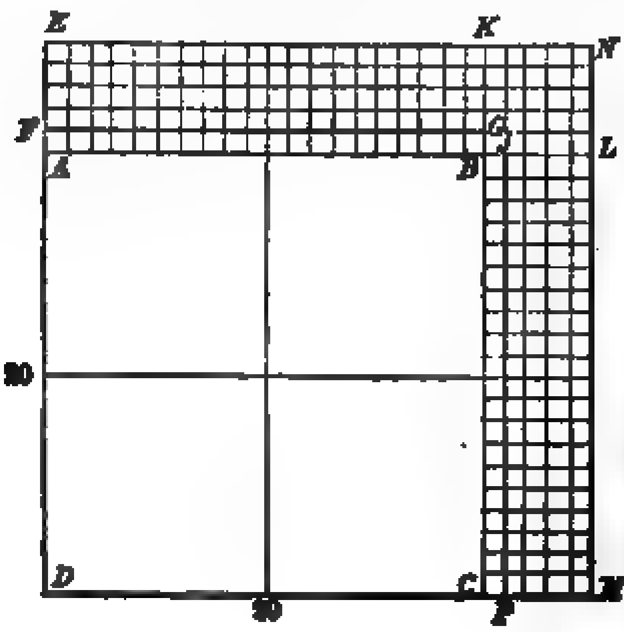
Ex. Find the square root of 625.

OPERATION.

$$
\begin{array}{rr}
 & \dot{6}2\dot{5}(25 \\
2^2 = & \underline{4} \\
20 \times 2 = & 40)225 \\
\overline{(2 \times 20) + 5} \times 5 & \\
= 2(20 \times 5) + 5^2 = & \underline{225}
\end{array}
$$



Explanation.—The number 625 may be regarded as referring to units of area, say sheets of tin, each 1 ft. square, and these are to be arranged in the form of a square. (Arts. 346, 1, and 349, 2.) We observe that there are 6 squares of 10^2 each, but only 4 of these can be used at first, as that is the greatest square in 6. The root of 4 is 2, so we construct a square, A B C D, 20 by 20, consisting of 4 squares of 10^2 each, and still have remaining 2 squares of 10^2 each, equal to 200 squares of 1^2 each, and 25 squares of 1^2 each, or 225 squares of 1^2 each to be added to the square already formed in such a manner as to still make the figure a perfect square. Additions may now be made on two sides of the square, requiring F B + B P, $(2 \times 20) = 40$ units in length, hence the addition may be $(220 \div 40)$, or 5 units in breadth, giving as the contents of the two side additions, A E K B and C M L B, $2 \times (20 \times 5) = 200$, while the remaining 25 squares of 1^2 each are arranged in the small square K B L N, a single square of 5^2; or as the rectangles A K, C L, and B N, have the same width, the contents of the three may be computed together, making $(\overline{2 \times 20}$ $\times 5 = 2(20 \times 5) + 5^2 =$



ditions which have been made to complete the square D N, of which the contents are $25^2=625$.

367. RULE.

1. *Separate the given number into periods of two figures each, commencing at units.*

2. *Find the greatest perfect square in the left-hand period and place its root on the right as the highest order of the root.*

3. *Subtract the square of the root figure from the left-hand period, and to the remainder annex the next period for a dividend.*

4. *Double the part of the root already found for a trial divisor, consider how many times it is contained in the dividend, exclusive of the right-hand figure, and write the quotient as the next figure of the root and also at the right of the trial divisor.*

5. *Multiply the divisor thus formed by the last found figure of the root and subtract the product from the dividend.*

6. *To the remainder annex the next period, and divide one-tenth of this number by double the root found or by the sum of the last true divisor and the last root figure, and proceed as before.*

NOTES.—1. The left-hand period often contains but one figure.

2. Twice the root already found is called the *trial divisor*, since the exact quotient may not be the next figure of the root. The quotient may be too large, in which case it must be made less. The true divisor is the trial divisor considered as tens, plus the new root figure.

3. When any dividend, exclusive of its right-hand figure, is not large enough to contain its trial divisor, place a cipher for the next figure of the root, and double the root thus formed for a new trial divisor, and form a new dividend by bringing down the next period.

4. When there is a remainder after all the periods are used, annex a period of two ciphers, and thus continue the operation until the requisite number of decimal places is obtained. In this case, there will be a remainder, how far soever the operation be continued, since the square of no one of the digits ends with a *cipher*.

5. The square root of a common fraction may be found by taking the root of both terms when they are perfect squares. When both terms of a fraction are not perfect squares, and cannot be changed to perfect squares, the root of the fraction cannot be exactly found. The approximate root, however, may be found by multiplying the numerator of the fraction by the denominator, and extracting the root of the product, and dividing the result by the denominator. By extracting the root to decimal places the error may be further lessened.

6. In finding the square root of a decimal or a mixed decimal, commence separating into periods at the order of units, and place a point over every alternate figure towards the right and left. If there be an *odd* number of decimal places, annex a cipher.

7. Mixed numbers must first be reduced to improper fractions or to mixed decimals.

368. EXAMPLES.

1. What is the square root of 133225?

	$1\dot{3}32\dot{2}5\dot{}$(365, *Ans.*
$3\times3=$	9
$3\times2=6$	6)432
$66\times6=$	396
$36\times2=72$	5)3625
$725\times5=$	3625

2. What is the square root of 62.8?

	$6\dot{2}.8\dot{0}$(7.924, *Ans.*
$7\times7=$	49
$7\times2=$	14.9)13.80
$14.9\times.9=$	13.41
$7.9\times.2=$	15.8 2).3900
$15.82\times.02=$	.3164
$7.92\times2=$	15.84 4).073600
$15.844\times.004=$	.063376
	.010224

What is the square root of—

3. 32041?
4. 492804?
5. 94249?
6. 2?
7. 234.09?
8. .0625?
9. 57600?
10. 425?
11. 18?
12. 3.272?
13. 176.89?
14. $\frac{25}{256}$?
15. $\frac{625}{2304}$?
16. $30\frac{1}{4}$?
17. $69\frac{4}{9}$?
18. 24^4?
19. $50625^{\frac{1}{2}}$?
20. 726^2?
21. 83^4?
22. $(\frac{4}{5})^6$?

CUBE ROOT.

. The ***Cube Root*** of a number is one of three equal that number. (Art. 358, 2.) Thus $\sqrt[3]{27}=3$; $(\frac{64}{125})^{\frac{1}{3}}=$ = 6.

. 1. A method of finding the cube root may be discovered mination of cube numbers, observing how they are de- d the relation of the orders of the root to those of the

$1^3=$	1	Observe that the cube of a unit of each higher order occupies three more places than the cube of the next lower order. (Art. 349, 3.)
$10^3=$	1000	
$100^3=$	1000000	
$000^3=$	1000000000	

he first nine numbers are—

2, 3, 4, 5, 6, 7, 8, 9,

r cubes,

8, 27, 64, 125, 216, 343, 512, 729.

he numbers in the second line are the only perfect cubes 000, and the cube root of any number between any two tive perfect cubes is between the two corresponding roots. 0 is not a perfect cube, and its approximate cube root is 6 and 7.

e cube of the least number of tens is $10^3=1000$, and the he greatest number of tens is $90^3=729000$, from which it hat the *cube of tens* gives no order below *thousands* or *ndred-thousands.*

the same manner it may be shown *that the cube of any must contain at least three times as many orders, less two, mber cubed.* Thus the cube of any number composed of ers must contain either ten, eleven, or twelve figures. , 2.)

nce if any number whose cube root is required be sepa- o periods of three figures each, the number of periods ate the number of figures in the root. Thus the cube 912673; of 13824; or of 3375 consists of *two* integral

NOTE.—If decimal periods be annexed to any number, the root will contain one decimal figure for every period of decimal places used.

371. Observe further, that the cube of any number may be represented by $(t+u)^3=t^3+3\,t^2\,u+3\,t\,u^2+u^3$. (Art. 355, 2.)

372. These two principles or facts concerning the number of orders in any cube, and the relation of the elements of the root to the cube, determine the methods of extracting the cube root.

373. The process of this evolution may be illustrated by the use of diagrams or blocks.

Ex. What is the length of one edge of the greatest cube that can be constructed from 15625 cubical blocks, each containing one cu. ft.?

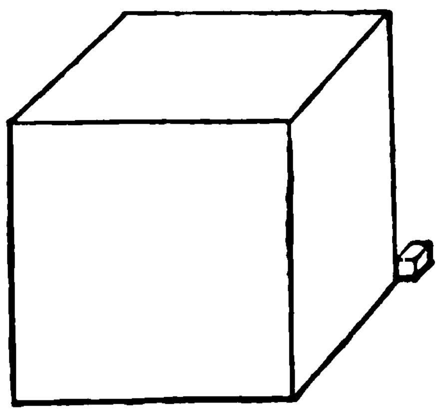
Since volume is the product of three dimensions (Art. 285), it is evident that the cube root of 15625 cu. ft. will be the dimension required. (Art. 369.)

OPERATION.

$$
\begin{array}{rll}
 & \dot{1}5625\dot{}(25 & \\
2^3= & \underline{8} & =\text{first complete cube.} \\
\text{Trial divisor, } 20^2\times3=1200) & 7625 & =\text{1st dividend.} \\
20^2\times3\times5= & 6000 & =\text{large additions on 3 sides.} \\
20\times5^2\times3= & 1500 & =\text{3 long corner pieces.} \\
5^3= & \underline{125} & =\text{small corner piece.} \\
\text{Subtrahend}= & 7625 & =\text{sum of all the additions to make 2d complete cube.}
\end{array}
$$

GEOMETRICAL ILLUSTRATION.

I.

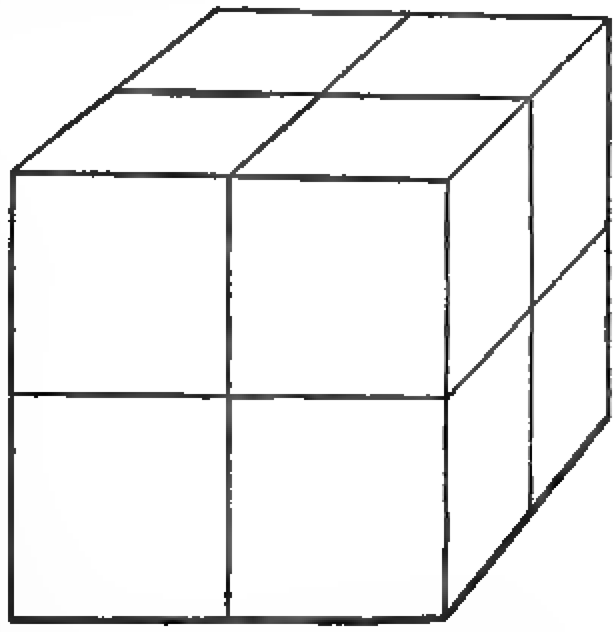


1. The ratio of units of volume is 1000 when that of linear units is 10; that is, the ratio of a 10 ft. cube to a 1 ft. cube is 1000, and the ratio of a 100 ft. cube to a 10 ft. cube is 1000. (Art. 349.) Now 15625 cu. ft. may be considered as consisting of 15 cubes, each containing 1000 cu. ft., and measuring 10 ft. on each edge (represented by the large cube sketched in Diagram I), and of 625 cubes, each containing 1 cu. ft. measuring 1 ft. on each edge (represented by the small cube sketched in Diagram I).

This may be conveniently indicated by separating 15625 into periods corresponding with the different classes of cubes, allowing three figures to a full period; thus, 15̇625̇.

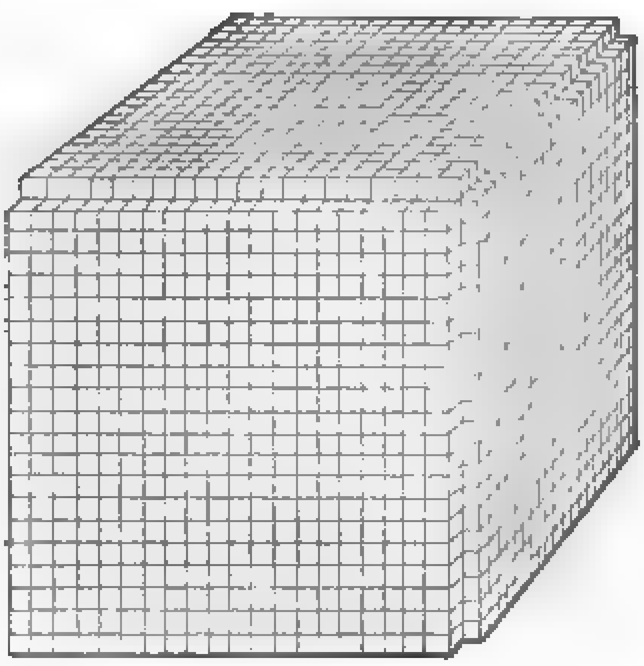
2. The largest cube that can be constructed from 15 *10-ft.* cubes contains 8 of these cubes; that is, it is a cube measuring 2 of these units on each edge, as shown in diagram II. Therefore 2 is written for the first figure of the root, and its cube, 8, is subtracted from the first left-hand period, 15, leaving a remainder of 7 *10-ft.* cubes. This remainder is reduced to 7000 *1-ft.* cubes, and combined with the period of the same order, which is now annexed to the 7 (Art. 62, Note), making 7625 *1-ft.* cubes to be so added to the first cube formed, 20^3, as to make an enlarged perfect cube.

II.



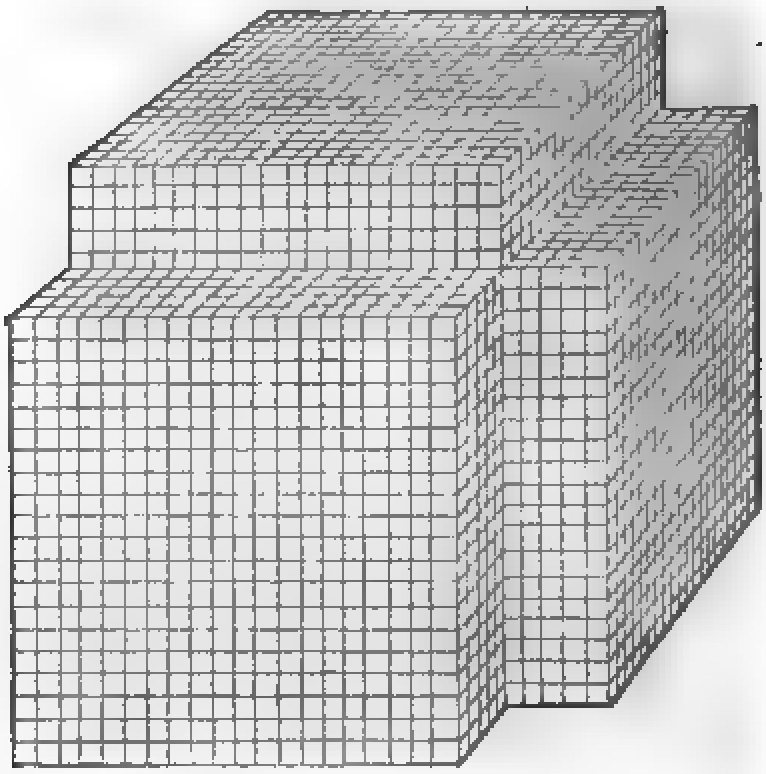
3. It is found necessary to make equal additions on at least three adjacent sides of the cube represented in diagram II, and it will evidently take 20 rows, each containing 20 *1-ft.* cubes to cover one of these sides, $20 \times 20 = 20^2 = 400$; and as the three sides to be covered are equal, it will take $3 \times 400 = 1200$ *1-ft.* cubes to cover them with a single layer of the smaller units, as shown in diagram III. This 1200 is called a ***trial divisor*** in the operation, because the quotient of 7625 cu. ft. divided by 1200 cu. ft. will evidently indicate about the number of layers of *1-ft.* cubes that may be added to the three sides, in order to properly arrange all of the units remaining, after the formation of the first cube.

III.



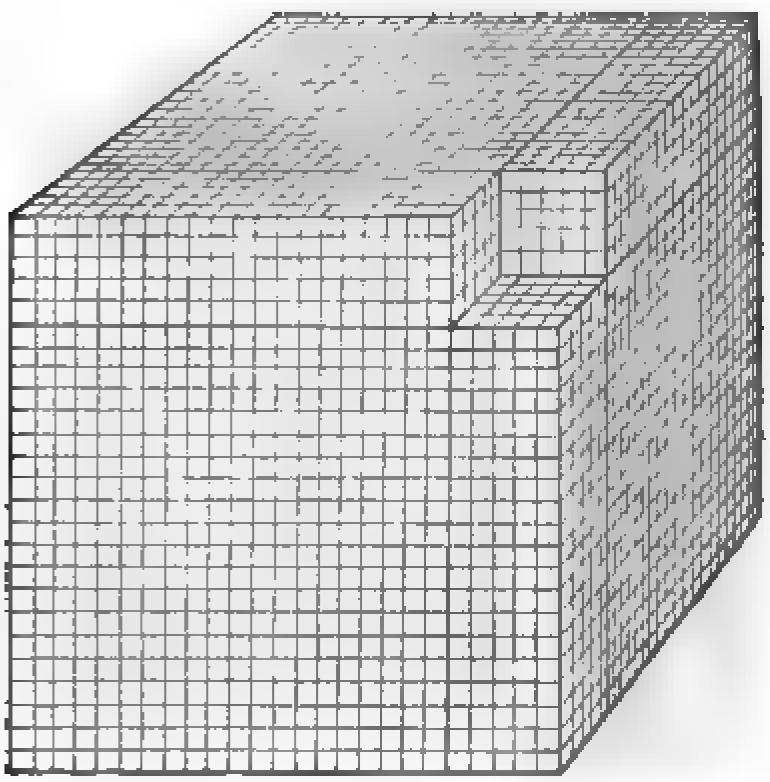
4. It is found that 5 layers of *1-ft.* cubes may be added; therefore 5 is written as the next root figure. The additions now made are represented in diagram IV, and since each of the 3 side additions is 20 units square, and is 5 units in thickness, the volume of

IV.



all of them is computed thus, $20^2 \times 5 \times 3 = 6000$, which is equivalent to multiplying the trial divisor by the last root figure found.

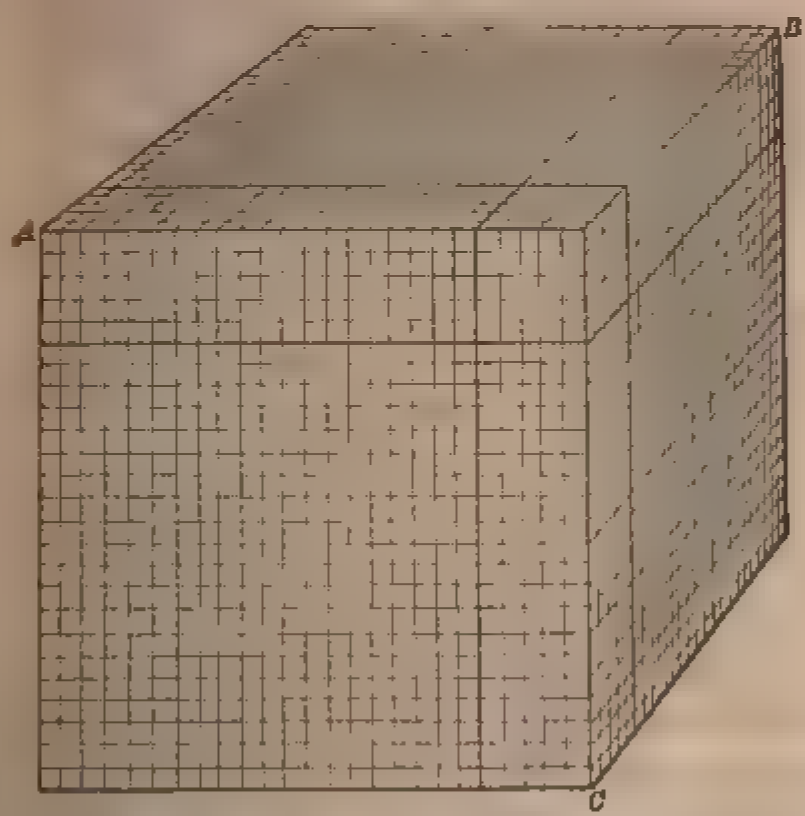
V.



5. The next additions toward completing the cube may be made most conveniently by constructing three long corner pieces to fill the spaces adjacent to the three side additions just made, as shown in diagram IV. These additions are represented in diagram V, and since each corner piece is 20 units (*1-ft.* cubes) long, 5 units wide, and 5 units high, the volume of all of them is computed thus, $20 \times 5^2 \times 3 = 1500$, which is equivalent to multiplying (a) all but the last root figure found, considered as *tens*, by the square of the last root figure found, and this product by 3.

In order to complete the cube it is necessary to make only one other addition, which shall fill the space shown at the corner adjacent to the last additions in diagram V. Since this must evidently be 5 units (*1-ft.* cubes) long, 5 wide, and 5 high, its volume is $5^3 = 125$, which is the cube of the last root figure found.

VI.

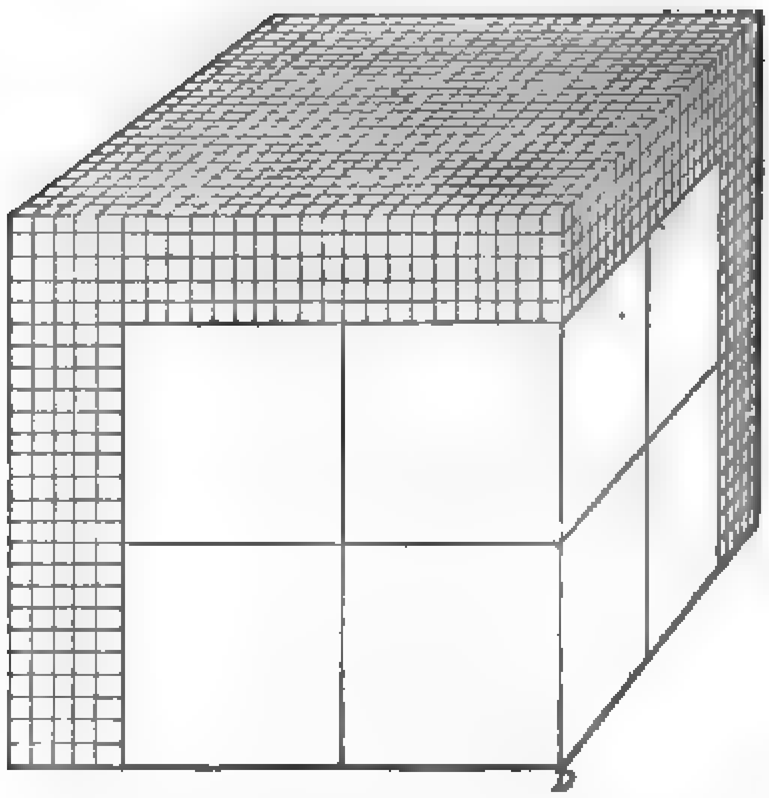


6. The completed cube is represented in diagram VI. The sum of all the additions made to the cube represented in diagram II is found to be 7625 cu. ft., which is equal to what remained after constructing cube II, hence the cube represented in diagram VI comprises $(8000 + 7625)$ cu. ft. $= 15625$ cu. ft., and is the cube required, each edge of it being 25 ft. long, and its volume being 25^3.

NOTE.—If the required root is to consist of more than two figures, the figures already found are to be considered as the first part of the root, and the next figure as the second part, the ratio of the units of these two parts always being 10; the trial divisor is 3 times the square of the first part considered as tens, the quotient is the next root figure, and the various additions are made in regular order, as shown by the operation and diagrams above, diagram II representing successively each completed cube.

7. The units bordering on one edge of the completed cube may all be regarded as of the smallest denomination (I), that is, 1-ft. cubes, 25 of which are shown to border on each of the nine edges shown in VI, three edges meeting in each of the three points, A, B, and C, or they may be regarded as consisting of 2 of the 10-ft.

VII.



cubes and 5 of the 1-ft. cubes that are shown to border on each of the three edges meeting in the point D, diagram VII, this diagram representing an opposite view of the completed cube as shown in VI. This last analysis properly represents the two parts of the root, $20+5=25$.

374. RULE.

1. *Separate the number whose cube root is required into periods of three figures each, by placing a point over the units' figure, and over every third figure from units towards the right and left.*

Find the greatest cube in the left-hand period, and place its ... the right, like a quotient.

Subtract the cube of this root figure from the first period, ... remainder annex the figures of the next period, and consider ... as a dividend.

Take three times the square of the root found considered as ... for a trial divisor.

Find how many times the trial divisor is contained in the ... lend, and write the quotient as the next figure of the root; ... multiply the divisor by this last root figure, placing the pro... under the dividend.

Multiply the square of the last root figure by the preceding ... figure or figures considered as so many tens, and this product ... ree, and place the product under the last; then under these ... products place the cube of the last root figure, and find their ... calling it the subtrahend.

Subtract the subtrahend from the dividend, and to the re... der bring down the next period for a new dividend, with ... proceed as before till the required root be found.

...TES.—1. When any dividend is not large enough to contain its trial ..., place a cipher for the next figure of the root, and take three times ...are of the root thus formed and considered as tens for a new trial ... Form a new dividend by bringing down the figures of the next ..., and in determining the quotient or next root figure, make some ...nce for the additions to be made.

When there is a remainder after all the periods are used, annex peri... ciphers and continue the operation until the requisite number of deci... ...aces be obtained.

Extract the cube root of both terms of a common fraction, when they ...fect powers; otherwise multiply the numerator by the square of the ...inator, and divide the root of the product by the denominator. The ... will be the root with an error, less than one divided by the denomi...

In extracting the cube root of decimals or mixed decimals, ciphers ... be added, to fill the periods.

When the root does not contain enough decimal places, ciphers must ...fixed.

Complete the last decimal period in the *number*, if necessary, by an... ciphers.

...**5.** 1. The process of evolution for the cube root may be ...y contracted by the following method, which may be illus-

trated by the formula for $(t+u)^3$, or by the use of diagrams or blocks.

2. If we let t or t represent the part of the root found, and u or u the next lower figure of the root, these will represent parts of the root, the ratio of whose units in each case is 10.

3. It may be sufficient simply to indicate the analysis of the operation by use of the formula, with only a partial explanation.

Ex. Find the cube root of 18609625.

OPERATION.

$$\overbrace{\mathrm{t}^3+3\,\mathrm{t}^2\,\mathrm{u}+3\,\mathrm{t}\,\mathrm{u}^2+\mathrm{u}^3}=\dot{1}8\dot{6}09\dot{6}25(\overset{\mathrm{t\,u}}{\overset{\frown}{\underset{}{t\,u}}}265$$

$$t=2 \qquad t^2=4,\times 2 \quad = \underline{8=t^3}$$

$$3\,t=60 \qquad 3\,t^2=1200 \qquad 10609$$

$$u=\underline{\ 6}$$

$$3t+u=66,\ \times 6=3t\,u+u^2 \left\{ \begin{aligned} &=\underline{396} \\ 3\,t^2+3\,t\,u+u^2\ &=1596,\times 6=\ \underline{9576}=3\,t^2u+3t\,u^2+u^3 \\ u^2\ &=\underline{\ 36} \qquad 1033625 \end{aligned}\right.$$

$$2\,u=\underline{12}$$

$$3t+3u$$

$$\text{or } 3\,\mathrm{t}=780.\ 3(t^2+2\,tu+u^2)=202800 \text{ or } 3\,\mathrm{t}^2$$

$$\mathrm{u}=\underline{\ 5}$$

$$3\,\mathrm{t}+\mathrm{u}=785,\times 5=3\,\mathrm{t}\,\mathrm{u}+\mathrm{u}^2=\underline{\ 3925}$$

$$3\,\mathrm{t}^2+3\,\mathrm{t}\,\mathrm{u}+\mathrm{u}^2=206725,\times 5=1033625=3\,\mathrm{t}^2\,\mathrm{u}+3\,\mathrm{t}\,\mathrm{u}^2+\mathrm{u}^3$$

NOTE.—$\mathrm{t}^3=(8000+9576)=17576=(t+u)^3=t^3+3\,t^2\,u+3\,t\,u^2+u^3$.

376. 1. The first term in the *first* column at the left may represent *linear units* in one edge of the cube. (Art. 373, diagram I.) The subsequent terms in this column may represent the *linear units* in one edge of each of the corner additions, that is of the three long pieces and one short piece. (Art. 373, diagram VI.)

2. When an addition is made to the first or to any successive complete cube, the sum of the linear units in the new corner pieces will equal the linear units in the previous corner pieces, plus twice the length of the short one, hence double the last root figure (2 × 6) is added to the last term in column 1st when a new figure of the root is to be found.

3. This sum considered as so many tens, increased by the next root figure, and multiplied by the same, gives the area of one side of all the corner additions made at one increase of the cube, and this area is added to the trial divisor in the 2d column to give the true divisor. Thus in the example above, 66 is the number of linear units in the corner additions to the first cube. In the next corner additions, one edge of each long corner piece, and three edges of the small corner piece, are to be covered. (Art. 374, diagram VI.) Now $66 = 60 + 6$, the combined length of the three long pieces and one edge of the small piece last added, hence if this be increased by (6×2), 12, the sum, 78, will represent the combined length of all the corner pieces for the new addition.

4. The first term in the *second* column may represent the *units of area* on one side of the cube. The subsequent terms in this column, marked as hundreds, and used as *trial divisors*, may represent the units of *area in three sides of the cube* to be covered by the addition of three large side pieces (Art. 374, diagram III), and the terms used as *true divisors* may represent the *area of one side of all the additions*, including the corner pieces. Thus in the example above, 1596 represents the area of the three large additions to the cube, and of one side of each of the corner additions. The next additions must cover one side of each of the three large additions last made, two sides of each of the long corner pieces, and three sides of the small corner piece. (Art. 374, diagram VI.) Now the area of one side of all the corner pieces is included in 396, the same is included again in 1596, so that only the square of the last root figure, 6^2, needs be added to these two terms to give the entire area to be covered. Thus each trial divisor after the first is found by adding the square of the last root figure found to the last two terms preceding in column 2d.

NOTE.—The special advantages of this method consist in continuing the computation of the linear units in the first column, and of the units of area in the second column. It is a contraction of Horner's method (Art. 382), and the process, as here given, was invented and first used by Rev. M. Earl Dunham, A.M., formerly Principal of Sauquoit Academy, Oneida County, N. Y., by whose permission it is here inserted.

377. RULE.

1. *Separate the number whose cube root is to be found into periods of three figures each, counting from the decimal point.*

2. *Find the greatest perfect cube in the left-hand period; and place its root at the right as the highest order of the root required. Write it also at the left as the first term of a first column, write its square as the first term of a second column, subtract its cube from the first period, and to the remainder annex the next period for a dividend.*

3. *Multiply the first term in each of the two left-hand columns by 3, and to the product in the second column annex two ciphers for the first* **trial divisor.**

4. *Find how many times the trial divisor is contained in the dividend, and write the quotient as the next figure of the root. Annex this figure also to the last term in column 1st, multiply this new term by the same figure, and add the product to the trial divisor; the sum will be the* **true divisor.** *Multiply the true divisor by the last root figure, subtract the product from the dividend, and to the remainder annex the next period for a new dividend.*

5. *Add twice the root figure last found to the last term in column 1st; then add the square of the same root figure to the last two numbers in column 2d, and to this sum annex two ciphers for* **a new trial divisor.** *Find how many times this is contained in the new dividend, and proceed as before with the quotient figure.*

NOTE.—Compute mentally when convenient.

Ex. $\sqrt[3]{122,615,327,232}=4968.$

		122615327232(4968
4	16	64
129	4800	58615
	1161	
	5961	53649
	81	4966327
1476	720300	
	8856	
	729156	4374936
	36	591391232
14888	73804800	
	119104	
	73923904	591391232

378. When only a limited number of decimal places is required in the square or cube root, the operation may be *contracted after obtaining one more than half the number of root figures required, by using the last remainder as a dividend, and dropping one figure from the last true divisor at each successive division, until the required number of places be found.* (Art. 263.)

Ex. Find the cube root of 3 to six places of decimals.

		3.(1.442249+
1	1	1
34	300	2.000
	136	
	436	1744
	16	256000
424	58800	
	1696	
	60496	241984
	16	14016000
4322	6220800	
	8644	
	6229444	12458888
	...	1557112
		1245889
		311223
		249177
		62046
		56064
		5982

379. EXAMPLES IN CUBE ROOT.

NOTE.—Let the pupil understand the analysis in Art. 373, or 375, and then use the best method for each particular problem.

1. Find the cube root of 912,673.
2. Of 128,024,064.
3. Of 48,228,544.
4. Of 3,048,625.
5. Of 8,365,427.
6. Of 517,781,627.
7. Of 12.167.
8. Of 15.32.

9. Of $39\frac{38}{125}$.
10. Of .000097336.
11. Of $11\frac{25}{64}$.
12. Of $\frac{216}{42875}$.
13. Of 14 to two decimals.
14. Of .015625.
15. Of $1{,}242\frac{19}{64}$.
16. Of 21, to six decimals.
17. What is the value of $14^{\frac{5}{3}}$?
18. $1.191016^{\frac{1}{3}}$?
19. $(1.7+.25)^{\frac{2}{3}}$?
20. $\sqrt[3]{3}$ to six decimals?

HIGHER ROOTS.

380. 1. It will readily be seen that $\sqrt[4]{10^4}=(\sqrt{10^4})^{\frac{1}{2}}=10$, that is, the square root of the square root is the fourth root. (Art. 350, 2.)

2. Examine the following:

$\sqrt{10^2} = 10^1$ because $(10^1)^2=10^2$.
$\sqrt{10^4} = 10^2$ " $(10^2)^2=10^4$.
$\sqrt{10^6} = 10^3$ " $(10^3)^2=10^6$.
$\sqrt[6]{10^6} = (\sqrt{10^6})^{\frac{1}{3}}$ " $(10^3)^2$ or $(10^2)^3=10^6$.
$\sqrt{10^8} = 10^4$ " $(10^4)^2=10^8$.
$\sqrt[8]{10^8} = (\sqrt{10^8})^{\frac{1}{4}}$ " $(10^4)^2$ or $(10^2)^4=10^8$.
$\sqrt[3]{10^3} = 10^1$ " $(10^1)^3=10^3$.
$\sqrt[3]{10^6} = 10^2$ " $(10^2)^3=10^6$.
$(\sqrt[3]{10^{18}})^{\frac{1}{3}} = 10^2$ " $(10^2\times10^2\times10^2)^3=10^{18}$ etc.

381. Hence, when the prime factors of the index of the root required consist of no numbers except 2 or 3,

RULE.

Make successive extractions of the square or cube root, the product of whose indices equals the index of the required root.

EXAMPLES.

1. $(279{,}841)^{\frac{1}{4}}$.
2. $(1{,}034{,}100{,}432{,}834{,}624)^{\frac{1}{6}}$.
3. $(14{,}774{,}554{,}437{,}890{,}625)^{\frac{1}{8}}$.
4. $(13{,}841{,}287{,}201)^{\frac{1}{12}}$.
5. $(26)^{\frac{3}{4}}$.
6. $(1.4)^{\frac{7}{3}}$.
7. $(.2)^{\frac{5}{3}}$.
8. $(142)^{\frac{5}{4}}$ to four decimals.

HORNER'S METHOD.

382. The following method of evolution is applicable to *all* cases. Its general principles were first elucidated in 1819 by Mr. Horner, of Bath, England, and the method of application is well stated by Mr. Greenleaf, of Massachusetts, in his National Arithmetic.

RULE.

1. *Commence as many columns as there are units in the index of the required root, by writing the given number as the first term of the right-hand column, and zero as the first term of each of the others.*

2. *Separate the given number into periods of as many figures as there are units in the index.*

3. *Find the nearest required root of the left-hand period; place this at the right as the first figure of the root; write the same figure in the first column, and, having added it to the term above it, multiply the sum by the same figure, and write the product in the second column; add this to the term above it, multiply the sum by the same figure, and write the product in the third column; add again, and thus proceed, writing the last product in the last column; subtract this from what stands above it, and to the remainder annex the next period for a dividend.*

4. *Add the root figure last found to the last term in column 1st, multiply the sum by the same figure, and add the product to the last term in column 2d, etc., writing the final product in the last column but one, and annex as many ciphers to this as may be equal to the number of the column. This will be the* ***trial divisor.*** *Repeat the process, stopping each time with one column farther to the left, annexing ciphers corresponding to the column, till the last term of the first column shall appear with one cipher annexed, after the last addition.*

5. *Consider how many times the trial divisor is contained in the dividend; write the quotient as the next figure of the root, and proceed as before.*

NOTES.—1. When any dividend will not contain the trial divisor used, write a cipher in the root, annex to the dividend another period, annex to the last term in each of the other columns a number of ciphers corresponding to the number of the column, and use the new form of the trial divisor.

2. When the root is required to many places of decimals, the work may be contracted by rejecting one figure at the right from the last term in the column next to the dividend, two from the last term of the next column to the left, etc., proceeding otherwise as direct in the rule, except that the new figure of the root is not added to the first column. When all the figures in the last term of the first column are rejected, proceed as in contraction of division of decimals. (Art. 263.)

383. EXAMPLES.

1. $(184528125)^{\frac{1}{5}}$.

1st.	2d.	3d.	4th.	5th.
0	0	0	0	$18\dot{4}528125\dot{}$(45
4				
$4\times4=$	16			
4	$16\times4=$	64		
$8\times4=$	32	$64\times4=$	256	
4	$48\times4=$	192	$256\times4=$	1024
$12\times4=$	48	$256\times4=$	1024	82128125
4	$96\times4=$	384	12800000	
$16\times4=$	64	640000		
4	16000			
200				
5				
$205\times5=$	1025			
	$17025\times5=$	85125		
		$725125\times5=$	3625625	
			$16425625\times5=$	82128125

NOTE.—In practice the work might be arranged in more condensed form.

2. $(35{,}081{,}000)^{\frac{1}{5}}$ to three decimals?

3. $(8{,}955{,}400{,}000)^{\frac{1}{10}}$ to four decimals?

4. $(572)^{\frac{1}{7}}$ to four decimals?

5. $10^{\frac{4}{5}}$ to four decimals?

NOTE.—Logarithms are especially convenient in higher involution and evolution. (Art. 361.)

SECTION XVII.

ARITHMETICAL PROGRESSION.

384. 1. When several numbers are so arranged as to increase or decrease in regular order by a common difference, they are said to be in ***Arithmetical Progression.***

2. When they increase by the *addition* of a constant number, it is called an *ascending series*, *e. g.*, 1, 3, 5, 7, 9, 11, 13, &c.

3. When they decrease by the *subtraction* of a constant number, it is called a *descending series*, *e. g.*, 19, 16, 13, 10, &c.

4. The numbers are called *terms*, the first and last being called *extremes*, and the intermediate terms the *means*.

385. 1. In Arithmetical Progression there are five elements so related to each other, that any three of them being given, the remaining two may be found. These relations give rise to twenty different cases or problems, only four of which will here be given.

2. These five elements and their common symbols are—

The *first term*,	generally	indicated by	a.
The *last term*,	"	"	l.
The *common difference*,	"	"	d.
The *number of terms*,	"	"	n.
The *sum of the series*,	"	"	s.

386. If the first term, or a, of a series be 5, and d 2, then the second term will be $5+2=7$, or the first term plus one difference; the third term will be $5+2+2=9$, or the first term plus two differences, etc., that is, in an ascending series.

This relation of any term to the first term, and the common difference, may be indicated thus:

I. *Formula*, $$l=a+d\,(n-1).$$

NOTES.—1. In a descending series, the term $d\,(n-1)$ is negative (Art. 29) in the formula given, that is, $l=a-d\,(n-1)$.

2. The pupil should be required to translate each formula into a rule.

387. If the last term be 56, the first term 8, and the number

of terms 13, then we know that $56-8$ must equal d multiplied by $(13-1)$ or 12; that is, $48\div12=4=d$; hence,

II. *Formula*, $$d=\frac{l\sim a}{n-1}.$$ (Art. 39.)

388. If $a=10$, $d=3$, *and* $l=55$, then as $55\sim10=$ a certain number of times d or 3, and as this is one less than the number of terms, we know that the number of terms must be 16. Hence,

III. *Formula*, $$n=\frac{l\sim a}{d}+1.$$

389. 1. By examining any series, as 7, 11, 15, 19, 23, 27, 31, etc., it may be seen that the sum of any two terms equally distant from the extremes is equal to the sum of the extremes, or double the average of the extremes. Thus,

$$\begin{array}{ccccccccccccc} 7 & & 11 & & 15 & & 19 & & 23 & & 27 & & 31 \\ \underline{31} & & \underline{27} & & \underline{23} & & \underline{19} & & \underline{15} & & \underline{11} & & \underline{7} \\ 38 & + & 38 & + & 38 & + & 38 & + & 38 & + & 38 & + & 38=266. \end{array}$$

Now observe that the sum of the equal amounts thus obtained is the sum of two series, each equal to the first one, hence one-half of this, that is, $266\div2=133$, equals the sum of one series. Hence,

IV. *Formula*, $$s=\frac{(a+l)\,n}{2}.$$

NOTE.—This is equivalent to the product of the average of the extremes by the number of terms.

2. By applying the ordinary principles for finding a missing term in an equation (Art. 41), and by practical analysis (Art. 334), the four formulas given may answer for *all* problems in Arithmetical Progression.

NOTE.—Let the pupil be questioned on the relations above indicated until he is familiar with them.

390. EXAMPLES.

1. A laborer agreed to dig a well 100 feet deep, for which he was to receive 1 cent for the first foot, 5 cents for the second, and so on, increasing the price 4 cents per foot for the entire depth. What would he get for the last foot?

2. If a man begin by lifting 200 lb., and make equal additions

to the weight daily for a year of 365 days, what must be the daily additions to reach 800 lb. at the end of the year?

Secondly. With what weight must he begin, so that the daily additions may be two pounds?

Thirdly. If he begin with 200 lb., and add $1\frac{1}{2}$ lb. daily, how many days would it require to reach 800 lb.?

3. How many strokes does the hammer of a clock make in 12 hours?

4. If 100 stakes be set in a straight line 10 feet apart, how much twine will it require to connect the first one in the line with each of the others separately?

5. A man agreed to contribute for a benevolent object one cent the first day, two cents the second day, three cents the third day, and so on through the year of 365 days. What was the amount of his donation?

6. Find a and s when $d=\frac{1}{3}$, $l=10$, and $n=8$.

7. Find d and n when $s=300$, $a=48$, and $l=12$.

8. Find s and a when $l=480$, $n=48$, and $d=10$.

9. Find d and n when $l=90$, $s=799$, and $a=4$.

10. If a man deposit $45 in a savings bank Jan. 1, and on the first of each month following $5 more than the previous month, and if he be credited *two cents* per month for every *five dollars* deposited, what is the amount of his deposit for Dec. 1, and what is the amount of his deposits and credits at the end of the year?

GEOMETRICAL PROGRESSION.

391. 1. When a series of numbers have a *constant ratio* between every two consecutive terms, they are said to be in ***Geometrical Progression.***

2. A series whose successive terms increase is called an *increasing series;* as, 3, 6, 12, 24, 48, 96, 192, etc.

3. A series whose successive terms decrease is called a *decreasing series;* as, 297, 99, 33, 11.

4. The first and last terms are called the *extremes,* the other terms are called the *means.*

5. The *common ratio,* as used in Geometrical Progression, is the *constant multiplier* of the terms of the series from left to right. Thus the ratio of the increasing series above is 2, and the ratio of

the decreasing series is $\frac{1}{3}$, and any consequent term of a series may be found by multiplying its antecedent term by the common ratio.

NOTE.—The definition of the *common ratio* here given may seem inconsistent with the definition of *ratio* before given and applied (Arts. 20, 321). It is to be observed, however, that as a ratio is the quotient arising from dividing the number compared by the number taken as the standard of comparison, both definitions are consistent with this general signification of the term *ratio,* the consequent term being taken as the standard in ordinary ratios and proportions, and the antecedent term being taken as the standard in Progression.

391a. 1. In Geometrical Progression, as in Arithmetical Progression, there are five quantities so related to each other, that any three of them being given the remaining two may be found. Of the twenty cases arising therefrom only four will here be noticed.

2. In the formulas expressing the relation of the five quantities referred to above, they are represented as follows:

a = The *first term.*
l = The *last term.*
r = The *common ratio.*
n = The *number of terms.*
s = The *sum of all the terms.*

392. If $a=5$, and $r=4$, then we know that the second term must be $5\times4=20$; the third term, $5\times4\times4=5\times4^2=80$; the fourth term, $5\times4\times4\times4=5\times4^3=320$, etc. Now we observe that each term is a composite number, of which the factors are the first term and some power of the ratio, or taking simply the first power of the ratio and repeating it when necessary, we find that the number of factors in each term equals the number of the term, and that each one of these factors, except the first term, is equal to the ratio; hence,

I. *Formula,* $l=a\,r^{(n-1)}$.

NOTES.—1. In a descending series r is a proper fraction.

2. As l is a composite number, either factor may be found if l and the other factor be known. Thus if $l=486$, $r=3$, and $n=5$, then r^{n-1} must equal 81, and consequently a must equal $486\div81=6$.

This formula is frequently used to advantage in problems in Compound Interest. (Art. 530.)

393. If the factor required be r^{n-1} it is readily found, and if n be known r may be found by evolution. Thus if $a=6$, $l=486$,

and $n=5$, then $486\div6=81=r^{n-1}=r^4$. Now 81 is the fourth power of the fourth root of 81, that is $r=\sqrt[4]{81}=3$; hence,

II. *Formula*, $r=\left(\frac{l}{a}\right)^{\frac{1}{n-1}}$ or $\sqrt[(n-1)]{\frac{l}{a}}$.

394. If r be known and n required, the problem becomes one of analysis and comparison in involution. Thus if $a=6$, $l=486$, and $r=3$, then $486\div6=81=3^{n-1}$. That is, 81 is a certain power of 3. Now involving 3 to a power equal to 81, we find that *it* is the 4th power of 3, and $n-1=4$, and $n=5$. Hence,

III. *Formula*, $r^{(n-1)}=\frac{l}{a}$.

That is, n minus 1 equals the index of the power to which the ratio must be involved to equal $l\div a$.

395. If the *sum of a series* be required, practical analysis will aid in discovering a method. If we take the series D, $5+20+80+320+1280$, and multiply each term by the ratio 4, we shall have a new series, which we will call

$$\begin{array}{ll} \text{F} & 20+80+320+1280+5120; \\ \text{subtract D} & \underline{5+20+80+320+1280;} \\ \text{then F}-\text{D}= & 5120-5. \end{array}$$

Now the sum of the series F is 4 times the sum of the series D; hence F−D must equal 3 times the sum of the original series D, and $(5120-5)\div3=1705$, must be the sum of the series D. Now observe that 5120 is the product of l in the series D and the ratio of that series, and that 5 is the first term of that series; hence, as r may be integral or fractional,

IV. *Formula*, $s=\frac{lr\sim a}{r\sim1}$.

NOTE.—The four formulas given may answer for most of the cases in this Progression; other cases belong properly to higher mathematics.

INFINITE SERIES.

396. 1. When the number of terms in a series is unlimited, it is called an ***Infinite Series.*** Thus 4, 7, 10, etc., or 4, 8, 16, etc., *ad infinitum.*

2. The *last term* in an *increasing* infinite series, either Arithmetical or Geometrical, is greater than any assignable number;

hence it is said to be infinitely great. It is called *infinity*, and is represented by the symbol ∞.

3. The *last term* in a geometrical *decreasing* infinite series is smaller than any assignable number; hence it is said to be infinitely small. It is called an *infinitesimal*, and is represented by the symbol zero, 0, which is to be regarded as its numerical value.

NOTE.—There cannot be an arithmetical decreasing infinite series.

4. The *sum* of a decreasing infinite series is sometimes required, in which case the formula given (Art. 396) becomes $s=\frac{a}{1-r}$.

397. EXAMPLES.

1. A man offered to purchase 10 cows, paying for the first 5 cents, for the second 15 cents, and so on tripling the amount for each succeeding cow. What would the last one cost him, and what would the whole cost him?

2. If the first term be 100, the common ratio 1.06, and the number of terms 5, what is the last term?

3. A gentleman offered for sale a lot of ten acres on the following terms: One mill for the first acre, one cent for the second, one dime for the third, and so on in geometrical progression. What was his price for the whole?

4. What is the sum of the series $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, &c., or .333, &c., carried to infinity?

NOTE.—It may be observed that the value of any *repetend* may be considered as the sum of a decreasing infinite series, of which the repetend is the first term and the ratio $\frac{1}{10}$.

5. What common fraction is equivalent to the repetend $.\dot{7}$?

6. At 12 o'clock the hour and minute hands of a clock are together. In what time will they be together again?

SOLUTION.—When the minute hand has performed one entire revolution around the face of the clock, the hour hand will be $\frac{1}{12}$ of a revolution in advance. When the minute hand shall have gone over this $\frac{1}{12}$, the hour hand will still be $\frac{1}{12}$ of that twelfth in advance, or $\frac{1}{144}$ of an entire revolution. When the minute hand shall have reached that point, the hour hand will be $\frac{1}{12}$ of $\frac{1}{144}$ in advance, and so the comparison of their relative position may be supposed to be made an infinite number of times. It is evident that for the minute hand to overtake the hour hand, it must perform as many revolu-

(and hence take as many hours) as would be the sum of the series 1, $\frac{1}{12}$, $\frac{1}{144}$, &c., continued to infinity, equal to $1\frac{1}{11}$ hours. With the above oning one might almost believe that the hour hand would always be d, but as a matter of fact we know that the minute hand does overtake pass the hour hand, and therefore at some point the distance between two must be nothing. Furthermore, as the series above represents the cessive distances apart in their actual progress, we have from this case clusive proof that the last term of an infinite decreasing geometrical ies is *absolutely nothing*.

7. If an ivory ball is let fall upon a marble slab, from a height of 10 feet, it rebounds 9 feet, and falling again, rebounds 8.1 feet, and so continues ays rebounding $\frac{9}{10}$ of the distance through which it fell last, will it ever e to rest, and if so, through what space will it have passed?

8. If $a = .05$, $r = 1.2$, and $n = 12$, what is the sum of the series?

9. Find how much must be deposited in a bank the third month, deposits ng made regularly for 10 months, the last deposit being \$3, and the ratio uccessive deposits being $\frac{1}{2}$. Find also the amount deposited in 10 months.

10. Find three means of the series of which 102 and 110.40808 are the remes.

PERMUTATIONS AND COMBINATIONS.

398. To find the number of *permutations* or different arngements of a given number of things when all are taken each e, but in a different order.

RULE.—*Find the product of the series of common numbers corponding to the number of things.*

Ex. 1. How many permutations can be made with the letters in the word ? NOTE.—$1 \times 2 \times 3 = 6$; thus *pay, apy, ayp, pya, ypa, yap.*

2. How many permutations of the nine digits are possible?

399. To find the permutations when only a part of all are en each time.

RULE. *Take the whole number of things as the first term of a reasing arithmetical series, and with a common difference of 1, m as many terms of the series as equal the number of things in h arrangement; then find the product of these terms.*

Ex. 1. How many permutations are possible with 7 musical notes, taking a time? NOTE.—$7 \times 6 \times 5 \times 4 =$ *Ans.*

2. How many numbers of five figures each can be formed from the nine its, not repeating any figure in the same number?

400. To find the number of *combinations* that can be formed hout respect to order, taking less than all at a time.

RULE.—*Divide the final product, as found in Art. 399, by the rmutations of the number in each combination.*

PART SECOND.

COMMERCIAL TRANSACTIONS AND COMPUTATIONS.

TRADE AND MONEY.

401. 1. ***Commerce*** or *trade* is the exchange of one or more articles of merchandise for others received as equivalents in value, or it is buying and selling goods for anything that represents or certifies the value transferred.

2. *Barter* is simply exchanging commodities for commodities; as, giving one bushel of wheat for two bushels of corn.

3. A *wholesale* merchant is one who generally sells goods in large quantities to other merchants. A *jobber* sells goods in smaller quantities to other merchants.

4. A *retail* merchant is one who generally sells goods in small quantities to consumers, or those who use them.

402. 1. ***Capital*** consists of any possession which is the result or accumulated product of labor, and which has an exchangeable value.

2. A *Capitalist* is a person whose capital is comparatively large, and of such a kind as may be readily exchanged, loaned, or otherwise actively employed in business transactions.

403. 1. ***Value*** is of three kinds; *real or intrinsic, commercial,* and *nominal.* Commercial value is generally indicated when only the term *value* is used.

2. The *intrinsic value* of anything is measured by the amount of labor and skill required to furnish it, and render its utility available.

NOTES.—1. *Utility* is a natural quality which labor and skill may develop but not create. It is mistaken for value by some.

2. The *cost* of anything is the expense of capital required to procure it without reference to its utility.

3. The *commercial value* of anything, called simply its *value* or *price*, is measured by its procuring or purchasing power, its exchangeability, or what it is *worth* in market. It is this which makes all trade possible.

NOTE.—*Value* seldom corresponds with *utility*. Thus water is of great utility, but it has no value unless furnished at some expense.

4. *Nominal value* indicates a value existing merely in name but not in fact, or a very small relative value. Thus the nominal value of a share of railroad stock may be one hundred dollars, while it may be *worth* any amount more or less than this; land at ten cents an acre would be said to have simply a nominal value.

404. 1. ***Money*** is any representative or symbol of capital or value used as "a common equivalent for all commodities and services."

2. Money may consist of any ordinary commodity which, by common consent, is recognized as of nearly a constant value, and is readily received by all in exchange for any kind of capital. Various articles have been used thus as money by different nations; as, slaves, cattle, sheep, fish, grain, shells, leather, salt, nails, tobacco, etc.

3. Money may also consist of some comparatively rare article not extensively used as a commodity, but having great value, being most convenient and safe as a standard and representative of value, and thus well adapted to commercial transactions.

4. *Gold* and *silver* have been used thus as money for many ages by nearly all nations. At first they were used as commodities of rare value, readily received by all in any exchange, and were estimated by weight, but later they were made into coins whose value was certified by the sovereign, and were used in this form as *money* only. This kind of money is now called *specie*.

5. Gold and silver have been found the most serviceable as money for the following reasons:

1st. They possess great value in small bulk.

2d. Their value remains quite uniform, changing only by slow degrees.

3d. They can be used or hoarded without much wear or decay.

4th. The pieces can be united or subdivided without loss of value.

5th. They are homogeneous in their structure, and easily identified.

6. Gold and silver, by being used as money, become the standard of reference for expressing the value of other commodities. The *price* of a commodity is usually its value expressed in denominations of money.

7. Gold and silver have an intrinsic value, depending upon their cost of production and uses. They derive a value from their capacity to facilitate exchanges, just as horses, mules, and railroads derive their value from facilitating transportation. Government can no more *create* a value to gold and silver than to sugar. It does, however, increase their value by coining them into pieces convenient for use, and declaring them legal tender in payment of debt.

403. 1. ***Legal Tender*** is anything with which debts may be lawfully paid, and is generally the same as the money of a country. Coins of small value, and bills of small denominations, are made legal tender for small debts, and those of larger value or denominations for larger debts.

2. Coinage being only a certificate of value already existing in the metal, it is not necessarily the work of government, neither is legal tender a *necessary* element of even a metallic currency, as is seen in the coinage of copper and nickle. Coining gold and silver enhances their value in the same way that manufacturing steel enhances the value of the iron used, or as the brand of an official inspector renders certain articles of merchandise more saleable. Making gold and silver coins legal tender increases their value because it increases their demand, for when other articles of value cannot be used in liquidating indebtedness, these will always answer the purpose.

3. Money, by being legal tender, becomes naturally a standard of value for other property; but money itself is not an invariable measure of value, for the reason that its value, like that of other kinds of property, is affected by cost of production, supply, demand, etc. The debasement of coins by government is not here taken into the account. If gold alone were used as money and

gal tender, its gradual change of value would be perceptible only it caused an increase or decrease of prices. A diminution in value of gold would raise prices, and vice versa. Generally both ld and silver are legal tenders. In making both legal tender, it necessary for government to establish their relative value. If legal relative value be the actual commercial relative value, n both will circulate equally well, except so far as convenience y dictate. But as both are constantly changing in their commercial value, while the legal relative value remains the same, the tal that has the greatest commercial value will be used to make eign purchases, while the cheaper metal will remain at home. is a general principle in currency, that if several different artis be allowed to circulate as money, the cheaper will displace dearer. If a thousand silver dollars will pay a larger debt in oreign country than a hundred gold eagles, then silver will be pped in payment.

4. By making silver legal tender for *small sums only*, its legal ative value, or mint valuation, may be considerably higher than commercial or marketable value, and still the currency will not burdened by its abundance, and gold will be retained for the son that it will be demanded for the payment of those debts t are above the silver limit. Whenever both are legal tender any amount, there must be frequent necessity for government change the legal or nominal value. In 1853, silver coins the United States were made "*legal tender for all sums not eeding five dollars*," and its nominal value was made so great t it is not probable there will be need of another change for y many years, if ever.

The relative values of gold and silver vary at different times. e dollar in gold in August, 1871, was worth \$1.12½ in curcy, and one dollar in silver was worth \$1.04 in currency; in tober, 1871, gold was worth \$1.13⅞, and silver \$1.10.

CURRENCY.

406. *Currency* is *any common* medium of exchange.

It is of three kinds, viz.: "*Value Currency, Credit Currency,* d *Mixed Currency.*"—(Walker's Science of Wealth.)

407. 1. ***Value Currency*** is real money or any certifies of value that can be exchanged at option for specie. This

comprises all standard coins of gold, silver, copper, and nickel, gold and silver ingots or bars, gold dust, and *gold certificates.*

2. The last are certificates of gold deposits, for which they can be exchanged on demand. These are used to some extent between associated banks in some large cities in settling their balances at the "Clearing-house," in some Custom-house transactions, and among gold and stock brokers. It has been strongly advocated by some that a "gold-note currency" on such a basis, might with many advantages be issued from the Sub-Treasury of the United States. Bullion banks might also be formed that would furnish the same kind of currency.

408. 1. ***Credit Currency*** consists of notes or certificates of indebtedness, commonly used instead of money. These are bank notes or bills. (Art. 573.)

2. The bills issued by the United States Treasury and national banks, called *greenbacks* and *national currency,* are a purely credit currency. They are paper notes that have comparatively no intrinsic value, and contain only a promise to "pay to bearer," or "to the bearer on demand," the amount for which the note is issued. Specie is not promised or expected in equal exchange for them, and this credit must be extended until the government makes these notes exchangeable for *money* itself, *dollar for dollar.* (*See* BANKING, 573.) Only, in a popular sense is *this* money; it is really *currency* only, its negotiability being its essential feature.

3. The "Treasury Note" is made "*a legal tender, at its face value, for all debts, public and private, except duties on imports and interest on the public debt.*"

4. Every note of the "National Currency," that is issued by any National Bank, is made "*receivable at par in all parts of the United States in payment of all dues to the United States, except duties on imports, and also for all debts owing by the United States to parties within the United States, except interest on the public debt.*" These notes are also used as legal tender between the different banks issuing them. Counterfeiters of these notes are liable to severe penalties. (See any "greenback" or National Bank Note.)

5. These notes are made of convenient denominations from "One Dollar" to "One Thousand Dollars."

6. *Fractional Currency* is issued only by the United States Treasury, and "is exchangeable for United States notes in sums not less than three dollars; receivable in payment of all dues to the United States less than five dollars, except customs." (See any note of Fractional Currency.) The denominations are 5, 10, 15, 25, and 50 cents.

409. 1. A ***Mixed Currency*** consists of specie, and bank notes that may be exchanged for specie on demand.

2. In a mixed currency there is frequently a slight depreciation of the value of the paper money, arising mostly from danger of counterfeiting, or a possible failure to redeem it.

3. Such a currency was formerly supplied by the various independent and State banks in the United States, and is still used in Canada, Great Britain, and some other countries.

410. 1. *Paper Money* (commonly bank notes), is simply certificates of indebtedness, with the promise of the maker to pay to the bearer the amount specified on demand. Hence it forms a most convenient and economical currency. In exchange for this the holder can procure any commodities or services he may desire if either the credit of the maker be good, or if it be convertible into specie at pleasure.

2. Bank notes, certificates of deposit, checks, bills of exchange, &c., are *used as money* in business, but are not money. They are *representatives of money* when an *equivalent amount* of gold and silver is lying idle, and the paper takes its place in the circulation;—otherwise they are *representatives of indebtedness* merely, and the man who receives them in payment of any debt has only given up one claim for another which may perhaps be more available. Bank notes, *actually representing gold or silver in store*, may be used with profit, for the reason that the coin lying in the vault is saved from wear, and the inconvenience and risk attending the transfer of large sums are to a great extent avoided.

3. When gold or silver is received, it is an ultimate payment, for they are supposed to contain intrinsically an equivalent value. The policy of a paper currency, beyond an actual specie basis, is a question upon which intelligent political economists disagree. The use of certificates of deposit, checks, bills of exchange, etc., greatly facilitates the transaction of business, and reduces the amount of metallic currency needed. To make them serviceable

and reliable, however, they should not be issued as a basis of credit, or for procuring loans, but should arise from legitimate business transactions in which the drawee has previously become actually indebted for the amount of the bill.

4. *Cash* is a term generally applied to money and currency, but it has also a broader meaning as applied to all kinds of notes, checks, drafts, bonds, mortgages, or other certificates of indebtedness, forms of money, or representatives of money that are readily convertible into money, at their face value.

5. *Money of account* is the kind or denomination of money used to indicate value in accounts or computations, as distinguished from denominations of currency. Thus, in the United States, *mills* are used as money of account but not as currency; in Canada many accounts are kept in sterling money, while the currency is decimal, like Federal Money; and in Hamburg the *mark banco* is used in accounts while the *mark current* of less value is used as currency; in the United States also the price of certain articles of merchandise, is frequently marked and mentioned as one or more *shillings*, which in some States indicates one-eighth of a dollar and in others a different amount, but the shilling has never been used here as currency.

UNITED STATES MONEY.

411. 1. ***United States Money,*** called also *Federal Money*, consists of *specie* currency or coin, and *paper money*, both of which are made legal tender in certain cases, although not having the same value.

2. The unit of this currency is the *dollar* (see Note, Art. 81, 2), which is divided into 100 *cents* (cts. or ¢; Latin *centum*, a hundred). In computation the cent is divided into 10 mills, (*m.* Latin *mille*, one thousand), the mill thus being one-thousandth of a dollar but not a denomination of currency. The dollar being regarded as the unit, cents are expressed decimally as hundredths and mills as thousandths. Thus seventeen dollars, twenty-nine cents = $17.29; eight dollars, ten cents, five mills = $8.105.

3. The coins are of gold, silver, nickel or copper.

Notes.—1. The principal gold coins are the $50, $20 or double-eagle, $10 or eagle, $5 or half-eagle, $2½, $3, and $1.

2. The silver coins are the dollar, half-dollar, quarter-dollar, dime (10¢), f-dime, and three-cent piece.

3. The nickel coins are the 5¢, 3¢, and 1¢ pieces, and the copper coins the 2¢ and 1¢ pieces.

4. In notes, bank checks, receipts, etc., cents are usually written as com- fractions. Thus, \$85.63 would be written \$85$\frac{63}{100}$, and read "eighty- , and sixty-three one-hundredths *dollars*."

4. Fractions of a cent are commonly mentioned as such instead the equivalent amount in mills: the grocer gives the price of s sugar as twelve and a half cents per lb. rather than twelve nts and five mills, and the merchant says calico is worth thir- n and a half cents per yd. instead of thirteen cents and five ills; but in computations, convenience may decide whether these actions be reduced to mills or retained as parts of a cent.

412. 1. The *Rules* for the fundamental operations in Federal oney, are essentially the same as those for the same operations Decimals (Arts. 232–258).

2. *Dollars may be reduced to cents by removing the decimal int two places towards the right—the equivalent for multiplying* 100; *and cents to mills by removing the decimal point one place wards the right.* Thus, \$12=1200 cents=12000 mills; \$7.25 725 cts.=7250 mills.

3. *Cents may be reduced to dollars by dividing by* 100, *or moving the decimal point two places towards the left; and mills dollars by dividing by* 1000, *or removing the decimal point ree places towards the left.* Thus, 4750 cts.=\$47.50; 3295 mills \$3.295.

4. The dollar sign, \$, should always be prefixed when figures e used and the word dollars does not follow. The same sign y be used instead of the word cents or mills. 62 cents may be ritten \$.62; and 9 mills, \$.009, 2¢ 3m as \$.023.

413. MENTAL PROBLEMS.

1. How many cents in \$2$\frac{3}{4}$?
2. How many mills in 37$\frac{1}{2}$ cts.?
3. How many mills in \$5$\frac{1}{4}$?
4. John buys a coat for 15 dollars, 60 cts., and a vest for \$5$\frac{1}{4}$. How much does he pay for both?
5. From \$9 take 9 mills.

6. Bought a barrel of potatoes for $4¾ and sold it for 5 dollars and 15 cents. How much was made by the transaction?

7. Mr. Brown had $75 in the bank. The next day he deposited $15.75, and subsequently gave Mr. Clark a check for 25 and 40 hundredths dollars; what was the balance in bank?

8. What will 40 bu. coke cost at 14 cents per bu.?

9. A merchant buys muslin at $5.75 per piece. How much must he pay for 7 pieces?

10. How many times 20 cents are 5 dollars?

11. I paid a grocer $11.55 for 7 lb. of tea. What did it cost per lb.?

12. A bought soft coal at $5 per T. and sold it at $6.50, and made $6 on all he bought. How many tons did he buy?

414. WRITTEN PROBLEMS.

1. How many mills in 28⅛ cents? In 4030½ cts.? In 106¼ cts.?

2. How many cents in 150 dimes? In 216½ dimes? In 25¾ dimes?

3. Reduce $12.50 to mills. $7050 to mills. $272.10 to mills.

4. Change $90 to mills. 7050 mills to dollars. 1682¾ mills to dollars.

5. How many cents in 2 eagles, 5 dollars, and 8 dimes?

6. Reduce 4360 cents to dollars. 118¾¢ to dollars. 23.5¢ to dollars.

7. Add the following: $9.60, $12.70, $45.37½, $.06, $1.50, $4.98, [illegible], $68.33, $8.39, $60, and $.80.

8. Sold a carriage for $120.75, a horse for $90.60, a harness for $15.60, and a saddle for $13.12½. What was the amount received?

9. From $108 take 12½ cents.

10. Bought a barrel of flour for $6.37½, and sold it for $5.87½. What did I lose?

11. Bought a house and lot for $1500. Paid $40 for a front fence, $110.90 for painting the house, $9.75 for fruit trees, and $15 for other improvements. I then sold the property for $1800. What did I gain?

12. What will be the cost of 45 barrels of flour at $5.80 per barrel?

13. What will 80 bushels of coal cost at 15 cents per bushel?

14. What will be the cost of 60 bushels of wheat at $1.12½ per bushel; 140 bushels of corn at 66⅔ cents a bushel; and 45 bushels of oats at 25 cents a bushel?

15. How many bushels of coal at 12½ cents a bushel can be bought for $125?

Suggestion.—The dividend and divisor must be reduced to the same denomination. Change both to thousandths. 125.000 ÷ .125 = 1000. *Ans.* 1000 bushels.

16. How many pounds of butter at 16 cents per pound must be given for 15 barrels of flour at $8 per barrel?

17. How many barrels of flour at $5.62½ per barrel can be bought for $225?

18. How many half-dimes would it take to pay for 16 cows at $16.37½ per head?

19. A drover bought 105 head of cattle at $57 per head. He paid for their pasturage one month $250, and then sold them at $[illegible]0 per head. What did he gain by the transaction?

20. A merchant has on hand goods to the amount of $27500, A's note for $1220, B's note for $1500, C's note for $167.85, cash $[illegible]01.08, various amounts due him as shown by his ledger accounts of $9610.80, and other property worth $20200. He owes $12506.18 for goods, $3500 for borrowed money, $4515 on real estate, and $[illegible]82.40 on sundry other accounts. Considering that he will lose [illegible]½% of the amounts due him on ledger accounts, how much do his assets exceed his liabilities?

NOTE.—*Assets* include a man's entire property and the actual value of whatever is due to him; *liabilities* include all that a man owes to others.

21. What is the sum of 69 dollars, 80 cents, 2 mills; 91 dollars, 3 cents; 43 dollars, 7 mills; 109 dollars, 15 cents, 8 mills; and 90 cents, 3 mills?

22. B owes me $35.42; D owes me $73.37½; and E owes me $[illegible]9. I owe G $15.62½ and L $55.09. How much do my assets exceed my liabilities?

23. Mr. Green bought his winter's fuel, as follows: 3 T. of hard coal at $8.37½ per T.; 2 cd. of wood at $9.75 per cd.; 50 bu. coke at 12½ cts. per bu.; and a load of kindlings for $3.62½. How much did he pay for the whole?

24. When corn is worth 57½ cts. per bu., how many bu. can be purchased for $1106.30?

25. Sarah bought 13 yd. of ribbon for $2.47, and sold a friend one-half of it at an advance of 2 cts. per yd. How much did she receive for the part sold?

26. Board for a man and his wife for the last six months of the year cost $391. How much was their board per day?

27. Mr. A bought a lot and built a house at a total cost of $5875. His lot was 45 ft. wide and cost him $62½ per front foot. He paid for lumber and carpenter work $1685.75; for chimneys and plastering $376.35; for painting and glazing $296.40; for bath tubs, water pipes and gas pipes $315.90; for blinds $76; for 2 marble mantles $62½ each; and the balance was expended in fencing the lot, making walks and preparing grounds. How much was thus expended?

28. After harvest a farmer sold grain as follows: 589 bu. winter wheat @ $1.05, 321 bu. spring wheat @ 87½ cents, and 732 bu. oats @ 43⅓ cents. He found that after paying the expenses of harvesting, threshing and marketing the grain, he had $783.25 left. How much were those expenses?

29. A grocer bought of a farmer 16 bu. potatoes @ 62½ cts., 25 doz. eggs @ 13½ cts., and 34 lbs. of butter @ 18¾ cts. He gave the farmer 6 dollars and paid the balance in sugar @ 12½ cts How much sugar did the farmer get?

30. A dealer bought chickens at 17½ cents apiece. He also bought three times as many turkeys as he did chickens, and paid the price of three chickens for one turkey; for both kinds of fowls he paid $10.50. How many turkeys did he buy?

31. A merchant sold 19 yd. of Brussels carpet and 22 yd. of ingrain carpet for $55.70; he received 45 cts. per yd. more for the Brussels than for the ingrain. What did he get per yd. for the Brussels carpet?

32. Bought 17560 bricks at $7.50 per thousand, and 15855 ft. lumber at $18 per thousand. How much did both cost?

NOTE.—Reduce the quantities to thousands and decimals of a thousand, before multiplying.

33. A gentleman bought for his horse 2890 lb. of hay for which he paid at the rate of $12.50 per ton, and 9680 lb. of straw at $6.75 per ton. What did he pay for both?

NOTE.—Reduce quantities as in preceding example and divide results by 2 before multiplying.

34. A miller sold 17 bbl. of flour at \$6.85 per bbl., 760 lb. flour at \$3.37½ per cwt., 535 lb. of meal at \$1.62 per cwt., and 1360 lb. bran at \$17 per T. How much did he receive for all?

35. What is the cost of 1600 lb. hay at \$13½ per ton, and 7¾ tons coal at the rate of 1250 lb. for \$5?

415. PROBLEMS RELATING TO MONEY AND CURRENCY.

1. If a pound of sugar be worth half a peck of wheat, what would be the price (in wheat) of 50 lb. of sugar? And how many quarter-dollars would pay for the wheat @ \$1.08?

2. If a pound of sugar be worth 8 pounds of wheat or 10 yards of tape, how much tape can be bought for a pint of wheat, a bushel of wheat weighing 60 lb.?

3. If an ounce of silver be worth 6400 ounces of iron, how many tons of iron can be bought with 3½ pounds of silver?

4. If the value of a bushel of wheat be represented by 1, what would be the value of 5 bu. 1 pk. 3 qt.?

5. Before the federal currency was established by Congress in 1786, and indeed for some time after, the denominations in the money of account in the United States colonies were pounds, shillings, and pence, as in England, while most of the *coin* in circulation consisted of Spanish silver dollars, their halves, quarters, and sixteenths. Owing to the scarcity of metallic currency, and the fact that the relative value of the money of account, compared with the silver dollar, had not been generally determined or agreed upon, remarkable fluctuations in the money of account arose, varying in different States, so that when it became necessary to fix their relative values, it was found that in the New England States £1 or 20 shillings = 3⅓ Spanish dollars, while in New York and Ohio £1 = only 2½ Spanish dollars. What per cent. below the New England standard was the money of account in Ohio?

6. Assuming the pound sterling of Old England to have been equal at that time to 4⁴⁄₉ Spanish dollars, as is stated by some, what per cent. below that standard had the New England money of account depreciated?

7. Assuming the Spanish dollar to equal a dollar in federal currency, how much less in cents would an article cost in New

York whose price was 7 shillings, than in New England where the price was 6*s.* 6*d.*?

8. Paper currency frequently occasions great fluctuations in the money of account. Continental money, when first issued, was very nearly par with silver. In 1778 its depreciation was as 6 to 1, in 1780 as 30 to 1, in 1781 as 1000 to 1. The money of account, however, would soon cease to follow such *extreme* fluctuations, and would adopt some other standard. Assuming the paper currency of Chicago to have depreciated the money of account $2\frac{1}{2}\%$ below that of New York city, how much more in New York funds would an article be worth in New York than in Chicago, the price in each place being $1000?

9. In 1837 the fineness of the silver dollar United States coin was changed from $\frac{8924}{10000}$ to $\frac{90}{100}$, but its weight, which was 416 grains, was so changed that the amount of pure silver in the coin remained the same as before. What was the weight after the change?

10. In 1853 the weight of the silver half-dollar was changed from $206\frac{1}{4}$ grains to 192 grains, the fineness remaining the same, viz., $\frac{9}{10}$. What is the value in new silver coin of the dollar coined before 1853?

11. From 1792 to 1834 the United States eagle weighed 270 gr., and was $\frac{11}{12}$ fine. Its weight was then reduced to 258 gr., its fineness remaining the same. In 1837 the fineness was reduced to $\frac{9}{10}$, the weight remaining the same, since which there has been no change. What is the present value of an eagle coined previous to 1834?

12. Augustus Humbert, United States Assayer in California, under a legal provision of 1850, issued fifty-dollar pieces of gold, purporting on their face to be 887 thousandths fine and weighing 1310 grains each. Assuming them to be of full fineness and weight, what is their value in United States gold coinage?

In the above examples no account is made of the alloy.

13. If I take 20 lb. of bullion of standard fineness to the mint to be coined, and pay a seigniorage of $\frac{1}{2}\%$, what amount of money, in gold coin, should I receive?

14. In A.D. 671 a pound sterling was equivalent to a pound Troy of silver. In the 14th century the same amount of silver was coined into £1 5*s.*, and a pound of gold into £15. After succes-

sive debasements for the profit of kings, a pound of silver now makes £3 11*s.* 2*d.*, and a pound of gold makes £50 9*s.* 5*d.* The average price of wheat in the 14th century was £1 for what now averages £2½. Prices of other staple commodities and wages have undergone a similar change. The conclusion is apparent, that though governments may depreciate the money of account, they can not force the sale of common merchandise at much less than its *value.* Though they may change the conditions of legal tender, so that 6 shillings worth of silver will pay a pound of debt, *new* contracts will recognize the change, and ultimately not be affected by it. Queen Elizabeth, using a pound of silver for coining £3 5*s.* for England, put no more than that into £8 for Ireland. What ought to have been the price of flour in Ireland for what in England cost £1?

15. James the Second manufactured four pennyworth of silver into £10, with which he paid off his soldiers. What per cent. of their just dues did they receive?

16. Suppose an estate to have been left, centuries ago, for the support of the dean of a cathedral and four choristers, the income then being £300 per year, out of which he was to pay each chorister £30. If the debasement of current coin has raised rents 250%, and the increased supply of the precious metals has raised them 150% more, how do the relative salaries of the dean and choristers compare with what they were evidently designed to be by the testator? How many times what a chorister receives *should* the dean receive?

17. July 11, 1864, \$100 in gold could be exchanged in New York for \$285 in currency. What was the gold value of a ten-dollar bill then?

18. Aug. 17, 1870, the ratio of gold to currency value was 1⅛. What was the currency value of £50 then?

NOTE.—The gold dollar contains 23.22 gr. pure gold and the £ 113 gr.

19. How many carats fine are United States gold coins? How much pure gold in 5 eagles? (See Ex. 11 and Arts. 292, 1, and 289, 3.)

20. What is the difference between the weight of 150 silver dollars and 600 quarter-dollars?

NOTE.—The amount of silver in the quarter is in the same proportion as the amount fixed for the half-dollar in 1853.

TRIAL BALANCES AND RULES FOR DETECTION OF ERRORS.

416. 1. In keeping accounts by Double Entry, each item appears in at least two different accounts, on the Dr. side in one and on the Cr. side in the other; hence the sum of all the debit entries in all the accounts will equal the sum of all the credit entries in all the accounts, and the sum of the Dr. balances will equal the sum of the Cr. balances, if all the entries are properly made.

2. ILLUSTRATION OF DOUBLE ENTRY.

Dr. CASH. Cr.

Apr.	10	To Mdse.	$ 75	Apr.	1	By Mdse.	$3000
"	12	" "	380	May	10	" "	500
May	2	" "	1200				
"	15	" "	725				
		Balance	1120				
			$3500				$3500

Dr. MERCHANDISE. Cr.

Apr.	1	To Cash	$3000	Apr.	10	By Cash	$ 75
May	10	" "	500	"	12	" "	380
				May	2	" "	1200
				"	15	" "	725
						Balance	1120
			$3500				$3500

417. ***A Trial Balance*** is a summary of the entire amounts entered on the Dr. and Cr. side of each account, or simply of the balances of all the accounts in detail, and if the sums of the debit and credit entries in the Trial Balance are not equal, then there is some error in the accounts or in making up the Trial Balance, which should be discovered.

418. The first rule of the book-keeper should be *to make no error;* but for such as are fallible the following suggestions may be of some practical utility.

1. If the error be found in one figure only, it is probably an error of adding or copying.

2. If it involve several figures it may have arisen from the omission of an entire entry or from making the same entry twice.

3. If it be divisible by 2, without a remainder, it may have arisen from posting an item to the wrong side of the account, in which case the item would be half of the apparent error.

4. If the error be divisible by 9, without a remainder, it may have arisen from transposition, three cases of which may be easily detected by rules founded on the peculiar property of the number 9. These cases are

1st. When two figures are made to exchange places with each other, the orders in notation remaining the same: *e. g.*, 372 made to read 327, or 732, or 273.

2d. When two or more figures are made to change their places in notation, their arrangement in respect to each other remaining the same: *e. g.*, \$4275 made to read \$42750, or \$42.75, or \$427.50.

3d. When *two* significant figures are made to change position both with respect to each other and also the orders of notation: *e. g.*, \$14 made to read \$0.41.

5. To detect the first and second cases of transposition *divide the amount of the error in the trial balance successively by* 9, 99, 999, 9999, *etc., as far as possible without a remainder, rejecting all ciphers at the right of the last significant figure in the error.*

The quotients that contain but one digit figure will express the difference between the two digit figures transposed, which will be adjacent to each other if the divisor consist of but one 9, separated by one other figure if it consist of two 9s, by two other figures if it consist of three 9s, and so on.

Those quotients which contain two or more figures will express the *number* itself, which is transposed in notation simply, the arrangement of the significant figures remaining the same. In either case the *order* of the last *significant* figure in the error will be the lowest order of the figures transposed. The orders of the other figures can be easily determined by referring to the error and applying the principles of notation.

6. To detect the *third* case, divide the error in the balance by as many 9s as is possible so as to give only a single figure in the quotient, and then the remainder in the same way, rejecting all ciphers at the right of the last significant figure in both dividends, after which there should be no remainder.

The first quotient will be the figure filling both the highest and lowest order in the transposition; the second quotient will be the other figure.

NOTE.—If the error of the trial-balance be not divisible by 9 it cannot be the result of transposition alone. But whenever the error becomes so reduced as to be divisible by 9 without a remainder, a transposition being then possible, the above tests should be applied.

419. To illustrate the application of the foregoing rules, four examples are given below, each one representing a balance-sheet taken from the ledger, but erroneous, from the fact that the footings of the Dr. and Cr. columns do not agree.

1.		*2.*		*3.*		*4.*	
DR.	CR.	DR.	CR.	DR.	CR.	DR.	CR.
25	34	184	74	100	22	184	22
100	981	24.50	10.25	320.60	36.40	23.50	185
87.50	73	39	200.75	400.90	20	126	71
300	90	20.40	80	10	31.20	81.44	137.80
18.40	92	120	110	10.44	10	326	323
7	93.50	100	50	495	800	3.51	6.44
94	12	90.60	33	450	200	74.25	100
81 50	310	75	25	100.16	120.50	353	40
144	86.24	201.75	40	30	200.10	25	10
63	122.22	8.25	30	20.10	10	350	290
922.40	11.84	75	333.92	8	49.50	24	35.99
1,842.80	1,905.80	855 25	929.50	1,945.20	1,499.70	1,570.70	1,221.23
63		74.25			445.50		349.47

1. The "errors" 63, 7425, 4455, and 34947 being each divisible by 9, transposition is possible. Taking the first example, we have $63 \div 9 = 7$. As this is the only division we can perform, we conclude the transposition can occur only in those amounts where the digit figures expressing the units and tens of dollars differ by 7. In the Dr. column there are three numbers answering these conditions, and in the Cr. column two, viz.: \$18.40, \$7, \$81.50, \$981 and \$92. The transposition could not have occurred in the third number, for the footing is already too small. If, then, either of the other numbers had been transposed from \$81.40, \$70, \$918, and \$29 respectively, the error is accounted for, a question easily settled by reference to the ledger.

2. In the second example, we have $7425 \div 9 = 825$ and $7425 \div 99 = 75$. The quotients containing two or more figures in the transposition must be in *notation* simply. By reference to the Dr.

nd Cr. columns it will be observed that these quotients occur four times in the former and once in the latter, viz.: \$0.75, \$201.75, \$8.25, \$75, and \$200.75. The transposition could not have occurred in the second number without displacing other significant figures, nor in the fourth, because the Dr. footing is already too small, nor in the fifth, because the Cr. footing is already too large. The only two numbers to be compared, therefore, are the first and third, which, perhaps, should have been \$75 or \$82.50, either of which would account for the error.

3. In the third example we have $4455 \div 9 = 495$, $4455 \div 99 = 45$. Here the transposition must be in notation simply, and may be found in one of two places only, viz.: \$495 or \$450.

4. In the fourth example we have $34947 \div 9 = 3883$, $34947 \div 99 = 353$, $34947 \div 9999 = 3$, with a remainder 495, which $\div 99 = 5$. We omit the division by 999, because the remainder is not divisible by 99 without a remainder. For the same reason we omitted it in the third example. In this case there could be no transposition in the notation of 3883, because the number does not occur. There may have been a transposition of \$353 from \$3.53, or the figures 3 and 5 may somewhere have been made to change places with respect to themselves and notation also; as, when \$0.53 had been made to read \$350.

NOTE.—In the use of these rules in practice, not only the balances of the ledger accounts as they appear on the balance-sheet should be examined, but also all the separate postings, as a transposition *there* will equally affect the final balance.

BILLS.

420. 1. ***Bills***, in a general sense, include all written statements of goods purchased or services rendered, and all statements of accounts.

2. *A Bill*, in a common and restricted sense, is a written statement of articles sold at one time or at different times by a retail dealer to a consumer. It should state the place and time of each sale, the names of the seller and buyer, the articles sold with the prices of each and of all, and any extra charges or discount to be allowed.

NOTES.—1 When any charge is made for the box, barrel, jar, etc., containing goods, it is customary to place its price above and to the

right of it, as in Bill 4, and its price is to be added to that of the goods it contains.

2. When goods are sold by the bulk, as sugar by the barrel, tea by the caddy, etc., the gross weight, tare, and net weight are usually given in the form of an equation, the word pounds being omitted, as in Ex. 4, 6.

3. *Tare* is an allowance made for the weight of the box, case, etc.; *net* weight includes the goods only; *gross* weight includes both the goods and the box or case.

3. A bill is said to be drawn *against* the purchaser, and in *favor* of the merchant or seller.

If the party against whom the bill is drawn is not able to pay it when presented, he may acknowledge the same by giving a *due-bill.* This will prevent all subsequent dispute as to the correctness of the claim. A bill may be receipted by means of a *due-bill,* as in Bill 7.

4. A bill is receipted by writing the words *Received payment* at the bottom and affixing the seller's name. A bill may be receipted by a clerk, agent, or any authorized person, as in Bill 2. A receipted bill is called a *Receipt.*

5. *An Invoice* is a Bill of merchandise sold to a jobber or retailer, or consigned to an agent.

6. *An Inventory* is a detailed and complete list of merchandise or other property in a particular store, or owned by a particular person, showing the value of each article or kind of property.

421. EXAMPLES.

1. *Bill.*

CLEVELAND, July 1, 1871.

MR. JOHN COOK,

Bought of SAMUEL BLISS.

15 lb. Rio Coffee,	@ 16c.	\$2.40
50 lb. W. I. Sugar,	@ $8\frac{1}{2}$c.	4.25
36 lb. Pearl Starch,	@ $12\frac{1}{2}$c.	4.50
8 gal. Molasses,	@ 40c.	3.20
90 lb. Butter Crackers,	@ 9c.	8.10
45 lb. Picnic Crackers,	@ 11c.	4.95
		\$27.40

Received payment,

SAML. BLISS.

2. BUFFALO, Jan. 1, 1871.

PETER HIND,

1870. *Bought of* JAMES FINK & Co.

July	15.	9 yd. Silk,	@ $0.95	.	
"	"	8 yd. Ribbon,	@ .45	.	
"	"	12 yd. Muslin,	@ .15	.	
Sept.	9.	3 yd. Cassimere,	@ 1.75	.	
"	"	2½ yd. Broadcloth,	@ 4.50	.	
"	"	6 yd. Doeskin,	@ 1.12½	.	
"	"	1 Cravat,	@ 1.25	.	
Oct.	15.	4 pr. Boots,	@ 5.20	.	
"	"	2 doz. Hose,	@ 2.40	.	
"	"	½ doz. Sleeve Buttons,	@ .48	.	
"	"	3¼ yd. Linen,	@ .60	.	
Nov.	30.	1½ doz. Collars,	@ 2.25	.	
"	"	2 doz. Handkerchiefs,	@ 1.40	.	
"	"	3 Vests,	@ 2.40	.	
		Received payment,			$79.765

JAMES FINK & Co.
per SMITH.

3. PORTSMOUTH, JULY 1, 1871.

MR. J. H. POE,

1871. *Bought of* WM. MILLER.

May	3.	75 lb. Sugar,	@ 6¼c.	.	
"	"	9 lb. Tea,	@ 65c.	.	
"	"	21 gal. Golden Syrup,	@ 70c.	.	
June	1.	10 lb. Spice,	@ 20c.	.	
"	"	12 lb. Pepper,	@ 25c.	.	
"	"	12 lb. Ginger,	@ 18c.	.	
"	"	15 lb. Coffee,	@ 12½c.	.	
"	10.	20 lb. Dried Apples,	@ 10c.	.	
"	"	18 lb. Dried Peaches,	@ 12½c.	.	
"	"	2 bu. Onions,	@ 80c.	.	
"	15.	13 lb. Mackerel,	@ 8c.	.	
"	18.	9 lb. Smoked Herrings,	@ 20c.	.	
"	20.	25 lb. Rice,	@ 5c.	.	
"	"	12 lb. Dried Beef,	@ 12½c.	.	
"	"	5 Sacks Table Salt,	@ 20c.	.	
"	"	5 bu. Corn Meal,	@ 80c.	.	
"	27.	17 lb. Soda Crackers,	@ 9c.	.	

Received payment, WM. MILLER.

4. CINCINNATI, Aug. 1, 1871.

MR. ROBERT SMITH,

Bought of JONES & CLARK.

3 bbl. Apples,	@	$2.75	
8 bskt. Peaches,	@	.62½	
9 bu. Potatoes,	@	.85	
1 tub .62 Butter, 49−12=37,	@	.23	
1 box .35 Eggs, 46 Doz.,	@	.17	
3 bu. Beans,	@	2.50	
1 jar .50 Lard, 57−15=42,	@	.16½	

Rec'd Payment,

JONES & CLARK,
by JOHN ALLEN.

CINCINNATI, Aug. 4/71.

5. CHICAGO, April 15, 1871.

MR. ISAAC WILLIAMS,

Bought of D. LOCK & Co.

650 lb. Sash Weights,	@	$.03¼	
8 " " Cord,	@	.30	
50 " Casing Nails,	@	.06	
1½ doz. Mortice Locks,	@	7.50	
¾ " Porcelain Knobs,	@	4.50	
¾ " Mineral "	@	2.00	
1½ " Sash Fasts,	@	1.50	
6 " Hooks,	@	.25	
24 pr. Butts,	@	.20	
12 " " Small,	@	.06¼	
10 " Plated Escutcheons,	@	.15	
2 gross Screws,	@	.62½	
18 set Blind Hinges,	@	.25	
2 " Gate "	@	.62½	
4 Mortice Latches,	@	.10	
2 set Sl. Door Sheaves,	@	1.12½	
24 ft. " " Track,	@	.06¼	
2 " " Locks,	@	1.37½	
4 Brass Catches,	@	.15	

Rec'd Paym't,

D. LOCK & Co.,
per SCOTT.

CHICAGO, Apr. 17th, 1871.

6. *Invoice.*

ST. LOUIS, Aug. 11, 1871.

[M]R. JAMES DREW,

Bought of C. RICE & Co.,

1 bbl. A Sugar,	251−19=232,	@	$.12¾	.
1 " C "	243−21=222,	@	.11¾	.
1 " Pulv'd "	269−20=249,	@	.13⅝	.
1 chest Y. H. Tea,	90−16= 74,	@	1.28	.
1 " Blk "	60−17= 43,	@	.98	.
1 caddy Jap. "	25− 6= 19,	@	1.17	.
1 sck. Rio Coffee, 132 lb.,		@	.19	.
1 mat Java " 42 lb.,		@	.27	.
1 sck. " Rice, 75 lb.,		@	.09	.
1 box P. & G. Soap, 60 lb.,		@	.10	.
1 " S. G. Starch, 36 lb.,		@	.10¾	.
1 mat Cinnamon, $4\frac{1}{16}$ lb.,		@	.70	.
½ doz. No. 2 Tubs,		@	8.50	.
2 " 3 Hp. Pails,		@	2.75	.
2 " Cl. Lines,		@	2.62½	.
1 Cheese, 52 lb.,		@	.17½	.
15 lb. Bi-carb Soda,		@	.09	.
50 " Sal "		@	.04	.

Rec'd Payment by note at 3 mo.,

C. RICE & Co.

7. CINCINNATI, June 20, 1871.

[A]MOS KENT, ESQ.,

To W. B. COOK & Co., *Dr.*

1 doz. Webster's Unabridged Dictionary,	@	$50.00	.
12 doz. Robinson's Arithmetic,	@	9.00	.
5 doz. Sanders' Fifth Readers,	@	7.20	.
9 doz. Wells's Grammar,	@	3.00	.
2½ doz. Small Testaments,	@	1.20	.

July 1/71. Settled by due-bill,

W. B. COOK & Co.

NOTE.—The form "To................Dr." is sometimes used in Bills [ins]tead of "Bought of." (Art. 421, 3.)

8. *Inventory.*

SMITH & JONES' INVENTORY OF MDSE., JAN. 1, 1871.

1 case Satinets,	500 yd.,	@	$.95
2 " Cassimeres,	900 "	@	1.75
5 ps. Extra Blk Cassimeres,	186 "	@	3.25
3 " Blue " B. Cloth,	75 "	@	3.75
2 " Extra F. Blk "	44 "	@	6.25
6 bales Tickings,	2500 "	@	.19
3 " Drillings,	2175 "	@	.13
2 " Cotton Check,	1800 "	@	.12
2 cases Prints,	1400 "	@	.09
1 " "	900 "	@	.11
3 " "	2600 "	@	.12¼
1 " Ginghams,	850 "	@	.18
1 " "	900 "	@	.18
1 " "	960 "	@	.16
3 gr. gross Coates' Thread,	432 doz.,	@	.65
2 " " Clark's "	288 "	@	.65
3 " " Assorted "	432 "	@	.35
18M Milward's Needles,		@	2.50
3 ps. Tap. Brus. Carpeting,	256 yd.,	@	1.12
2 " Extra 3-ply "	198 "	@	1.37
4 " Fine Ing. "	368 "	@	.95

9. INVENTORY OF A RETAIL GROCERY.

1⅓ bbl. A Sugar,	316 lb.,	@	$.12¼
⅔ " C "	169 "	@	.11⅓
1 " Gran. "	223 "	@	.13¼
¼ " Pulv. "	68 "	@	.13¼
1½ scks. Rio Coffee,	216 "	@	.21
3 mats Java "	119 "	@	.27
169 lb. Rice		@	.09
65 " Blk. Tea,		@	.87
83 " Jap. "		@	1.13
45 " Y. H. "		@	1.25
23 " Imp. "		@	1.35
116 " Cheese,		@	.15½

395 lb. Dr. Apples,	. . . . .	@	$.06\frac{1}{2}$	.
75 " " Peaches, peeled,	. . .	@	.23	.
136 " " " unpeeled,	. .	@	.14	.
30 " Prunes,	. . . . . .	@	.11	.
45 gal. N. O. Molasses,	. . . .	@	.78	.
38 " Silver Drip Syrup,	. . .	@	1.06	.
5 kits Extra Mackerel,	. . . .	@	2.75	.
3 $\frac{1}{4}$-bbl. White Fish,	. . . .	@	4.10	.
73 lb. Codfish,	. . . .	@	.08	.
2 bxs. Herring,	. . . . . .	@	.45	.
$1\frac{1}{12}$ doz. 3 Hp. Pails,	. . . .	@	3.00	.
$1\frac{1}{4}$ " 2 " "	. . . . .	@	2.25	.
$\frac{7}{12}$ " No. 1 Tubs,	. . . .	@	9.75	.
$1\frac{5}{12}$ " Brooms,	. . . . .	@	3.25	.
85 lb. Starch,	. . . .	@	$.07\frac{1}{2}$	.
65 " Corn Starch,	. . . . . .	@	.11	.
55 bars Soap,	. . . .	@	.07	.

ACCOUNTS.

422. 1. An Account (%) is a written statement of money due for merchandise, services or money furnished or received.

2. Every complete Account consists of two parts, *debits* and *credits*.

3. *The debits* show what is due from the party named at the head of the account, for what has been furnished *to* him, and he is called a *debtor* (Dr.) *to* the articles and amounts mentioned.

4. *The credits* show what is due to the party named, for what has been furnished *by* him, and he is called a *creditor* (Cr.) *by* the articles and amounts mentioned.

Thus, if I furnish a carriage worth \$200, and a harness worth \$80 to Thos. Brown, and he furnishes me with a bookcase worth \$120 and cash \$160, the account would be

Dr.		Thos. Brown.		Cr.
To 1 carriage, . .	\$200.		By 1 bookcase, . .	\$120.
" 1 harness, . .	80.		" cash, ...	160.

NOTE.—This illustrates simply the relation of Dr. and Cr. without giving the exact form of % in full, which would include the date.

5. Any person or thing that produces debits or credits may be designated by some *name* which is used as the title of the account in an account-book called the Ledger. Hence arise accounts with cash, expense, merchandise, etc., as well as with persons.

6. *The balance* of an account is the difference between the amounts of the debits and credits, and when this is received or paid, the % is said to be *balanced.* Computing the balance is called *balancing* an account. The balance is said to be *against* an % when the Dr. side is greater than the Cr. side, and *in favor* of the % when the Cr. side is the greater.

7. *A statement* is a brief description or memoranda of the debits and credits of an %, generally furnished, if required, when an % is to be closed or balanced.

When debits only are to be made out it is similar to a bill.

8. *An account current* is a detailed statement of items included in an % which is not to be balanced but continued.

423. EXAMPLES.

1. New York, Jan. 1, 1859.

Mr. John Smith,

To Hurd & Brothers, *Dr.*

1858.			
Aug. 20.	To 12 yd. Broadcloth,	@ $3.50	
" "	" 16 yd. Cassimere,	@ 1.12	
" "	" 17 yd. Drilling,	@ .11	
Sept. 25.	" 12 doz. Spools Cn. Thread,	@ .60	
" "	" 7 yd. Gingham,	@ .25	
" "	" 34 yd. Fine Muslin,	@ .18	
" "	" 5 yd. Red Flannel,	@ .62½	
" "	" 2½ yd. Silk Velvet,	@ 4.00	
Oct. 9.	" 12 gross Shirt Buttons,	@ .75	
" "	" 15 doz. Wool Hose,	@ 3.00	
" "	" 8 pr. Kid Gloves,	@ 1.25	
" "	" 2 doz. Linen Napkins,	@ 2.40	
" "	" 2 doz. Shirt Bosoms,	@ 4.80	
Nov. 1.	" 11 yd. Drilling,	@ .10	
" "	" 5 yd. Jean,	@ .75	
" "	" 2 Silk Hdks.,	@ 1.00	
" "	" 12½ yd. Vel. Ribbon,	@ .20	

Received payment,

John Still,
for Hurd & Brothers.

2. CHICAGO, July 1, 1859.

JOSEPH CAMP,

1858.		*To* GEO. W. COLBURN, *Dr.*		
Apr. 8.	To 8 doz. Scythes,	@	$9.00	
" 8.	" 1½ doz. Hoes,	@	5.00	
May 1.	" 6 doz. Rakes,	@	1.75	

Received payment by due-bill,

July 15, 1859. GEO. W. COLBURN.

3. ST. LOUIS, Dec. 31, 1859.

REED & SPRY,

1859.		*To* HALL SMITH & Co., *Dr.*		
July 7.	To 15 yd Cambric,	@	9c.	
" "	" 50 yd. Print,	@	12½c.	
" "	" 6 yd. Cassimere,	@	$1.60	
" 20.	" 33 yd. Sheeting,	@	11c.	
" "	" 6½ yd. Broadcloth,	@	4.37½	
" "	" 3 yd. Velvet,	@	3.00	
Aug. 30.	" 20 yd. French Print,	@	17c.	
" "	" 15 yd. Lyonese,	@	70c.	
1859.	*Cr.*			
Sept. 1.	By 40 bu. Coal,	@	11c.	
" 9.	" 6 Cords of Wood,	@	$ 3.00	
Oct. 20.	" Cash,	@	16.00	
Nov. 25.	" 8 Days' Labor,	@	1.50	
	Balance due,			

Received payment,

HALL, SMITH & Co.,
per HIBBS.

4. CHICAGO, Feb. 1, 1871.

DAVID S. SPAULDING,

1871.		*To* A. B. COOK & Co., *Dr.*		
Jan. 5.	To 25 yd. Pep. E. Br. Sheetings,	@	$.15	
" "	" 2 sps. Thread,	@	.08	
" "	" 2 pap. Needles,	@	.10	
" 11.	" 20 yd. Canton Flannel,	@	.27	
" 18.	" 11 " Cassimere,	@	1.75	
" 21.	" 30 " N. Y. Mills Shirting,	@	.28	
" 23.	" 13 " All Wl. Delaine,	@	.75	
" 26.	" 5 " Ribbon,	@	.18	
" "	" 2 pap. Pins,	@	.10	
" "	" 2 cds. Hks. and Eyes,	@	.06	
" 30.	" 9 yd. Gingham,	@	.19	

1871.		Cr.		
Jan. 9.	By 10 lb. A Sugar,	@	$.13½	
" "	" 3 3/16 " Butter,	@	.32	
" 11.	" 3 " Jav. Coffee,	@	.35	
" "	" 2 doz. Eggs,	@	.26	
" 18.	" 9¼ lb. Codfish,	@	.11	
" 20.	" ½ gal. Syrup,	@	1.30	
" 25.	" 3 lb. Rice,	@	.14	
" "	" 3 " Pulv. Sugar,	@	.16	
" 29.	" 30 " B. Flour,	@	.04½	

Balance due, ...

Rec'd Payment,

A. B. Cook & Co.

5. **Expense Account.** *Dr.*

1871.		
May 1.	To Calcimining and Cleaning Store,	$13.75
" "	" ½ ton Coal,	4.1[illegible]
" "	" 47 lbs. Wrapping Paper,	2.[illegible]
" "	" 5 Blank Books,	15.[illegible]
" 3.	" Printing Bill Heads,	9.[illegible]
" "	" Note Paper and Ink,	[illegible]
" 5.	" Advertising in Tribune,	11.[illegible]
" "	" Postage Stamps,	1.[illegible]
" 9.	" Express Chgs.,	2.4[illegible]
" 11.	" Red Ink and Rule,	.6[illegible]
" "	" 1 Gross Pens,	1.3[illegible]
" 16.	" Mending Show Case,	2.2[illegible]
" "	" 1 Broom and Sprinkler,	.8[illegible]
" 19.	" Envelopes and Postage,	1.1[illegible]
" "	" Cartage,	2.8[illegible]
" 20.	" Adv't in Evening Papers,	9.56
" 25.	" Fixing Shelves,	7.7[illegible]
" "	" Express Chgs.,	5.8[illegible]
" "	" Wrapping Paper and Twine,	3.6[illegible]
" 26.	" Cartage,	1.87
" 27.	" Bill for Gas,	3.45
" 31.	" Store Rent,	62.50
" "	" Bookkeeper's Salary,	75.00
" "	" Salesman's "	83.33
" "	" Ice Bill,	3.75

6. *Mdse. % for one month.*

MERCHANDISE. *Dr.*

1871.			
Jan.	2.	To Inventory,	$13,965.85
"	4.	" Brown & Claflin,	967.62
"	6.	" Stuart & Leiter,	1,365.38
"	9.	" Jones & Johnson,	675.85
"	12.	" Brown & Claflin,	65.83
"	13.	" A. B. Field & Co.,	835.28
"	16.	" Stuart & Leiter,	868.15
"	17.	" " "	115.92
"	20.	" James Smith & Co.,	450.47
"	"	" Jones and Johnson,	381.16
"	23.	" A. B. Field & Co.,	675.25
"	24.	" Martin & Rose,	103.43
"	"	" Brown & Claflin,	195.85
"	26.	" " "	237.67
"	27.	" James Smith & Co.,	610.20
"	28.	" Jones & Johnson,	417.53
"	30.	" Stuart & Leiter,	69.29

Cr.

Jan.	2.	By Cash Sales,	$49.15
"	"	" Booked " (i. e. chgd in %,)	63.19
"	3.	" Cash "	85.43
"	"	" Booked "	115.63
"	4.	" Cash "	106.19
"	"	" Booked "	123.09
"	5.	" Cash "	103.38
"	"	" Booked "	147.88
"	6.	" Cash "	89.85
"	"	" Booked "	163.50
"	7.	" Cash "	187.59
"	"	" Booked "	193.48
"	9.	" Cash "	119.29
"	"	" Booked "	126.47
"	10.	" Cash "	89.95
"	"	" Booked "	133.84
"	11.	" Cash "	135.42
"	"	" Booked "	109.18
"	12.	" Cash "	151.92
"	"	" Booked "	162.17
"	13.	" Cash "	99.35
"	"	" Booked "	167.45
"	14.	" Cash "	180.96

Jan. 14.	By	Booked Sales,	$213.15
" 16.	"	Cash "	142.44
" "	"	Booked "	163.65
" 17.	"	Cash "	129.88
" "	"	Booked "	174.88
" 18.	"	Cash "	108.53
" "	"	Booked "	144.86
" 19.	"	Cash "	129.27
" "	"	Booked "	163.69
" 20.	"	" "	85.56
" "	"	" "	96.14
" 21.	"	Cash "	215.81
" "	"	Booked "	280.18
" 23.	"	Cash "	176.89
" "	"	Booked "	184.90
" 24.	"	Cash "	192.22
" "	"	Booked "	105.13
" 25.	"	Cash "	88.48
" "	"	Booked "	195.18
" 26.	"	Cash "	169.89
" "	"	Booked "	98.36
" 27.	"	Cash "	86.47
" "	"	Booked "	77.41
" 28.	"	Cash "	193.18
" "	"	Booked "	276.77
" 30.	"	Cash "	169.89
" "	"	Booked "	254.58
" 31.	"	Cash "	185.83
" "	"	Booked "	193.36
		Balance,	

7. By balancing my accounts, I find James Johnson's balance in my favor is $396.63; David Ely's balance in his favor is $212.98; Charles Field's balance in my favor is $796.08; Isaac North's balance in my favor is $71; Brown & Claflin's balance in their favor is $968.77; Jacob Smith's balance in my favor is $519.28; Stuart & Leiter's balance in their favor is $623.10; Franklin Jones' balance in my favor is $437.71; Abram Lee's balance in my favor is $211.43; Joseph Allen's balance in my favor is $95.89; Sweet & Swift's balance in their favor is $191.29; Hugh Wood's balance in my favor is $187.15; Peter Drake's balance in my favor is $418.95; Simon Gay's balance in my favor is $229.45; Willard Kean's balance in my favor is $679.23; R. M. Scott & Co.'s balance in their favor is $397.22; Thomas Hood's balance in his favor is $73; Henry Snyder's balance in my favor is

; Mrs. Jane Adams' balance in my favor is $49.20; William Noyes' ce in my favor is $129.81; Daniel Brown's balance in my favor is $286.58; Cook's balance in my favor is $19.25; Homer Wilson's balance in favor is $311.97; Asa Clark's balance in his favor is $161.74; Bank balance in my favor is $965.13; if I collect all that is owing me, how money will I have, after paying my debts?

REDUCTION OF CURRENCIES.

424. 1. ***Reduction of Currencies*** is computing the s of money expressed in the units or denominations of one try in those of another. This is used mostly in bills or ices of foreign goods, and in foreign exchange. (Arts. 489, 622.)

2. The general standards for comparison of values are *gold* *silver*.

425. 1. The *legal value* of foreign *coins* is the rate of valua- fixed by law. The value of other currency and money of ount is certified in each case by the American consul at the icular place from which the invoiced goods are exported.

2. The *intrinsic value* of foreign coins is their value depending he weight and fineness of the metal of which they are made; is also called the *par of exchange* except in case of the English sovereign.

NOTES.—1. At the United States mint pure gold is now estimated at $\$$? per troy ounce, and pure silver at about $\$1.30\frac{?}{?}$ per troy ounce, a r deduction being made for alloy in weighing, and 1 oz. of pure gold worth about 15.182 oz. of pure silver; and 1 oz. of standard American .900 fine, being worth 15.988 oz. of Standard American silver.

2. Gold and silver foreign coins are *purchased* at the mint at $\frac{1}{2}$ of one cent less than the values mentioned.

3. *The commercial value* of coins is what they are worth in the market.

GENERAL RULE.

426. 1. To reduce foreign money to United States money.

Multiply the amount expressed in units of the foreign money the value of one of these units expressed in United States money.

2. To reduce United States money to foreign money.

Divide the amount expressed in United States money by the ited States value of a unit of the kind of money required.

NOTES.—1. Compound numbers should be first reduced to units whose value is known.

2. When the United States value of any foreign money is not given, it should first be reduced to sterling money and then to the denomination required.

3. In exchange, *par value* is a certain value generally equal to the *intrinsic value*; *premium* is any per cent. advance on the par value; and *discount* is any per cent. deduction from the par value.

427. 1. The *old par value* of the pound sterling is $\$4\frac{4}{9}$ and *this* is still used as the basis of computation, although the *legal value* is now 9% above this, and the *par of exchange* $9\frac{1}{2}\%$ above it. (Art. 606.)

2. At the old par value, observe that as $£1=20s.=\frac{40}{2}s.=\$\frac{40}{9}$, therefore $\frac{1}{2}s.=\$\frac{1}{9}$.

428. SPECIAL RULE FOR STERLING MONEY.

Reduce to half-shillings and divide by 9. Then compute any premium or discount on the old par value.

NOTE.—It is generally most convenient to first reduce the pounds to shillings and express the entire number of shillings and pence; then multiply the shillings by 2 and divide the pence by 6 to get half-shillings, etc. Thus £3 5s. 11d.=65s. 11d., and $(65\times 2)+(11\div 6)=131.833\frac{1}{3}$ half-shillings, and this divided by 9 gives \$14.648+. If exchange was at 10% premium this would be worth (\$14.648+\$1.464)=\$16.112. At 9%, add to the *par value* 1% of the *number* of half shillings.

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429. EXAMPLES.

NOTE.—See tables in Part Third.

1. What is the old par value of £172 8s. 9d.?

2. What is the legal value of £200 18s. 7d.?

3. What is the par of exchange value of £500 5s. 5d.?

4. What is the difference between the legal and par of exchange values of £225 2s. $2\frac{1}{2}d.$?

5. What is a similar difference for 5000 francs?

6. What is the par of exchange value of \$1000 in francs?

7. What is the legal value of 100 marcs banco, 12 shillings, 10 pfennings of Hamburg? (Art. 879.)

8. What is the par of exchange value of \$2500 in rix dollars of Bremen?

9. What is the legal value of 100 florins, 50 kreutzers, 3 pfennings of Austria?

10. What is the par of exchange for 2000 silver rubles of Russia?

PROFIT AND LOSS.

430. 1. ***Profit*** is any gain realized from an *appreciation* or increase of value in commodities.

2. ***Loss*** is any loss realized from a *depreciation* or decrease of value.

3. Variations in value may arise from modifications by natural causes, by labor and skill, by the variation of supply and demand, or by speculation.

4. A large part of the profit of the merchant is really his wages or the compensation paid him by his customers for his services in furnishing the goods they want; the rest is made up of compensation for risk assumed and for skill in doing business, determined chiefly by the merchant himself and limited by competition.

431. 1. The price paid for an article, or the total expense of producing it, is its *cost;* the amount received for an article by the vender is its *selling price.* It is evident, from this, that the *selling price* of the vender, or salesman, may be the *cost* of an article to the purchaser.

2. When an article is sold for more than its cost, there is a *profit*, or *gain;* when it is sold for less than its cost, there is a *loss.* The actual gain or loss is the amount of this increase or decrease.

3. Profit or loss is generally computed as a given amount upon every hundred, or at a given rate per cent. The rate per cent. is the number of hundredths of the *cost* gained or lost.

4. Profit and Loss, though usually, are not always limited to transactions in money. When money, goods, time, distance, or anything else, undergoes an increase or decrease, there is gain or loss, and it may be computed at a rate per cent.

432. The terms concerned in Profit and Loss, with convenient symbols and the corresponding terms in Percentage, are as follows: (Art. 271.)

	Terms.	Symbols.	In Percentage.
1.	Cost, ...	c. . . .	Base.
2.	Per cent. of gain or loss, .	$\%g$. or $\%l$.	per cent.
3.	Gain or loss, ...	g. or l. .	Percentage.
4.	Price, ...	p. . . .	Amount, or Difference.
5.	Amount per cent. of gain, .	$A\%$ . .	Amount per cent.
	Difference " " loss, .	$D\%$ . .	Difference per cent.

433. The following formulas are adapted to the solution of all problems in Profit and Loss when reduced by analysis (Art. 304) to simple terms and conditions.

$$\text{I. } c + g, \text{ or } c - l = p.$$
$$\text{II. } c \times \%g = g, \text{ and } c \times \%l = l.$$
$$\text{III. } c \times A\%, \text{ or } c \times D\% = p.$$

NOTES.—1. Observe that in I. the *price* appears as a *sum* or a *difference*, one of the two elements of which, in each case, is the *cost*.

2. In II. and III. the gain, loss, or price appears as a *product* of two factors, one of which in each case is the *cost*.

3. In all *operations* the per cent. should be expressed decimally.

4. Let the pupil be questioned on the relations of the terms mentioned, and required to translate each formula.

434. MENTAL PROBLEMS.

1. If you gain 15% in selling a melon that cost you 40¢, how much is your profit?

2. If you buy a ball for 30¢ and lose $33\frac{1}{3}$% in selling it, what is your loss?

3. If you gain 7¢ in selling a book at $12\frac{1}{2}$% advance, what was the cost?

4. If by selling at a loss of 20% you lose 10¢ on a book, what did it cost?

5. It is said that soda water, with the syrup in it, costs the druggist $1\frac{1}{4}$¢ per glass. If he sells it at 10¢ per glass, what is the per cent. of gain?

6. If you lose 14¢ on what cost you 70¢, what is your % of loss?

7. If you gain 7¢ on 40¢, what is the price?

8. If you lose $12 on $80, what is the price?

9. If the price be $20 and the gain $4, what was the cost, and what is the % of gain?

10. If the cost be \$15 and the price \$24, what is the gain, and % of gain?

11. If the price be 80¢ and the cost 96¢, what is the loss, and the % of loss?

12. If the price be \$54 and the loss \$18, what was the cost, and the % of loss?

13. If the cost be \$80, and the amount % be 120%, what is the selling price?

14. If the gain % be 30% and the price \$26, what was the cost?

15. If the cost be \$40, and the price \$48, what is the amount per cent.?

16. If the cost be \$15, and the difference % be 80%, what is the price?

17. If the difference % be 90% and the price \$3.60, what was the cost?

18. If the cost be \$2.40 and the price \$1.68, what is the difference %?

19. If the gain at \$55 be 10%, what would be the gain at \$60?

20. If a man sell 40% of his goods at a profit of 15% and 60% of them at a loss of 10%, how much does he gain or lose if all the goods be sold for \$1200?

NOTE.—Similar problems for mental solution may be composed by the teacher or pupil until the processes shall become familiar.

433. 1. In *marking prices* on goods, they are marked at about a certain % advance on the cost, the rate being by no means uniform, and in the retail trade inconvenient fractions being frequently avoided by marking a little higher. Profits vary from 0 to 250 per cent., the ordinary range being from 5 to 50%; losses range from 0 to 100%. *Query*—Why not more than 100% loss?

2. Generally both the price and the real or assumed cost are marked on goods, the difference between the cost and the price being a "margin" for profit or variation.

3. *Prices* do not vary in proportion to the actual cost of goods, but they are determined generally by the ratio of supply to demand, the grade, the pattern, the amount of competition, and the desirability of making sales.

4. Sometimes goods are marked with a "wide margin" to allow salesmen to safely "fall in price," to suit customers. Thus a margin of 50% will allow a "reduction" of 30% and still secure a

profit of 5%. *Query*—What would be the result of selling the same goods 50% "under price"?

5. Sometimes in "closing out" a stock of goods "at great bargains," some leading articles are sold at from 1% to 5% below actual cost, while the larger part may be sold at from 1% to 10% advance.

6. Various devices are employed to render cost and price marks unintelligible to all not employed in the house where the goods are sold. Thus arbitrary values are assumed for certain figures, letters, or symbols, and these are changed whenever necessary. This may be illustrated by using the following letters or symbols for the figures standing above them.

1,	*2,*	*3,*	*4,*	*5,*	*6,*	*7,*	*8,*	*9,*	*0.*
d	*o*	*n*	*t*	*b*	*e*	*l*	*a*	*z*	*y*
♃	∠	θ	>	□	Δ	♆	▽	⊙	♀

NOTE.—Other devices may be invented or learned in business.

436. When it is desirable to change the price of goods, marking them higher or lower, at a certain per cent. on the old price, the required price may be found by the following

RULE.

1. *Find the difference between the old and new per cent. of gain or loss.*

2. *Divide this by the old amount or difference per cent.; the quotient will be the per cent. advance or discount from the old price.*

3. *Multiply the old price by the amount or difference per cent. just found and the product will be the required price.*

NOTE.—If the cost is to be taken as the standard for marking the new price, apply Formula III., Art. 433.

Ex. 1. What advance or discount must be made on the price of cloth marked @ \$2.60 to secure a profit of 26% or 14% if the present price affords a profit of 20%? What price required?

Ans. $\begin{cases} 5\%. \\ \$2.73 \text{ or } \$2.47. \end{cases}$

OPERATION.

$$(.20 \sim .26) \div 1.20 = .05;\ \$2.60 \times 1.05 = \$2.73.$$
$$(.20 \sim .14) \div 1.20 = .05;\ \$2.60 \times\ .95 = \$2.47.$$

Ex. 2. At what price must cloth now marked @ \$2.80, be sold to lose 26% or 14% if the present price is at 20% loss.

$(.20 \sim .26) \div .80 = .07\frac{1}{2}$; $\$2.80 \times .925 = \$2.59$.

$(.20 \sim .14) \div .80 = .07\frac{1}{2}$; $\$2.80 \times 1.075 = \$3.01$.

NOTE.—Profit and Loss are *very* widely applied, as will appear in several articles following. The problems given in the next article show some of their more common applications.

437. PROBLEMS IN PROFIT AND LOSS.

NOTE.—When more convenient, use the equivalent common fraction instead of the %.

1. If broadcloth cost \$4 per yard, for what must it be sold to gain 30%? (Art. 433, III.) $\$4. \times 1.30 = \text{price}$; that is, the price is 130 per cent. of the cost.

2. For how much must flour be sold to gain $12\frac{1}{2}\%$, if it cost \$5.60? ($\$5.60 \times \frac{9}{8}$.)

3. A man paid \$7950 for a farm, and offered to sell it for 15% less; what was the price asked? ($c \times D\% = \$7950 \times (1.00 - .15)$.)

4. A grocer paid \$11.25 per cwt. for C sugar and \$12.50 per cwt. for A sugar. At how much per lb. must he retail each to gain 12%? Also what price of a mixture of the two would afford the same gain? (Art. 336.)

5. Bought linen for 65¢, 85¢, and \$1.05 per yd. Having been damaged, it is to be sold at 28% loss. What is the price of each?

6. A man gains 30% by selling tea at $\$1.62\frac{1}{2}$ per lb. What was the cost?

NOTE.—Cost × 1.30 = Price.

7. A man was obliged to sell his house and lot for \$4250, which was 15% below cost; what was the cost?

8. By selling cloth at an advance of 27%, a merchant gained \$2.43. What was the cost?

NOTE.—$c \times .27 = g$.

9. How much must be received for goods in a year to afford a net profit of \$6000, at an average gain of 18%, the expense of their purchase and sale being \$3000?

10. A farm that cost \$7680, was sold for \$8640. What was the per cent. of gain?

NOTE.—Cost × Amount % = Price.

11. What per cent. was lost on a gun bought for \$32.50 and sold for \$25 ?

12. What is the % loss in buying coffee @ 30¢ and selling the same @ 24¢ ?

13. Bought a horse for \$130, paid for its keeping, two months, \$6, and then sold it for \$124; what per cent. was my loss.

14. A merchant made a profit of \$156 by selling a quantity of silks at a gain of 12 per cent. What was the cost of the silks, and for how much were they sold ?

Explanation.—Since he gained 12 per cent., or $\frac{12}{100}$ of the cost, \$156 must be $\frac{12}{100}$ of the cost, which is \$1300; \$1300 + \$156 = \$1456, selling price.

15. A grocer bought a lot of apples, and sold them at 30 per cent. profit, by which he gained \$36.60. How much did they cost him, and for how much did he sell them ?

16. Sold a cargo of wheat for \$16000, at a profit of 25 per cent. What was the cost of cargo ?

Explanation.—\$16000 is 25 per cent. *more* than what number? Or thus: Since I gained 25% per cent. or $\frac{25}{100} = \frac{1}{4}$, I must have sold it for $\frac{5}{4}$ of the cost.

17. Gould & Brown sold a lot of goods for \$16500, at a profit of $33\frac{1}{3}$ per cent. What did the goods cost them ?

18. Sold tea @ \$1.35 and gained 20%. What per cent. would have been gained by selling the same @ \$1.50 ?

NOTE.—Many problems of various kinds are most readily solved by proportion. Thus, \$1.35 : 1.20 :: \$1.50 : $(1.33\frac{1}{3})$; hence, *Ans.* $33\frac{1}{3}$%.

19. Sold a lot of books for \$480, and lost 20 per cent.; for what should I have sold them to gain 20 per cent. ?

20. If tea, when sold at a loss of 25 per cent., brings \$1.25 per lb., what would be the gain or loss per cent. if sold for \$1.60 per lb. ?

21. A merchant marked a piece of carpeting 25 per cent. more than it cost him, but, anxious to effect a sale, and supposing he should still gain 5 per cent., sold it at a discount of 20 per cent. from his marked price. Did he gain or lose ?

NOTE.—125% − 20% of 125% = 100% of cost for the price received.

22. If the retail price of tea @ \$1.20 affords 40% profit, what is the per cent. of profit at a wholesale price 12% below the retail ?

23. My retail price for broadcloth is \$4.75 per yard, by which I make a profit of $33\frac{1}{3}$ per cent. I sell a wholesale customer 100 yards at a discount of 30 per cent. from the retail price. What per cent. do I gain or lose, and what do I receive per yard?

24. A merchant asked for a quantity of dried fruit 22 per cent. more than it cost him, but being a little mouldy, he was obliged to sell it for 10 per cent. less than his asking price. He gained \$98 by the transaction. How much did the fruit cost? For how much did he sell it? What was his asking price?

25. I bought a horse of Mr. A for 15 per cent. less than it cost him, and sold it for 30 per cent. more than I paid for it. I gained \$15 in the transaction. How much did the horse cost Mr. A? How much did it cost me? For what did I sell it?

26. By selling Java coffee @ 27¢, there is a profit of 20%; what must be the price to afford $16\frac{2}{3}\%$ gain?

NOTE.—By Art. 486, $(20 \sim 16\frac{2}{3}) \div 1.20 = .02\frac{7}{9}$ and $27¢ \times .97\frac{2}{9} = 26\frac{1}{4}¢$. By Formula III, Art. 433, $(27¢ \div 1.20) \times 1.16\frac{2}{3} = 26\frac{1}{4}¢$ *Ans.* The more convenient method should be used in each particular case.

27. The cost of purchasing and transporting a quantity of goods from New York to Chicago is 9 per cent. of the first cost of the goods. If a merchant in Chicago wishes to make a profit of 25 per cent. on the full cost of the goods, what per cent. gain on the *first cost* must he ask for them? What amount of goods must he purchase in New York to realize a profit of \$3625 on the *first cost?* What would be the real profit on full cost?

28. What must be the asking price of cloth costing \$3.29 per yard, that I may deduct $12\frac{1}{2}$ per cent. from it, and still gain $12\frac{1}{2}$ per cent. on the cost?

29. I bought a lot of coffee at 12 cents per pound. Allowing that the coffee will fall short 5 per cent. in weighing it out, and that 10 per cent. of the sales will be in bad debts, for how much per pound must I sell it to make a clear gain of 14 per cent. on the cost?

30. What must be the asking price of raisins costing \$7.364 per box, that I may fall 10 per cent. of it and still gain 10 per cent. on the cost, allowing 10 per cent. of sales to be in bad debts?

31. The direct distance from Chicago to Washington is about 600 miles; the distance by railroad is 845 miles. Suppose that on account of curves the cars run 10% slower than they would on a

straight track, what per cent. of time is lost by the indirectness of the route, if the cars could go 30 miles per hour by an "air line?"

32. A grocer and a merchant sell to each other at cost. The merchant sells the grocer $2\frac{1}{4}$ yd. of broadcloth, the profit on which at the regular price of $7.30 per yd. is 25%. The grocer pays for the cloth with sugar and coffee in equal quantities; the profit on the sugar at the regular price of $13\frac{1}{2}$¢ being 8%, and the profit on the coffee at the regular price of 28¢ being $16\frac{2}{3}$%. How many pounds of each does the merchant receive? Which one lost the greater amount of profit by the trade, and how much?

33. A tailor charges a hatter $48.375 in % for a suit of clothes of which the regular price is $45. What should the hatter charge the tailor in % at the same advance, for a hat, the regular price of which is $10?

34. At what price per bushel must 3000 bu. wheat be sold to gain $4\frac{6}{11}$%, if 2000 bu. of the lot were purchased @ $1.09 and 1000 bu. @ $1.12?

35. If 20% be gained by selling cloth @ *b.le*, what would be the % gain @ *e.yy*? (Art. 435, 6.)

COMMISSION AND BROKERAGE.

438. 1. ***Commission*** is a compensation allowed for dealing in merchandise or other property, collecting debts, or transacting any similar business for another.

NOTES.—1. A *commission* is really a *service* or *transaction* by one party for another, but the definition given above is the one commonly accepted in the commercial world.

2. *Merchandise* (mdse.) is a term generally applied to everything of value excepting money, bills of exchange, real estate, stocks, insurance, and services.

2. Commission is generally computed at a certain per cent. of the money expended or collected; for some articles, however, it is computed at a certain rate per unit of weight or measurement; as, one cent per bushel for grain, etc.

439. 1. The one for whom the business is transacted is called a *Principal* and the one who operates for him is called a *Commission Merchant*, or *Factor*.

2. A quantity of goods sent or committed to a Commission

Merchant is called a *Consignment*, especially when the Principal and his Agent do not reside in the same place.

3. When a consignment is made, the Principal is called the *Consignor* or *Shipper*, and the Agent is called his *Consignee* or *Correspondent*.

440. 1. The account rendered to a consignor by a consignee is called an Account Sales. (Art. 698.)

NOTES.—1. The usual rates of commission for some ordinary transactions are as follows Auction Sales 5 @ 10 , Coffee 1, Cotton Goods 2 , Flour 2½, Fruit 5 @ 10, Grain 1¢ per bu., Iron 2 , Lumber 2½, Molasses and Syrups 1, Produce 5, Sugar 1, Wool 2¢ per lb. or 5% (all expenses), Woolen Goods 5%.

2. In some kinds of business, especially Insurance, the Agent receives a commission on the amount of money secured or collected by him from the same parties each successive year, the amounts received after the first year being called *renewal commissions*, or simply *renewals*. The usual rates of first commissions in Life Insurance are 10 @ 40%, and renewals 1 @ 10%; in Fire and Marine Insurance first commissions are usually 10 @ 20%, and renewals 1 @ 10%.

2. In addition to the commission the consignor is usually charged with the incidental expenses of transportation, storage, handling, inspection, insurance, etc., special rates for these charges being established in particular cities.

NOTE.—*Del credere* is a *guaranty* given by an agent for the security of sales made on credit, for which an additional percentage of 1½ @ 3% is allowed.

3 The *net proceeds* of a commission sales or a consignment is the amount of money due the consignor from the consignee after paying the commission and all other charges

441. 1 ***Brokerage*** is a single commission paid for simply securing or effecting a sale or purchase of mdse., stocks, gold, land or other property, for exchanging money, negotiating any bargain between a seller and a buyer, or transacting any similar business for another.

2. A *Broker* is a person who transacts business for a brokerage.

3. A commission merchant or factor transacts business for but few consignors, sometimes for only one, while a broker acts as a common agent for any and all who desire his services, except in cases of special contract.

4. The thing bought or sold is not consigned to the broker, but is generally transferred directly from the seller to the buyer in the most convenient manner. Brokers generally effect trans-

fers except in mdse., but without extra charge except for legal fees, as in real estate transfers.

5. Brokerage on mdse. is generally paid by the seller, but that on stocks or gold is paid by the particular person *for* whom the sale or purchase is effected.

NOTE.—The usual rates of brokerage are for Grain ¼ @ ½¢ per bu., Merchandise ½ @ 1¼%, Stocks ⅛ @ ¼%, Real-estate 2 @ 3%, Loans 2½ @ 5%, Rents 1 @ 2%, Collections 1 @ 5%.

6. Many stock brokers are also actual *dealers* in money, purchasing uncurrent money for that which is current at about ½% below the commercial value; they also frequently conduct a private banking business, receiving deposits, making loans, and dealing in bills of exchange. (Art. 563.)

NOTE.—The rate for exchanging money is not uniform. Thus, if gold is worth 112½ in New York, a broker outside will pay 112 for it; if Canada currency is worth 98¾ in United States gold, the broker will pay 98¼ @97¾ for it, making the "margin" equal to ½ @ ¾¢.

7. A *Pawnbroker* is a person who loans money for which he receives a compensation in advance, and for the secure payment of which he receives from the borrower certain personal property of considerably greater value than the money loaned. The personal property thus deposited for the security of the debt is called a *pawn* or pledge, and if not redeemed within a certain specified time by the payment of the money due, after proper notification, the broker may sell at auction or at a fair market value, the property pawned, and apply the proceeds to liquidate the debt, and he is required by law to pay the balance to the borrower.

The pawnbroker usually makes extra charges for insurance, storage, etc.

442. In *computing* commission and brokerage, the *amount of money received or expended* for the seller or buyer is taken as the *base* of the percentage allowed, except in the case of brokerage on *stocks* and *money*, of which the *par value* is usually taken as the base.

443. 1. To find the commission.

I. *Base* × *rate* = *commission.*

2. To find the amount required to be invested and to pay the commission.

II. *Base* × *amount per cent.* = *amount required.* (Art. 272.)

NOTES.—1. *The amount minus the base gives the commission.*

2. *The amount divided by the amount per cent. gives the investment.*

444. EXAMPLES.

1. A commission merchant in New Orleans purchased cotton for a manufacturer in Lowell to the amount of $10576. What is his commission at $2\frac{1}{2}$ per cent.? (Art. 156.) $2\frac{1}{2} - \frac{1}{4}$ of 10.

2. Paid a broker $\frac{1}{4}$ per cent. for exchanging $750 Ohio money for Eastern funds. How much was the brokerage?

3. My agent charges me $25 for collecting $800. What is his rate of commission?

4. An architect charges $\frac{3}{8}$ per cent. for plans and specifications, and $1\frac{1}{4}$ per cent. for superintending a building which cost $32000. What is his fee?

5. I collected 65 per cent. of a note of $87.50, and charged [illegible] per cent. commission. What is my commission and the sum paid over?

6. My agent in Baltimore has purchased goods for me to the amount of $1250, for which he charges a commission of $1\frac{1}{4}$ per cent. What sum must I remit to pay for goods and commission?

7. Sent to my agent in Cincinnati $765 to purchase a quantity of bacon; his commission is 2 per cent. on the purchase, which he is to deduct from the money sent. What is his commission, and what does he expend for bacon?

Remark.—The $765 sent includes the sum to be invested in bacon and the 2 per cent. commission on the money thus invested. For every 102 cents sent, he will lay out 100 cents for bacon; hence the $765 is $\frac{102}{100}$ of the amount invested. (See Case 4th, Percentage.)

8. I have received $11200 from my correspondent in Boston with directions to purchase cotton, first deducting my commission, $2\frac{1}{2}$ per cent. What is my commission, and how much must I expend for cotton?

9. My agent at Chicago writes that he has purchased for me [illegible]000 bushels of wheat at 85 cents a bushel, and wishes me to send him a check on New York. He can sell the check to a broker at a premium of $\frac{3}{4}$ per cent. How large a check shall I send him, his commission being 3 per cent.?

10. Field & Parsons sell for H. Johnson & Co. 3500 lb. of butter at 20 cts. a lb., 2580 lb. of cheese at 9 cts per lb., at a commission of 5 per cent. They invest the balance in dry goods, after

deducting their commission of $2\frac{1}{2}$ per cent. for purchasing. How many dollars worth of goods do Johnson & Co. receive? What is the entire commission of Field & Parsons?

11. I received of Brown & Lincoln \$560 in uncurrent money to purchase books. I pay a broker $3\frac{1}{2}$ per cent. for current funds, and invest the balance, after deducting my commission of 2 per cent. What do I pay for books, and what is my commission?

12. A broker bought 5 shares of R. R. stock at 35 per cent. discount. What is the brokerage at 5 per cent. on the par value, the shares being \$100 each?

13. What amount of money must be sent to an agent to purchase 500 bbl. of flour @ \$7.75 and pay storage on the same for 10 days at 3¢ per bbl., also 2000 bu. of wheat @ \$1.15, and pay a brokerage of 1% on the flour and 1¢ per bu. on the wheat?

14. What would be the net proceeds for selling 25 cwt. sugar @ \$$9\frac{5}{8}$, allowing a commission of $2\frac{1}{2}$%, and \$$12\frac{1}{2}$ for other charges?

15. If a consignee receive \$500 as commission and for guaranty, the rate of commission being $2\frac{1}{2}$%, and the net proceeds \$12000, what is the per cent. charged for the guaranty?

FREIGHT AND STORAGE.

445. 1. ***Freight*** is any merchandise or goods carried from one place to another; it usually refers to such goods as are carried by vessels or cars.

2. *Freight*, or freight charges, is a term commonly used also to signify the money paid for the transportation of merchandise, etc.

3. A *common carrier* is any person or company engaged in transportation of freight as a business; as a transportation company, a freight company, or an express company.

446. 1. Freight charges are usually computed at a certain rate per cwt., car load, bbl., bu., etc. Lumber freight is computed at a certain rate per thousand feet (M).

2. Freight is sometimes paid by the seller or shipper, and sometimes by the buyer or receiver, as determined by local contract or custom.

3. Freight rates vary according to distance, the quantity and kind of goods, and competition.

NOTE.—For table of weights of freight, see Part Third, Art. 882.

447. 1. ***Storage*** is storing or keeping goods in some depository until required for use or transportation.

2. The term *Storage* is also used to signify the *compensation* allowed for the ordinary safe keeping of mdse. in any government or private depository.

NOTE.—A government depository is also called a "*bonded warehouse*," as in it are stored imported goods on which duties have not been paid. Any other depository for storage is called a freight-house, warehouse, storehouse, or simply a store. A warehouse to and from which *grain* is shipped is generally called an *elevator*.

448. 1. Storage is generally estimated at a certain rate for each package of goods, bushel, or other convenient unit, for a certain time. The "storage time" is generally one week, 10 days, 20 days, or 30 days.

2. Sometimes the different rates and special methods of computation in different cities are determined by Boards of Trade, Chambers of Commerce, associations of warehousemen, or by leading warehousemen, and in some cases by the State legislature.

3. When goods are withdrawn before the close of the month no deduction is made, but storage is charged for the full month. In some cases, after the first month, for a part of a month less than one-half, no charge is made, but for a part greater than one-half, charge is made for a month. In some cities all fractional parts of a month are considered full months, and other "times" are treated in a similar manner.

CASH STORAGE.

449. Storage is generally paid on presentation of the receipt of the goods in store, when they are to be taken out of store, the charges on each receipt being made separately.

NOTE.—It is always desirable to have goods delivered on % of the oldest receipt on hand.

EXAMPLE.—A certain warehouse, July 5, received 550 bbl. sugar, July 22 delivered 400 bbl., July 26 received 450 bbl., Aug. 2 delivered 200 bbl., Aug. 6 delivered 400 bbl., all for the same merchant, at the rate of 6¢ per bbl. for the first 10 days or fraction thereof, and 3¢ per bbl. for each additional ten days or fraction thereof.

Date.	Rec'd.	Delvd.	Paid.
July 5	550		
22		400	$36
26	450		
Aug. 2		200	$21
6		400	$24

Explanation.—On the 400 bbl. delivered July 22, the chgs. are @ 6¢ for first 10 days, and 3¢ for next 7 days, hence 9¢ × 400 = $36 *chgs. paid July 22*, leaving 150 bbl. still in store. The 200 bbl. delivered Aug. 2 are made up of 150 bbl. on % of rec't of July 5, and 50 bbl. on % of rec't of July 26; on the 150 bbl. the chgs. are @ 6¢ for first 10 days, and 6¢ for next 18 days; and the chgs. on the 50 bbl. are @ 6¢ for first 7 days; hence (12¢ × 150) + (6¢ × 50) = $18 + $3 = $21 *chgs. paid Aug. 2.* On the 400 bbl. delivered Aug. 6, the chgs. are 6¢ × 400 = $24 *paid Aug. 6.*

RULE FOR CASH STORAGE OF MERCHANDISE.

450. *Multiply the number of cases, barrels, etc., by the rate for the first "storage term" or by the sum of the rates for the first and any additional "storage terms," concerned, at the time of delivering the goods, for the particular goods delivered.*

NOTE.—If the goods have been in store for any part of a "term" the charge is to be computed for the entire "term."

Ex. When is storage due and what amount is due on 300 bbl. flour received Aug. 1, 200 bbl. delivered Aug. 9, 250 bbl. received Aug. 15, 150 bbl. received Aug. 22, and 500 bbl. delivered Aug. 27, at 5¢ for the first term of 10 days or any part of it, and 2¢ for each additional term of 10 days or any part of it?

451. ***Grain Storage*** is more complicated in computation.

NOTE.—The following statements are according to the rules of the Chicago Board of Trade in 1871.

1. The rates for "winter storage," Nov. 15 to Apr. 15, differ from the rates for "summer storage," Apr. 15 to Nov. 15, and a receipt sometimes covers a part of both.

2. The rates differ for different kinds and grades of grain, and for different modes of delivery or shipment by cars or vessels.

NOTES.—1. July 1, 1871, the storage rates in Chicago for grain in good condition, received in bulk from cars, were "two cents per bushel for the first twenty days, or part of the same, and one-half cent per bushel for each additional ten days or part of same."

2. In the Grain Trade especially, storage is charged to each receipt by

ber, and these receipts are bought and sold as representing the amount grain in store, which is really the merchandise concerned, although more s are made by speculators without using any receipts even.

3. All grain sold in store is subject to regular rates of storage, d the storage is actually paid to the warehousemen on delivery the grain from the warehouse. A grain receipt may pass rough several different hands by sale, but in every case the ller deducts from the gross value of the grain sold any storage e 5 days after the date of sale, *excepting* the "regular storage" r the first term of 20 days or other term which is assumed by e buyer. Storage for terms after the first is called "extra orage," and the seller must provide for the receipt to run 5 days thout "extra storage" to the buyer.

NOTES.—1. Receipts having 15 or more days to run on the first term of rage are called "strictly fresh"; receipts having 10 to 14 days to run on storage are called "fresh"; and those having 5 to 9 days to run on any rage without "extra" charge are called "regular," and on none of these the seller chargeable with storage

2. Sales are estimated in bushels and lbs., a bu. of wheat weighing lb.; and 215 bu. 40 lb. would be written 215^{40}.

452. EXAMPLE 1. "Regular storage" Receipt No. 1285 for 0^{20} bu. No. 2 spring wheat, dated July 20th, is sold Aug. 31st, 1.10 for the wheat; what is the net amount due the seller?

Solution.—The grain is in store 11 days in July and 31 in gust; 11 + 31 = 42 days' storage accumulated, but the buyer umes the storage for the first term of 20 days, hence the seller st allow 22 days' storage in adjusting his account with the yer. Now 22 days' storage at ½¢ for every 10 days or part reof is $\frac{3}{2}$¢ or 1½¢ per bu., but by the payment of this it leaves days for the receipt to run without extra storage, hence it is gular."

Storage @ 1¢ =	$3.60	360^{20}	
" " ½¢ =	1.80	1.10	
Entire extra storage =	$5.40	$396.00	for the bushels.
		.36	" " lbs.
Value of the grain		$396.36	
Extra storage allowed, to be paid to the elevator by the last buyer		5.40	
Net am't due the seller =		$390.96	

1871.	Received.	Sold for Delivery.	Entire Chgs.	Extra Storage.
	Bushels.	*Bushels.*		
Aug. 1.	8000			
14		5000	$100	"Regular."
18	3000			
24	4000			
Sept. 4	5000			
6		5000	$155	$55
10		10000	$225	$25

Ex. 2. Required the storage on the receipts and deliveries of bushels of wheat here indicated at the rates above mentioned, the grain being shipped or taken out of store within 5 days from the date of sale.

Explanation.—On 5000 bu. sold Aug. 14, storage is due @ 2¢ for (13+5) 18 days to Aug. 19, 2¢ × 5000=$100. The 5000 sold Sept. 6 would be made up of the 3000 remaining on % of Rec't for Aug. 1, and 2000 on % of Rec't for Aug. 18. On the 3000, storage is due @ 2¢ for 20 days to Aug. 21, and @ ½¢ × 3 for the next 10+11 days to Sept. 11, that is one term @ 2¢ and three terms @ ½¢; 3½¢ × 3000=$105. On the other 2000, storage is due @ 2¢ for 20 days to Sept. 7, and @ ½¢ for 4 days to Sept. 11; 2½¢ × 2000=$50, hence the entire chgs. due Sept. 6=$105+$50=$155; $55 "extra storage" being paid by the seller. The 10000 sold Sept. 10 is made up of 1000 on % of the Rec't of Aug. 18, and the full am't of the other two rec'ts. Computing time to Sept. 15, the storage due on the 1000 @ 2¢ to Sept. 7, and @ ½¢ to Sept. 15 is 2½¢ × 1000=$25. On 4000 @ 2¢ for 20 da. to Sept. 13, and @ ½¢ for 2 da. to Sept. 15, the am't due is 2½¢ × 4000=$100. On 5000 @ 2¢ for 11 da. to Sept. 15, the am't due is 2¢ × 5000=$100, hence the chgs. on the 10000 are $25+$100+$100=$225; $25 "extra storage" being paid by the seller and $200 "regular" by the buyer.

Ex. 3. What is the storage on each transfer of the following receipt and on the final delivery of the grain: A's warehouse receipt No. 1480, for 5000 bu. wheat, dated July 20, sold to B July 26, to C Aug. 10, to D Aug. 23, to E Aug. 28, and E calls for the delivery of the grain Sept. 7?

A sells July 26	"Fresh."
	"Extra."
B " Aug. 10	1 d. ½¢
C " " 23	4 d. ½¢
D " " 28	4 d. ½¢
E ships Sept. 7	30 d. 2¢

NOTE.—A sells while the receipt has 14 days to run, as "fresh;" B, C, and D each allow ½¢ for extra storage, and E bears the expense of the regular 2¢ storage, although he pays the entire 3½¢ to the elevator.

A sells to B before the first term of 20 days expires, and has to make no allowance

for storage, as the receipt is "fresh" and the final shipper of the grain is subject to pay 2¢ storage, which covers the first term. B sells to C one day after the expiration of the 20 days, and must allow ½¢ "extra" storage or $25 to C in %, which carries the rec't to Aug. 19. When C sells to D he allows him 1¢ or $50 in %, but C really only bears an extra ½¢ as a half-cent was allowed him, and this 1 cent carries it to Aug 29. When D sells he allows 1½¢ to E (½¢ really paid by himself), and this carries the rec't to Sept. 8, while E pays the full storage due, of 3½¢, only 2¢ of which is really borne by himself.

Ex. 4. If, on Aug. 23, A sells to D a rec't for 5000 bu. wheat, dated July 20, what would be A's extra storage?

Computation.—Days remaining in July from date of rec't	11
" in Aug. to date of sale to D . . .	23
" for rec't to run without extra storage	5
	39
Deduct first term	20
	19

19÷10=1 and 9 rem., hence B must allow for 2 terms of extra storage or 1¢ × 5000 = $50.

NOTE.—In selling grain on which the first term has not expired, the seller counts only on 15 days to hold the rec't without extra storage.

452a. *Seller's storage on "regular sales"* of warehouse receipts for all grain in good condition, that is of any grade above "rejected," when the rates and conditions are as above stated, may be computed by the following

RULE.

Compute the whole time of storage from the date of the receipt to the time of sale; add the time (say 5 days) *during which the buyer may hold the receipt without extra storage; deduct the time* (say 20 days) *for which the buyer assumes the storage; divide the remainder by the number of days* (say 10 days) *in each term after the first, adding 1 to the quotient if there be any remainder; the result will be the number of terms of "extra storage." Multiply the rate* (say ½¢) *for one of these terms by their number; the product will be the entire rate of extra storage; multiply by the amount of grain in store and this final product will show the extra storage to be allowed or deducted from the gross value of the grain.*

NOTES.—1 This rule may be adapted to various rates and terms of storage.

2 Sales are frequently made which are not "regular," and would require special computation in each case.

Ex. Compute storage on A's warehouse rec't No. 17409, for 5000 bu. wheat dated June 5, sold to B June 30, to C July 7, to D July 19, D calling for delivery of grain from the warehouse July 23.

453. An account of sales of grain generally shows also the storage account. The following example was copied from the books of a leading firm in actual business.

NOTE.—The dates are the dates of the several receipts; the abbreviations refer to particular elevators; the amount of wheat is entered in bu. and lb.; the rates are made out at 4¢ for entire "winter storage," Nov. 15 to Apr. 15, and at 2¢ for the first 20 d. and ½¢ for each succeeding 5 days or part thereof, for "summer storage"; the computations for half cents are made separately and written immediately under the amount computed for the whole cents in the rate due; the wheat was sold at $1.28¼ per bu. and the seller deducted from the bill all but 4¢ storage.

Ex. CHICAGO, Apr. 10, 1871.

Messrs. PENNY, SMITH & Co.,

Bought of RUMSEY, BRO. & CO.

Aug.	23	N. W.	348 30	8½¢	$27 1	87 74
"	"	Dubuque.	350 50	"	28 1	07 75
Sept.	22	N. W.	349 10	5½	17 1	46 74
Oct.	17	" "	379 10	3	21	25
"	18	Galena.	329 10	"		
"	22	A. D. & Co. "A."	682 10	2½	33 8	67 42
"	"	" "	329 50			
"	"	" "	332 10			
"	25	" "	339 30			
"	27	" "B."	341 10	2	14	10
"	28	Central. "B."	364 10			
Nov.	4	A. D. & Co. "B."	307 30	1½	3 1	07 54
"	7	City.	332 30	1	3	32
"	"	H. Wheeler.	220 10	1	2	20
			5005 30		$166	20
			1.28¼			
			107			
			1252			
			640			
			640000			
			$6419.99			
		Deduct	166.20			
			$6253.79 due.			

454. STORAGE ON ACCOUNT.

1. If goods are received and sold on account, as in the commission business, or are received and delivered at the pleasure of the consignor, in some cases an account is kept, showing the date and number of casks, etc., received, and the date and number sold or delivered. In computing the storage on such an account it is customary to average the time, and charge a certain rate per month of 30 days. If there is a fractional part of a barrel, etc., in the average, it is treated as in the case of parts of terms above.

2. EXAMPLES.

1. What will be the storage of 150 barrels of flour at 4 cents per barrel from May 20 to June 6.

$$150 \times .04 = \$6. \textit{ Ans.}$$

2. What will be the cost of storing salt at 2 cents per barrel, received and delivered as follows: June 6, 1871, 120 bbl.; June 16, 140 bbl.; June 26, 600 bbl.; July 5, 300 bbl.; July 16, 180 bbl.; July 20, 160 bbl. All delivered Aug. 1.

OPERATION.

1871.			bbl.	d.	Prod.
June	6.	Rec'd.	120 ×	10 =	1200
"	16.	"	140		
			260 ×	10 =	2600
"	26.	"	600		
			860 ×	9 =	7740
July	5.	"	300		
			1160 ×	11 =	12760
"	16.	"	180		
			1340 ×	4 =	5360
"	20.	"	160		
			1500 ×	11 =	16500
Aug.	1.	Deliv.	1500		
					30)46160

Bbl. chargeable for 1 month, $1538\frac{2}{3}$

$$1545 \times .02 = \$30.78 \text{ storage.}$$

Explanation.—The storage of 120 bbl. for 10 d. is the same as the storage of 1 barrel for 1200 days; the storage of 260 for 10 days, the same as the storage of 1 barrel for 2600 days, and so on. Hence the amount of storage is 1200 ÷ 2600 + 7740 + 11600 + 6700 + 16500 days storage = 46340 days storage for 1 bbl. = 1544⅔ months' storage for 1 bbl., or calling ⅔ an entire month, it = 1545 months' storage = 2¢ × 1545 = $30.90.

3. What will be the storage of flour at 5 cents per bbl. per month, received and delivered as follows?

Received July 1, 1871, 400 bbl.; July 15, 350 bbl.; July 26, 450 bbl. Delivered July 12, 200 bbl.; July 20, 400 bbl.; Aug. 1, 200 bbl.; and Aug. 8, 400 bbl.

OPERATION.

1871.				
July 1.	Rec'd.	400	× 11 =	4400
" 12.	Deliv.	200		
	Bal.	200	× 3 =	600
" 15.	Rec'd.	350		
	Bal.	550	× 5 =	2750
" 20.	Deliv.	400		
	Bal.	150	× 6 =	900
" 26.	Rec'd.	450		
	Bal.	600	× 6 =	3600
Aug. 1.	Deliv.	200		
	Bal.	400	× 7 =	2800
" 8.	Deliv.	400		
				30)15050

1 bbl. chargeable for 502 months.

502 × .05 = $25.10 *Ans.*

Explanation.—From July 1 to July 12, 400 bbl. were stored; from July 12 to July 15, 200 bbls; from July 15 to July 20, 550 bbl.; from July 20 to July 26, 150 bbl.; from July 26 to Aug. 1, 600 bbl.; from Aug. 1 to Aug 8, 400 bbl.

3. RULE.

Commencing with the first date and ending with the last, multiply the number of barrels, or other articles in store from each

date to the one NEXT *following it, by the number of days between these dates. Divide the sum of the several products by 30, and the quotient will be the number of months storage for one article, and this number multiplied by the rate of storage for each article will give the amount of storage charged.*

455. 1. When the account is of much length, the following method may be used to great advantage in keeping the account and determining the amount of storage.

2. EXAMPLE.—Storage of goods on account of C. T. Wilder & Co., Chicago, Ill., at 5 cents a bbl. per month, by Hubby & Hughes, Cleveland, O.

Received Jan. 1, 1871, 350 bbl.; Jan. 12, 650 bbl.; Feb. 5, 500 bbl.; Feb. 10, 320 bbl.; Feb. 28, 440 bbl.; March 15, 850 bbl.; March 30, 200 bbl. Delivered Jan. 20, 700 bbl.; Jan. 31, 200 bbl.; Feb 24, 800; March 20, 350 bbl.; March 25, 700 bbl.; April 5, 400 bbl.; April 8, 100 bbl. What is the storage on this account, closed April 12, 1871, and how many barrels are on hand?

OPERATION.

DR. C. F. WILDER & CO. CR.

Time.		Rec'd.	Total Rec'd	Da.	Products.	Time.		Deliv.	Total Deliv.	Da.	Prod.
Jan.	1	350	350	11	3850	Jan.	20	700	700	11	7700
"	12	650	1000	24	24000	"	31	200	900	24	21600
Feb.	5	500	1500	5	7500	Feb	24	800	1700	[illegible]	40800
"	10	320	1820	18	32760	Mar.	20	350	2050	5	10250
"	28	440	2260	15	33900	"	25	700	2750	11	30250
Mar.	15	850	3110	15	46650	Apr.	5	400	3150	3	9450
"	30	200	3310	13	43030	"	8	100	3250	4	13000
		3310			191690			3250			133050
		3250			133050						
		60			58640						

Apr. 12, 60 on hand. 58640 days ÷ 30 = 1955 mos.; 5¢ × 1955 = $97.75 *Ans.*

Explanation.—If there had been none delivered, C. F. Wilder & Co. would be charged for storing one barrel for 191690 days, and have in store 3310 bbl; but upon the number delivered they are entitled to a credit of the storage of one barrel for 133050 days; this subtracted from the debit side of the account leaves the storing of one barrel for 58640 days or 1955 months. At 5¢ per month this = $97.75. There remain 60 bbl. in store.

3. RULE.

Compute the days storage on the d· according to the preceding rule and th subtract the number of days storage · number on the debit side; divide the r· are stored by the week, or by 30 if by · price per week or month by this quo· the storage due.

NOTE.—A good test of the correctn· bers in the columns of "total received· the sum of the numbers in the colu· the sum of the "days" column shou· date up to the time when the accou·

456. 1. Butchers and d· pastured or fed on account, · circumstances may require. manner as an account of st··

2. Account of pasturag· for Lewis & Vincent, Port··

Received June 3, 18·· head: July 1, 20 head: 3, 12 head; Aug. 16, 1· Sept. 30, 3 head; O· head; June 7, 4 head· 21, 10 head; July 3· July 28, 2 head; · 9 head; Sept. 28, · 15, 13 head. Wh·

457. *In*

459. 1. The *Policy* is the contract of insurance. This indicates the amount assured in case of loss, the premium, time of payment, the parties concerned, the particular risk and the conditions on which it is assumed.

NOTE.—A *policy fee* is a charge sometimes made for writing the policy.

2. *Certificates of Insurance* under any Policy issued *on account of "whom it may concern,"* are negotiable

460. 1. The *assured* is the party to whom is guaranteed the amount specified in the Policy; he is also called the *policy-holder*. He is legally required to have an interest in the thing insured to the full amount of the assurance. The premium is paid in the name of the assured or on his account.

2. The thing *insured* is the life, property or other subject of the risk assumed.

461. The *term* of insurance or rather of the policy, is the time during which the policy is to be in force.

462. 1. An *underwriter* is a person or company who issues policies of insurance and becomes responsible for the amount assured.

2. An *insurance agent* is a person who acts for an insurance company, in assuming risks, collecting premiums, and adjusting losses. An insurance *broker* merely negotiates policies, for securing which he receives a single commission or brokerage.

463. 1. Insurance may be applied to any kind of risk, venture, or contingency pertaining to *persons* or *property*.

2. Insurance of persons is effected with reference to contingencies of *life, health*, or *accident*. (For Life Insurance, see Art. 740.)

3. Insurance of property is effected with reference to contingencies of loss by *fire, marine* disaster, or *inland* disaster.

4. An *insurance surveyor* is a person who examines buildings and other property proposed for insurance, and classifies risks.

464. *Insurance Companies* (Ins. Co.) are of two kinds, *mutual* and *stock*.

1. A *Mutual Ins. Co.* is one in which the profits, deducting only the expense of the business, are divided among the insured, or so applied in renewal of insurance, in reduction of premiums, that the insured pays only for the amount of his actual risk.

2. A *Stock Ins. Co.* is one in which the capital stock is owned

by persons who compose the company, who are called stock-holders—the losses being paid from the premiums received, or if these are insufficient, from the capital stock. The stockholders alone are liable for the losses and alone share the profits. Premiums are generally payable in cash in advance.

465. 1. The real security of an Ins. Co. consists in having a large capital stock well guaranteed or invested, or well-invested accumulated premiums of sufficiently large amount to cover any extraordinary loss or series of losses; in having sufficiently high rates of premiums to fully cover the ordinary average of losses on risks assumed; in carefully selecting risks, and in guarding against excessive insurance.

2. The security of Insurance Companies is now largely provided for by insurance laws in most States, referring to the amount of capital and accumulated premiums or *reserve* required, imposing certain restrictions in assuming risks, and requiring regular reports of their condition. Thus, many Life Ins. Cos. especially, and some Fire Ins. Cos., are very carefully guarded and their policy-holders well secured.

FIRE INSURANCE.

466. Fire Insurance is the assurance of a certain amount of money payable to the policy-holder in case of the loss of specified property in consequence of fire. When a loss occurs it becomes a *claim* of the assured on the insurance company.

467. 1. Policies of Fire Insurance are issued on buildings and their contents, stocks of goods on hand, or other exposed property of definite value on which there is a known average risk, and the premium on which is definitely stated and generally paid in advance.

2. Policies are not generally granted for more than three-fourths the appraised value of the property, and the definite amount that may be assured by any one company without reinsurance is limited by its charter.

3. If only a part of the property insured is destroyed or injured, the claim is adjusted by paying only a part of the amount assured, and in some cases, especially in adjusting claims on build-

ings injured by fire, the insurance companies concerned repair or replace the property, instead of paying the amount claimed.

468. Premiums are estimated at a certain rate on each hundred dollars of value insured, which is called the *rate* of insurance; this may also be expressed as a certain rate per cent. Thus if the rate is 50¢ (on $100), it is $\frac{1}{2}\%$.

NOTES.—1. The *ordinary* rates on dwelling houses are $\frac{1}{4}\%$ @ 2%; on warehouses and stores $\frac{1}{2}\%$ @ 2%; on merchandise $\frac{1}{2}\%$ @ 3%.

2. Some risks are not regarded as insurable, and no rates are fixed for them; occasionally a great risk is taken at a very high premium.

3. Rates for particular classes of property are modified by the common exposure or protection of it, and facilities for removing it or for extinguishing any fire that may endanger it.

MARINE AND TRANSIT INSURANCE.

469. 1. ***Marine Insurance*** is the assurance of a certain amount of money, determined before or after the loss, payable to the assured in case of any loss of property by fire, storm or other disaster to it while "in board of" a vessel, "at, to, and from" the places of lading and unlading.

NOTE.—Insurance on vessels themselves is called *Hull Insurance.*

2. *Ocean Marine Insurance* covers risks at sea.

3. *Inland Marine Insurance* covers risks of transportation by inland waters.

470. Transit Insurance refers especially to risks of transportation by land only.

NOTES.—1. A single Policy may cover both Marine and Transit Insurance, and this is frequently required for insuring goods that are transported partly by water and partly by land.

2. The amount assured is generally from 5% to 10% advance on the cost of the goods, to cover all charges, and sometimes even a greater advance is secured to cover a part of anticipated profits.

471. Policies of Marine and Transit Insurance are of two kinds.

1. A *valued* or *closed Policy* is one in which the property insured is distinctly specified; and the amount assured, the premium, and the conditions definitely stated when the policy is issued.

2. An *open* or *open cargo Policy* is one in which the conditions are stated, the rates of premium indicated, and the property to be insured referred to in the general terms "*as endorsed*," but in which the value of the property, the amount assured, and the actual premiums are not entered until the policy-holder has an interest at risk in the property. Values and premiums are generally entered in the policy on receipt of the invoice of the goods insured, but if the policy contain the words "entered or not entered," the company is liable for any loss covered by the general contract, although the loss occur before the value of the goods be known to the parties concerned, or entered or endorsed on the policy. The cost of goods "in store" includes transportation.

Such a Policy is available for the insurance of all merchandise or other property in transit by land or water during an entire season, or it may cover the time of an entire voyage or several voyages at sea, during which different cargoes may be at risk. (Art. 475, 16 and 17.) It is also used to cover the cost of goods in store.

472. 1. Premiums on "open policies" are paid at the end of a voyage, or monthly, weekly, or at the time of each entry, as may be agreed upon.

2. As security for anticipated premiums, sometimes a premium note of sufficiently large amount is given, and this is surrendered when the actual premiums due are paid; or these are endorsed on it, and it is still held as security for farther premiums.

NOTE.—The *ordinary* rates on "open policies" are from $\frac{1}{4}\%$ to 5%.

INSURANCE PROBLEMS.

473. 1. Most of the ordinary problems referring to insurance are simple problems in Percentage. (Art. 272.)

2. The *amount of insurance* corresponds to the *base*, the *rate of premium* to the *per cent.*, and the *premium* to the *percentage*.

474. If the policy is required to cover both the amount assured on the property and the premiums paid, the face value of the policy multiplied by the difference per cent. of the rate (Art. 272, 1) will give the amount assured on account of the property.

2. To find the face value of a policy that shall insure both the property and the premiums.

RULE.

Divide the amount of insurance required on the property by the difference per cent. of the rate of insurance.

EXAMPLE.—For what amount must a policy be issued to cover both 70% of mdse. valued at $7000 and the premium thereon, if the rate of insurance be 2%?

The premium must evidently constitute 2% of the amount assured, hence the amount insured on the property can be only 98% of the face of the policy. $7000 × .70 = $4900 the am't insured on the property, and 4900 ÷ .98 = $5000, the face value of the policy required. Proof, Premium on $5000 @ 2% = $100, and $4900 + $100 = $5000.

475. EXAMPLES.

1. What is the premium for insuring goods for $4500, at $2\frac{1}{2}$%?

2. What is the expense of obtaining a policy of insurance on a hotel worth $15000, insured for $\frac{2}{3}$ of its value at $\frac{3}{8}$%, if $1.50 be charged extra for the policy fee and survey?

3. An insurance company insured a block of buildings for $350000 at $\frac{3}{5}$ per cent., but thinking the risk too great, they *re*insured $150000 of it at $\frac{3}{4}$ per cent. in another company, and $100000 of it at $\frac{5}{8}$ per cent. in another. How much premium did the company receive? How much did it pay to both the other companies? How much did it clear? What per cent. of premium did it really receive on the part not *reinsured?*

NOTE.—All property in one block, or in adjacent buildings, having communications, or on one vessel, is considered as *one risk*, and insurance companies seldom take more than $10000 in one risk. Some companies of very large capital take $20000, but small companies do not take more than from $3000 to $5000 in one risk, without reinsuring a part in some other company.

4. A ship valued at $40000 is insured for $\frac{3}{4}$ of its value at $1\frac{1}{2}$ per cent., and its cargo, valued at $36000, at $\frac{4}{5}$ per cent. What is the cost of insurance?

5. A merchant paid $1450 premium for the insurance of a cargo of cotton, shipped from New Orleans to Boston, the rate of insurance being $2\frac{1}{2}$ per cent. What was the value of the cargo?

6. Paid $7.20 for the insurance of a house at $\frac{3}{5}$ per cent. If

the policy and survey cost \$1.50, for how much was the house insured?

7. I pay \$50 for an insurance of goods valued at \$32500, and shipped from New York to St. Louis. What was the rate of insurance?

8. A house valued at \$1200 has been insured for $\frac{2}{3}$ of its value for 3 years at 1 per cent. per annum. Near the close of the third year it is destroyed by fire. What is the *actual loss* to the owner, no allowance being made for interest?

NOTE.—The insurance company must pay him \$800; but of this sum he has paid to the company \$24 premium; hence he actually receives but \$800−24=\$776.

9. My house was insured for \$45000 for 5 years. The first year I paid \$1.50 for policy and survey, and $\frac{5}{8}$ per cent. premium; each succeeding year I paid $\frac{1}{2}$ per cent. premium. What was the total cost of insurance? The house was burned during the fifth year; what was the actual loss of the *company*, no allowance being made for interest?

10. A merchant ships \$31360 worth of wheat from Chicago to Buffalo. For what must he get it insured at 2 per cent. so as to *cover* both the value of the wheat and the premium paid for its insurance?

11. For what must a cargo of R. R. iron worth \$115200 be insured to cover both the value of the iron and premium, the rate of insurance being 4 per cent.?

12. A merchant shipped a cargo of flour worth \$47880 from Chicago to San Francisco via New York. To insure it from Chicago to Buffalo he paid $1\frac{1}{2}$ per cent.; from Buffalo to New York $\frac{1}{4}$ per cent.; from New York to San Francisco $3\frac{1}{4}$ per cent. For what sum must it be insured to cover value of flour and premium for the voyage?

13. A policy covering property and premium is taken for \$12045. What is the value of the property insured, the rate being $\frac{3}{8}$ per cent.?

14. A merchant insures a cargo of goods for \$81800, covering both the value of the goods and the premium. What is the value of the goods, the rate of insurance being $2\frac{1}{4}$ per cent.?

15. The owners of the steamer Florence have, for the past 20 years, paid 5 per cent. per annum for her insurance. She was sunk

this morning. Have they gained or lost by having the steamer insured?

16. An *open policy* was issued by the "Independent Fire Ins. Co.," to A. Worthy & Co., "on merchandise hazardous and non-hazardous, their own, or held by them in trust or on commission, or sold but not delivered, contained in any elevator, packing-house, store, or warehouse, attaching only to risks endorsed on the book attached to this policy by the agents of this company in the city of Chicago." What was the premium due on the articles mentioned in the following entry:

Aug. 1, 1871, 3 mos. to Nov. 1, 1871, $5000 @ 40 less 10%?

NOTE.—The book referred to in this example is a small book into which the policy is fastened, and in which all entries are made in order. The rate 40 is 40¢ on $100. The 10% discount is a matter of special agreement.

17. An *open cargo policy* was issued to H. P. Jones & Co. by the "Ætna Ins. Co.," "on property as endorsed, from ports and places to ports and places" mentioned. This was to "cover all shipments of merchandise by the assured, in board of standard vessels, at and from all Lake Ports to Chicago, during the season of navigation expiring the 30th day of November, 1871. All such shipments of the assured to be so covered, entered or not entered, and the assured to report at least once a week particulars of such shipments as they may be advised of." Compute the premiums on the following entries under this policy, for which they are not computed?

1871.	(Vessel or Line.)	(Ports.)	(Entry)	(Mdse.)	(Value.)	(Rate.)	(Prem.)	(Signature of Sec'y)
Aug. 1	Polaris.	Buffalo to Chicago.	1	100 bbl. Sugar	$3000	25	$7.50	C. W. [illegible], Sec'y.
5	N. Trans. Co.	Ogdensb'g "	2	20 bbl. Apples.	125	35		
9	W. Trans. Co.	Buffalo "	3	Mdse.	200	25		
17	Anchor Line.	Erie "	4	Mdse.	1600	25		

TAXES.

476. ***A Tax*** is a legally required contribution to a common fund. It is generally designed to meet certain obligations of the tax-payer as a citizen, or to secure certain privileges or advantages to him as a member of the community by whose authority the tax is required.

477. 1. Taxes are generally *assessed* or *levied* upon *persons* or *property*, but sometimes also upon products, occupations, sales, incomes, certificates of business transactions, and legal instruments.

2. *Personal taxes* are called *Capitation* (Latin *caput*, head) or *Poll Taxes*, and are levied at a certain amount for each person or head (German *polle*, a head).

NOTE.—Poll taxes are only required of legal voters generally.

3. *Property taxes* are levied on *real estate* or on *personal property*, the former including land, houses, and other immovable property, and the latter including all other property, such as money, merchandise, furniture, etc.

4. *Specific taxes* include all taxes levied on anything except persons or property.

478. 1. Taxes are lawfully assessed or levied, and collected only by the highest recognized authority of the nation, state, county, town, city, or other community whose interests are directly concerned.

2. Taxes levied by and for the general government constitute a part of the national *revenue* or income.

Illicit traffic or manufacturing is that which is conducted without paying the revenue required.

3. *Internal revenue* includes all general taxes on persons, domestic property, products, etc.

4. *Customs revenue* includes all taxes on imported goods, called *imposts* or *duties*, and taxes on foreign vessels entering American ports, called *tonnage*. (Art. 483.)

NOTE.—The term *tonnage* is also applied to the capacity of the vessel and to the computation of the same.

5. *Stamp duties* refer to such taxes as are paid by means of revenue stamps purchased from the proper government agents. These stamps are affixed to the things taxed and specify the amount paid.

479. 1. *Assessors* are government officers who ascertain or appraise the value of property to be taxed, and apportion the taxes *pro rata*, that is in proportion to the value of each man's property.

2. *Collectors* are government officers who collect taxes.

3. Taxes are generally payable in stamps, or money only; but in local taxation, as for "road taxes" in some towns, they are payable in "day's work."

480. To assess a county, town, or city tax.

RULE.

1. *Find the number of polls and the value of the property to be taxed, and the net amount to be raised for actual use.*

2. *Divide the net amount required by the difference per cent. of the cost of collection* (Art. 265.); *the quotient will indicate the whole tax to be raised. From this subtract the whole amount of the poll taxes; the remainder will indicate the tax to be assessed on the property.*

3. *Divide the property tax required by the value of all the taxable property; the quotient will show the rate per cent. of taxation.*

4. *To the product of the value of each man's property multiplied by the rate per cent. expressed decimally add his poll tax; the amount will indicate his entire tax.*

NOTES.—1. When a certain per cent. of any tax is regarded as uncollectible, the whole amount to be raised including the cost of collection, divided by 1.00 less the per cent. uncollectible will give the total amount to be *assessed.*

2. Taxable property is such property as may be legally assessed; generally public schools, churches, asylums, etc., are exempt.

3. The rate is generally a certain number of mills or parts of a mill on each dollar of value.

After finding the rate, a convenient table may be constructed for the ready computation of each particular assessment. (Art. 481, 2.)

4. A *Rate Bill* of taxes for the expenses of a school may be made out as follows: *From the entire expense of the school deduct any public money or other aid furnished; divide the remainder by the number of days attendance of all the pupils liable to assessment, the quotient will be the rate per day; this multiplied by the number of days attendance of the pupils from any family will show the amount due from that family. If the cost of collection is to be included, see the 2d section of general rule above.* Rate bills are seldom required now in this country, as public funds are provided for the support of nearly all our common district schools and many higher schools.

481. For internal revenue tax on real estate, personal property and other items specified by law.

RULE.

1. *From the total amount of the value of the real estate and personal property, subtract the amount exempt by law; the required percentage of the remainder will be the tax required on valuation.*

2. *To this add any specific taxes for certain articles mentioned; the sum will be the whole tax due.*

NOTE.—When taxes on real estate are not paid as required by law, the property assessed may be sold for the taxes, according to certain conditions and restrictions; this more frequently occurs in city taxation.

482. EXAMPLES.

1. The taxable property of the city of Cleveland for a certain year was $21648938. The taxes were assessed as follows:

For State purposes,	. . .	3.1 mills on a dollar.
" County purposes,	. .	2.5 " " "
" Corporation purposes,	.	8. " " "

What was the amount of tax assessed for each purpose? How much will be collected, allowing 8 per cent. to be *uncollectible?*

2. The taxable property of the city of B. for 1870 was $35500000; the assessment was 15 mills on a dollar. What was the total tax of the city? What tax was assessed upon each of the following citizens?

Mr. A	who paid tax on		. .	$13560.
Mr. B	"	"	. .	9850.59.
Mr. C	"	"	. .	450.87.
Mr. D	"	"	. .	60850.
Mr. E	"	"	. .	119380.
Mr. F	"	"	. .	1000000.

3. Find from the following table the tax assessed upon

E. G.	who paid tax on		. .	$35867.50
H. E. S.	"	"	. .	115380.
A. K.	"	"	. .	586789.99
R. S.	"	"	. .	480.48

TABLE. (Art. 480, Note 8.)

Rate of tax 15 mills on a dollar.

Prop.	Tax	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax
$ 1	$.015	$21	$.315	$41	$.615	$61	$ .915	$ 81	$1.215
2	.03	22	.33	42	.63	62	.93	82	1.23
3	.045	23	.345	[illegible]	.645	63	.945	83	1.245
4	.06	24	.36	44	.66	64	.96	84	1.26
5	.075	25	.375	45	.675	65	.975	85	1.275
6	.09	26	.39	46	.69	66	.99	86	1.29
7	.105	27	.405	47	.705	67	1.005	87	1.305
8	.12	28	[illegible]	48	.72	[illegible]	1.02	88	1.32
9	.135	29	.435	49	.735	69	1.035	89	1.335
10	.15	30	.45	50	.75	70	1.05	90	1.35
11	.165	31	.465	51	.765	71	1.065	91	1.365
12	.18	32	.48	52	.78	72	1.08	92	1.38
13	.195	33	.495	53	.795	73	1.095	93	1.395
14	.21	34	.51	54	.81	74	1.11	94	1.41
15	.225	35	.525	55	.825	[illegible]	1.125	95	1.425
16	.24	36	.54	56	.84	76	1.14	96	1.44
17	.255	37	.555	57	.855	77	1.155	97	1.455
18	.27	38	.57	58	.87	78	1.17	98	1.47
19	.285	39	.585	59	.885	79	1.185	99	1.485
20	.30	40	.60	60	.90	80	1.20	100	1.50

Explanation of the Table.—Suppose we wish to find the tax of Mr. A in the above example. $13560 = $13000 + $500 + $60. The tax on $13000 is found from the tax of $13 (.195) by removing the decimal point *three* places to the right ($195.); the tax on $500 is found from the tax of $5 (.075) by removing the point *two* places to the right ($7.50); the tax on $60 is found in the table ($.90). $195. + $7.50 + $.90 = $203.40; tax on $13560. B's tax in Ex. 2 is found in the same manner. Thus: tax on $9 = .135, tax on $9000 = $135; tax on $8 = .12, tax on $800 = $12; tax on $50 = .75; tax on 59 cents (found from tax of $59 by removing point two places to the *left*)—.00885 = .009 nearly. Hence $135 + $12.75 + $.009 = $147.759 or tax on $9850.

4. The cost of maintaining the public schools of the city of B. for 1871 was estimated at $56000. The taxable property of the city was $22400000. How many mills tax on a dollar must be assessed for school purposes? Suppose the uncollectible tax will equal 10 per cent. of the tax assessed; how many mills on a dollar must in this case be assessed?

5. In a certain district the school expenses were for teacher's

salary $800, for repairs $210, for fuel and incidentals $30. The amount received from the general school fund was $150 and the total number of days' attendance was 6500. What was the rate per day required for a rate bill, and what was the amount due from Mr. Brown who sent 2 pupils 80 days each, and 1 for 60 days?

DUTIES OR CUSTOMS.

483. 1. ***Duties*** or ***Customs*** are sums of money assessed by government upon imported goods.

NOTE.—In some countries duties are also assessed upon exported goods.

2. Duties upon goods are collected at their port of entry, by officers appointed by government and called *custom-house officers.* At each port of entry for foreign goods is a *custom-house,* where all custom business is done.

484. Duties are of two kinds, *specific* and *ad valorem.*

1. *Specific duties* are assessed upon goods at a certain rate per tun, hogshead, bale, gallon, etc., without reference to their value.

2. *Ad valorem duties* are a certain percentage of the value of goods as shown by the *invoice.*

485. 1. An *Invoice* or *Manifest* is a written account of the particulars of goods shipped or sent to a purchaser, consignee, factor, etc., with the actual cost or value of such goods made out in the currency of the place or country from whence imported.

2. The invoice is exhibited at the custom-house by the master of the vessel, or the owner or consignee.

3. When an invoice has not been received, the owner or consignee must testify to the fact under oath, and then the goods are entered by appraisement.

4. When the currency of a country has a depreciated value compared with that of the country into which they are imported, a consular certificate showing the amount of depreciation is attached to the invoice.

486. 1. In assessing *specific duties,* certain allowances are made, called draft, tare, leakage, breakage, etc., before the duties are estimated.

2. *Draft* is an allowance for waste. It must be deducted before other allowances are made.

3. *Tare* or *Tret* is an allowance for weight of box, cask, etc., containing the goods. It is generally computed at a given rate per box, cask, etc.

4. *Leakage* is an allowance for the waste of liquid.

5. *Breakage* is an allowance on liquors transported in bottles.

6. *Gross Weight* is the weight of goods before any allowances are made.

7. *Net* or *Neat Weight* is the real weight of goods after the allowances have been deducted.

NOTES.—1. The ton of weight used at the United States Custom-House is 2240 lb., and the ton of capacity for vessels is 40 cu. ft.

2. In *ad valorem* duties no allowances are made for draft, tare, or breakage.

3. *Warehousing* is depositing imported goods in a government or bonded warehouse.

487. The following are the principal United States customs officers and their duties.

1. The *Collector of the Port* supervises all entries and papers pertaining to them, estimates all duties, receives all moneys and securities, and employs all weighers, gaugers, measurers and inspectors. The *storekeeper* has charge of the warehouse.

2. The *Naval Officer* receives copies of all manifests and entries, estimates all duties, countersigns all documents issued by the Collector, and certifies his estimates, accounts, etc. The Naval office thus acts as a check on the Collector's office.

3. The *Surveyor* superintends the employees of the Collector, inspects all vessels and cargoes, revises all entries and permits, and is directly responsible to the Collector.

4. The *Appraiser* appraises or decides the market value and dutiable character of all imports, so that *ad valorem* duties may be correctly laid. He is accountable to the United States Appraiser-General (residing at New York), to whom he makes regular reports.

NOTE.—It is stated on good authority that "the customs on imports are collected in the New York custom-house at a cost of from 1% to 1½%, just half of what it costs to collect imposts in England."

5. Besides the officers and their employees, there is generally found at or near every custom-house the *Custom-House Broker*. He is familiar with the complicated details of custom-house busi-

ness, and is employed by most merchants as a special agent to enter goods for them.

488. 1. *Smuggling* is bringing foreign goods into the country without paying duties on them, either by not entering them at any custom-house or by showing less than their real value in the invoice. It is a crime for the prevention and punishment of which stringent laws are made and a careful supervision exercised.

489. EXAMPLES.

1. A portion of the cargo of the ship Europa from Liverpool to New York was invoiced as follows:

650 yd. Broadcloth, . . .	cost 13s.	sterling	per yd.
1246 yd. Lace, ...	" 2s.	"	"
1200 yd. Coach Lace, . . .	" 11d.	"	"
1950 yd. Ingrain Carpeting, .	" 3s.	"	"
2560 yd. Drugget, ...	" 2s. 4d.	"	"

The duty on the broadcloth was 30 per cent.; on lace 25 per cent; coach lace 25 per cent.; carpeting 30 per cent.; drugget 30 per cent. What was the amount of duty in our currency, allowing the pound sterling to be $4.84?

2. C. Hartwell & Co., of Baltimore, have imported from Havana

100 hogsheads of Molasses, 63 gal. each,	cost	25 cts. per gal.
50 hogsheads of Sugar, 500 lb. each .	"	5 cts. per lb.
150 boxes of Oranges, ...	"	$2.50 per box.
300 boxes of Cigars, ...	"	$8 per box.
160 boxes of Bananas, ...	"	$1.75 per box.

The leakage of molasses is 2 per cent.; duty on same 30 per cent.; duty on sugar 30 per cent.; on oranges 20 per cent.; on cigars 40 per cent.; on bananas 20 per cent. What was the duty on each article? What was the amount of duties?

3. A wine merchant in New York imported from Havre

100 baskets Champagne, ...	at $13	per basket.
80 casks Madeira, ...	at $42	per cask.
56 casks Oporto, ...	at $45	"
50 casks Sherry, ...	at $25	"

If an allowance of 3 per cent. for leakage is made on the wine in casks, what will be the amount of duty at 40 per cent.? For what must the wine be sold per basket or cask to make a clear profit of 25%?

4. Invoice of Merchandise forwarded to D. C. McIver, Liverpool, for shipment per "*Batavia*," to and for account and risk of Messrs. J. V. Farwell & Co., New York.

						£	s.	d.			
J. V. ⟨F⟩ & Co.		693 & 694 *Batavia.*									
693	⅜	18", 19" in. Camb. Hdkf.	242	50 Doz.	2/7	6	9	2			
			253	50	2/11	7	5	10			
			262	75	3/3	12	3	9			
			272	50	3/8	9	3	4			
	⅝	19/20 in.	278	25	5/6	6	17	6			
			282	25	6/6	8	2	6			
			6	275 doz.		50	2	1			
					Disc't 2½%	1	5	1	48	17	0
694		12/14 in. Hemstitched Hdkfs.	684	100 Doz.	3/5	17	1	8			
		16/18 in. Cloth "	686	50	3/10	9	11	8			
		1¼ & 1⅝ Hems. 1 doz. pl'n cen.	688	25	4/2	5	4	2			
			690	25	4/6	5	12	6			
				200		37	10	0			
					Disc't 2½%		18	9	36	11	3
		E. & O. E.							£85	8	3
		BELFAST, Apr. 6, 1871.									
		HENRY MATIER & Co.									

Charges and Duties paid in New York for ⅔ of Invoice consigned to J. V. Farwell & Co., Apr. 25, 1871.

		35%			40%					
J. V. ⟨F⟩ & Co.	"*Batavia.*" Apr. 25, 1871.									
	693 / 694 } ⅔ Linen Hdkfs.	34	4	6	14	12	0			
					36	11	3			
	NOTE.—Classified by the appraiser.	34	4	6	51	3	3	85	8	3
	Shipping		1			1	2		2	3
	(Agts) Com. 2½%		17	2	1	3	7	2	2	9
		35	2	8	52	10	6	87	13	2
	[Even dollars]	$170			$254			$424		

J. V. ⟨F⟩ & Co.	Apr. 25, 1871		
	Duty on ⅔ pr Batavia,	$161	10
	Prem. 11% on 161.10 [NOTE.—Gold premium],	17	73
	Permits 1.60, Ctge. 1.50, C. H. Ex. 1.00, Bonds .50, Stamps, .80, (Broker's) Com. 4.00,	9	40
	Freight £1 15 1 122⅔,	9	54
	NOTE.—122⅔ includes exchange and premium on gold.	$197	78
	693 694		
	$170 @ 35% = $59.50		
	254 @ 40% = 101.60		
	$161.10		

NOTES.—1. The student should verify the computations in these examples.

2. The mark 2/7 (Ex. 4) indicates 2s. 7d.

3. On some goods both a specific and an *ad valorem* duty is required as in 1 case in Ex. 5.

4. See Part Third for the picul of weight in Ex. 6.

5. Invoice of Merchandise forwarded to D. C. McIver, Liverpool, per steamer *Hansa*, to and for account and risk of Messrs. J. V. Farwell & Co., New York, by S. W. Kaston, Paris, No. 11 Rue Bergere.

J. V. <F> & Co.	(*Kilos.*)						
794	148^{500}	5 Maroon. 5 Magenta. 16 Azuline. 30 Ponceau. 5 Rose. 6 Sultan. 5 Saumon. 5 Vert. 8 Chamois.					
		85 Mouseline de laine, 28 in. 3007^{4} .88				2646	60
795	143	85 Meine asstd. " 3004^{7} .88				2644	15
796	141^{500}	84 " Moire sultan 28 in. 3001^{2} .88				2641	05
797	140	6 Maroon. 6 Magenta. 6 Rose. 6 Sultan. 16 Azulin. 5 Saumon. 30 Ponceau. 8 Chamois. 3 Vert.					
		85 id. " 2600^{9} .88				2543	79
						10475	59
		Disct. 2%				209	50
						10266	09
		Disct. 2%				205	33
						10060	66
		36 Cartons @ 1.50,	54				
		Legalization (consul's certif.)	13				
		Emballage (packing, etc.),	81	10		148	10
						10208	76
		(Broker's) Com. 3%				306	26
						f. 10515	02
		PARIS, Aug. 21, 1871. S. W. KASTON.					

Charges and Duties paid in New York for % of Invoice consigned to J. V. Farwell & Co., Sept. 8, 1871.

J. V. <F> & Co. Chicago, Ill.	794 @ 797	Per steamer *Hansa*. (Classified by appraiser.) 8¢	40%	
		Three cases Worsted, ... 1 case. 8315 yd.	f. 10508	02
		Shipping, ...	61	50
			f. 10569	52
		f. @ .18⁶.	$1965	60
J. V. <F> & Co.	794/797	*Hansa*. 4 cases.		
		Duty on 8315 yd. @ 8¢ yd., ...	665	20
		" " $1965 @ 40%, ...	786	00
			1451	20
		Permits, 1.00 Cartage, 3.00 C. H. Ex., 2.50 Bonds, .50 Stamps, 1.90 Com's'n, 4.00 12.90		
		Gold Prem. 13% . .	197	73
		(Gold & Exch.) Freight £8 0 3 133%	43	96
		Charges, . . .	12	90
			$1705	79

6. Invoice of *28 half chests Japan Tea,* shipped per steamer "*America*" to San Francisco, and thence overland per rail to Chicago, consigned to *order* and for account and risk of *whom it may concern.* (Sent to N. Sherwood & Co., Chicago.)

N. S. & Co.	(*Picul.*) $1. 28 hf. ch. Ps. 8^{82}; before firing, Ps. 9^{62} @ $41 p. pcl., ... Each hf. ch. cont. 42 lb., in all 1176 lb.			$394	42
	CHARGES.				
	Firing, Boxes & Packing, Ps. 9^{62} @ $4 p. pcl.	38	48		
	Matting, Rattaning & Marking @ 30¢ p. pkg.	8	40		
	Shipping @ 5	1	40		
	Godown Rent (freight & stor.) @ 8	2	24		
	Export Duty, ...	11	42		
	Consular Fee (in quadruplicate), ...	4	50		
	Fire Insurance, ½%, ...	1	97		
	Inspecting Commission, 1%, ...	3	94		
	Post and Petty, ...	1	00	73	35
				$467	77
	Buying Commission, 5%, . .			23	39
	Bill Brokerage, ¼%, . . .			1	23
	Mexican.			$492	39
	Marine Ins. effected through open policy to Chicago. E. & O. E. YOKOHAMA, 16th June, 1871. JAMES C. FRASER & CO.				

7. Estimate of cost of 165 pkgs. Japan Tea imported by N. Sherwood & Co., per Pacific Mail Steamship "*Japan.*"

N. S. & Co.	100 hf. ch. 33^{76} pcls., before firing 37^{12} @ $46¼, 65 pckgs. 13^{12} " " " 15^{38} @ "			$1716	80
				711	33
		Lacquered.		*Bulk.*	
		$711	33	1716	80
	Firing, Boxes & Packing, 37^{12} @ 4 per pcl., .			148	48
	Matting, Rattaning & Marking, @ 30¢, .			30	00
	Shipping, @ 5 .			5	00
	Godown Rent, @ 8 .			8	00
	Consular Fee, ...			2	50
	Export Duty, ...			39	65
	Inspection Commission, 1% (on 1716.80), .			17	17
	Fire Insurance, ½% " " .			8	58
	Bank Advances in Yokohama, . . .			9	00
	Buying Commission, 5%, ...			99	38
	Bill Brokerage, ¼%, ...			5	22
	Petty Charges, ...			2	50
	Lacquered boxes, lead, firing, &c., . . .	327	79		
	Yokohama cost, in Mexican dollars, . .	1039	12	2092	28
	Mexican. Sterling. Back chgs. 3%. 1039.12=£230 12 5, add £6 18 4=£237 10 9= (£ @ $4.90			$1163	98
	2092.28= 464 4 6, " 13 18 7= 478 3 1= Am. gold)			2342	95
	3131.40= 694 16 11, " 20 16 11= 715 13 10=			3506	93
	(*Policies.*) Insurance { 1339, @ 1¾, ...	1163	98	2342	95
	Insurance { 2406, @ "	23	43	42	10
	Freight, Yokohama to Chicago, . . .	250	00	285	00
	Petty Charges, Custom House, &c., . .	3	00	7	00
	Duty 15¢ per lb., ...	279	00	675	00
	Total cost in U. S. gold. -	$1719	36	3352	05
	" per lb., "		92⅓		74⅓
	CHICAGO, Nov. 26, 1871. JAMES C. FRASER & CO.'s Agts.				

GENERAL AVERAGE.

490. 1. **When, for the safety of a ship in distress, any destruction of property or expense is necessarily and voluntarily incurred, either by cutting away the masts, throwing goods overboard, or otherwise, all persons who have goods on board, or property in the ship, bear their proportion of the loss.**

2. The method of apportioning the loss among the several interests, sacrificed or benefited by the sacrifice, is called ***General Average,*** and the property thus sacrificed is called ***Jettison.***

491. 1. In ascertaining the amount of *loss* to be averaged, not only the amount of goods thrown overboard is considered, but also all damages to the ship, cost of repairs, and expense of detention for making repairs, including the wages of officers and crew; also the expense of entering a harbor to avoid peril, or of setting afloat when stranded; also towage in case of being disabled, or salvage paid another vessel for affording relief, etc.

2. When the repairs made consist of new masts, rigging, etc., a deduction of $\frac{1}{3}$ of their cost is usually made, since they are considered better than the old.

3. In estimating the value of the three *contributing interests*—vessel, freight, and cargo—it is customary to value the cargo at the price it would have brought at its port of *destination.* It is sometimes valued at its *invoice* price at the port of *lading.*

4. As the wages of seamen, pilotage, etc., are paid out of the freight, a deduction is made from the gross freight for this purpose. The amount to be deducted is not determined in a uniform manner. According to some authorities, the gross freight less $\frac{1}{3}$ is the net freight, except in New York, where $\frac{1}{2}$ is deducted. The general practice, however, is to ascertain what sum will actually be left to the vessel as *net* freight, after paying seamen's wages, etc. Sometimes the vessel earns a net freight of $\frac{2}{3}$ the total amount, and sometimes the seamen's wages, etc., absorb the *whole* of a very low freight. Each case is estimated by its *attendant circumstances.*

5. The practical difficulty in General Average is to determine whether the loss is subject to a general average. In some cases the loss is borne by only a part of the contributory interests.

6. When either a part or the whole of the ship or cargo or both is insured, the insurers bear their proportion of the loss as

found by average. In some instances the adjustment of the insurance becomes a very intricate problem.

7. In order to a valid claim for a General Average three considerations are necessary: 1st. *An imminent common peril and a necessity for some sacrifice;* 2d. *A voluntary sacrifice of a part to save the rest;* 3d. *The success of the effort to save a part, as a result of the sacrifice made.*

492. RULE.

Divide the total loss subject to average by the sum of the values of the contributory interests, and multiply each interest by the per cent. thus found.

NOTE.—The *jettison* must be included in the contributory interests, and bear its proportion of the loss.

493. EXAMPLES.

1. The ship Western World, in her passage from New York to Aspinwall was struck by a severe gale near the island of Cuba. After throwing overboard cargo amounting to \$4650, she made the port of Havana. Here the cost of the necessary repairs of the vessel was \$1800, and the cost of detention in port \$450. The contributory interests were as follows: value of ship \$35000; value of cargo \$24000; net freight \$4000. Of the cargo, \$8500 was shipped by Terry & Wheeler; \$7500 by Morse & Duty; \$5000 by T. C. Hood & Co.; and \$3000 by P. Kinney & Co. How ought the loss to be averaged?

OPERATION.

Vessel, ...	\$35000	Jettison, ...	\$4650
Cargo, ...	24000	Repairs, less $\frac{1}{3}$, . .	1200
Net freight, ...	4000	Cost of detention, .	450
Total contrib. interests,	\$63000	Total loss, ...	\$6300

$6300.00 \div 63000 = .10$; loss 10 per cent.

\$35000 × .10 = \$3500, loss borne by ship.
24000 × .10 = 2400, " " " cargo.
4000 × .10 = 400, " " " freight.
8500 × .10 = 850, " " " Terry & Wheeler.
7500 × .10 = 750, " " " Morse & Duty.
5000 × .10 = 500, " " " T. C. Hood & Co.
3000 × .10 = 300, " " " P. Kinney & Co.

2. The steamship Asia sailed from Liverpool to Boston with a cargo as follows: shipped by T. S. Foot & Co. \$45500; by C. S. Moore & Co. \$10500; by T. Hope & Sons \$7450; by C. White & Co. \$12550. During a storm the captain was obliged to throw overboard cargo amounting to \$8500, and the necessary repairs of the ship cost \$2700. In addition to repairs, the charges for seamen's board, dockage, etc., were \$500. How is the loss to be shared, the value of the ship being \$40000, and the net freight \$4000?

Remark.—The following example will give the student some idea of Insurance as connected with General Average.

3. The schooner Michigan sailed from Chicago for Buffalo with the following cargo: 25000 bushels of wheat owned by Smith & Dewy; 18500 bushels of corn owned by Fisk & Hunter; 850 barrels of flour owned by T. Ford & Co. The schooner is insured in company A for \$30000, which is $\frac{3}{4}$ of its value, at 3 per cent.; the wheat in company B for \$22500 (invoice price) at 2 per cent.; the corn in company C for \$9250 (invoice price) at $1\frac{1}{2}$ per cent.; and the flour in company D for \$4250 (invoice price) at $2\frac{1}{2}$ per cent. The gross freight was \$6000, and seamen's wages, etc., $\frac{1}{3}$ of the gross freight. During a severe storm the flour was thrown overboard. How is the loss to be borne? How is the payment of the sum for which the flour is insured to be adjusted?

Explanation.—By general average we find that the *average loss is 5 per cent.*, and that the schooner must sustain \$2250 of the loss; the cargo \$1800; and the freight \$200. Insurance Company A must pay 5 per cent. of \$30000=\$1500; Company B, 5 per cent. of \$22500=\$1125; Company C, 5 per cent. of \$9250=\$462.50; and Company D, 5 per cent. of \$4250=\$212.50.

4. Suppose, in the above example, that when the schooner reached her dock in Buffalo the flour could have been sold for \$6120; the wheat for \$35780; the corn for \$11100. How is the insurance to be adjusted?

5. The ship Speedwell worth \$38000, en route from New Orleans to New York, being in peril, sacrificed \$7200 worth of the entire cargo valued at \$22000. The net freight was valued at \$3500. A's interest in the cargo was \$6000. B's \$8000. C's \$4500 and D's \$3500. The cost of repairs was \$1650 and of detention \$480. How should the loss be adjusted?

BANKRUPTCY.

494. 1. *Bankruptcy* or *insolvency* is a failure in business and an inability to pay indebtedness.

2. A *Bankrupt* or *insolvent* is a person who fails in business and has not means to pay all his debts.

3. The entire property of an insolvent is called his *assets;* and the amount of his indebtedness his *liabilities.*

4. An *Assignment* is the transfer of the property of a bankrupt to certain persons called *assignees,* in whom it is vested for the benefit of creditors.

5. A *Dividend* is the amount paid to a creditor.

495. 1. The Common Law allows an insolvent to assign his property in favor of a certain creditor, or class of creditors, to the exclusion of others; but the United States Bankrupt Law of 1867 regards such an assignment as fraudulent.

2. Claims are paid in full for amounts of less than $50 to a "preferred class" of creditors, including operatives and employees to whom wages are due for services rendered within 6 months of the assignment.

3. The assignee may, on petition of the creditors granted by the court, defer settlement for the purpose of making a profit from the assets by using them for the creditors to whom he makes a final statement of his accounts.

4. After the assets of a bankrupt have been applied to meet his liabilities, still he remains liable (for a time) for the balance due, unless he has voluntarily or involuntary gone through a Court of Bankruptcy and complied with its conditions.

496. To apportion dividends in Bankruptcy.

RULE.

Divide the net proceeds of the assets by the amount of liabilities, and the quotient will be the per cent. of the indebtedness (or the number of cents on a dollar) *that can be paid.*

To find each creditor's dividend, multiply his claim by the per cent. thus found.

497. EXAMPLES.

1. A merchant failing in business owes A $950, B $2500, C $1500 and D $3050. His assets are $6000, and the expense of

settling will be $800. What per cent. of his indebtedness can he pay? What dividend will each creditor receive?

A's claim,	$950	$950 × .65 =	$617.50 A's div.
B's "	2500	2500 × .65 =	1625.00 B's "
C's "	1500	1500 × .65 =	975.00 C's "
D's "	3050	3050 × .65 =	1982.50 D's "
Liabilities, . . .	$8000	Proof,	$5200.00
Assets, ...	6000		
Expense of settling,	800		
Net proceeds, . . .	$5200		

$$5200.00 \div 8000 = .65, \text{ or } 65 \text{ per cent.}$$

Explanation.—Since his liabilities are $8000 and the net proceeds of his assets $5200, he can pay $5200 on $8000, or 65 per cent. of his liabilities. Hence each creditor can receive 65 per cent. of his claim, or 65 cents on a dollar.

2. Best & Foster became embarrassed and failed in business. Their indebtedness was $65000. The firm had cash and goods convertible into cash, $12500; building and lot, $40000; bills collectible, $2100. If the expense of settling is 5 per cent. of the amount *distributed to creditors*, what per cent. of their indebtedness can they pay? What will C. Greene & Co. receive, whose claim is $25800?

NOTE.—Divide assets by 1.05; the quotient will be net proceeds.

3. C. Smith & Co. have become insolvent. They owe A $3500, B $1500, C $1450, D $850, E $350 and F $450. Their effects (assets) amount to $4981.50. The charges of the assignees will be $2\frac{1}{2}$ per cent. of the amount distributed to creditors. What per cent. of their indebtedness can they pay? What will each creditor receive?

INTEREST.

498. 1. ***Interest*** is really the *use* of another's money, capital, or credit.

2. *Money* paid or allowed in account *for such use* is also called interest, and this is commonly understood in speaking of interest.

3. Interest arises from loans, from certain investments, and from delay in paying acknowledged debts.

499. 1. The *Principal* is that of which the borrower or creditor has the use.

2. The *Amount* is the sum of the principal and interest.

500. Interest is computed at a certain *rate per cent.* for some particular interval of time, generally *for one year*, and this per cent. is called the *rate of interest*. Thus, if the rate be 6% per annum, it signifies that a sum equal to six hundredths of the principal is to be paid for the use of it one year, and for other intervals of time in the same ratio; as, .18 for 3 years, .01 for 2 months, .005 for 1 month, etc. If the rate be 2% *a month*, then two hundreths of the principal is to be paid for its use each month; $\frac{1}{10}$% *a day* requires .001 of the principal for each day's use, etc. If no time is specified one year is understood.

501. The prevailing rate of interest indicates the worth of money or its price. This is controlled by the condition of public credit, supply and demand, and other considerations similar to those that control the prices of merchandise, and sometimes identical with them.

2. *Legal interest* is the maximum rate allowed by law. Higher or lower rates are frequently secured by special agreement.

3. *Usury* is *illegal interest*, or a higher rate of interest than is allowed by law.

Usury formerly was synonymous with *interest*. England having abolished all usury laws, has no further use for that term. The practicability of voluntary contracts in loaning money, restricted only as other contracts are restricted, is gaining increased favor among intelligent political economists, as well as among money *borrowers*. When the rate has not been previously agreed upon, a *legal rate* is desirable, to avoid contention or oppression. Government regulates all weights and measures, but not the prices of the articles weighed and measured. So it regulates the weight and fineness of coins, but it should not dictate the price paid for the use of them.

(For usury laws and rates of interest, see Part Third. Art. 783.)

NOTE.—Usury really means *use*, and was formerly applied to all interest, which was regarded as a kind of extortion from the opinion that money was dead capital and of no value to the lender, and he was considered as unjust in requiring the borrower to return more than the principal.

502. 1. ***Simple Interest*** is interest on the original

principal only, and this is generally understood by the term *interest.*

2. ***Compound Interest*** is interest on the *Amount,* computed at regular intervals. In this case the interest is said to be *compounded* (*mixed* with the principal) annually, semi-annually, etc.

3. The consideration as to *when interest is justly due* has given rise to a variety of laws and usage concerning it. Simple interest is based upon the idea that the interest is not to be considered as any part of the principal or loan due at any time, while in compound interest the interest is considered as constituting a part of the loan, if not paid at certain intervals of time. Practically, however, in most cases, this presents no difficulty, as interest is generally considered due at the end of the time to which the rate refers, unless otherwise specified, and only simple interest is allowed, except by special agreement.

NOTE.—Formerly, in some States, if a note read "with interest payable annually," or "with annual interest," the interest might be collected at the close of each year; but, if not paid, the interest due drew only *simple interest* to the time of maturity, or until paid, and this was called *annual interest.* Properly a new note should be given for each amount of interest due and unpaid, when "annual interest" is to be allowed.

4. *Accrued interest* is interest accumulated on account of any obligation, due or not due.

503. 1. ***Discount*** is interest paid in advance.

2. *Bank discount* is discount on the entire principal, and this is commonly understood by the term *discount.*

The *proceeds* are the principal less the bank discount.

3. *True discount* is discount on such a sum of money as would *amount* to the principal if put at interest at the given rate and for the given time. The *present worth* is the principal less the true discount.

COMPUTATION OF INTEREST AND DISCOUNT.

504. 1. Simple *Interest* is the *product* of the *Principal, rate per cent.,* and *time*; hence either of these four terms may be found if the other three are known. (Art. 159, 11.)

2. All *interest rules* are based on these relations, different methods of operation being used to obtain results by convenient contractions.

505. The *principal* (p) corresponds to the *base* in Percentage; the *decimal rate* × *time* ($r \times t$) to the *per cent.*; and the *interest* (I) to the *percentage*; hence the formula ($B \times \% = P$, Art. 272,) becomes $p \times (r \times t) = I$.

Ex. What is the interest on \$50 for 3 y. 6 m. at 6%?

Solution.—If the interest be .06 (six hundredths) of the principal for one year, $3\frac{1}{2}$ y. would afford $.06 \times 3.5 = .21$ of the principal; hence the interest required is $\$50 \times .21 = \$10.50$, or $\$50 \times .06 \times 3.5 = \$10.50$.

NOTE.—The *rate* should be expressed decimally, and the *time* should be expressed in the units of time to which the rate refers.

506. The terms derived from the four mentioned above (Art. 504), and their corresponding terms in Percentage are as follows:

1. *Amount* (A) $= p + I$, and corresponds to *Amount* in Percentage.
2. *Amount per cent.* $= 1. + (r \times t)$, and corresponds to $A\%$ in Percentage.
3. *Difference* (D) $= p - I$, and corresponds to *Difference in* Percentage.
4. *Difference per cent.* $= 1. - (r \times t)$, and corresponds to $D\%$ in Percentage.

507. The three formulas following are adapted to the solution of all problems in Simple Interest and Discount.

$$\text{I. } p \times r \times t = \textit{Interest}.$$

$$\text{II. } p \times \overline{1. + (r \times t)} = \textit{Amount} = p + I.$$

$$\text{III. } p \times \overline{1. - (r \times t)} = \textit{Difference} = p - I.$$

508. TABLE OF CORRESPONDING TERMS.

	I. Percentage. Art. 274.	II. Interest Art. 505, 506.	III. Bank Discount Art. 508, 512.	IV. True Discount. Art. 508, 512.
1	Base.	Principal.	Principal.	Present worth.
2	Per cent.	Rate × time.	Rate × time.	Rate × time.
3	Percentage.	Interest.	Discount.	Discount.
4	Amount.	Amount.		Principal.
5	Difference.	Difference.	Proceeds.	

509. By substituting in the three formulas given, the proper terms for Bank Discount and True Discount, all problems to

which they apply may be solved in accordance with the following simple principles.

1. *A required product is found by multiplying together known factors.* From this may be found terms 2, 3, 4, or 5 in the table above if the proper factors be known.

2. *A required factor is found by dividing the known product of all the factors by the product of the known factors.* From this may be found 1 or 2, and r or t, if the proper product and all but one of its factors be known.

3. *A required amount is found by addition, and a required difference by subtraction.* From this may be found 4 and 5 if 1 and 3 be known.

510. *Three days of grace* are to be added to the time specified, in computing *bank discount.* These were originally allowed as an accommodation to the maker of a note or draft for its convenient payment, without any extra charge; but for some time interest has been charged for these days of grace, and a note, etc., is not regarded as legally due till the last day of grace.

NOTE.—Three days are the common term of grace in the United States; the same allowance is made in Great Britain, but more time is allowed in most other European countries.

511. Bank discount is really more than interest compounded in advance. Thus if a note for \$50,000 be discounted for 57 days at 6%, the borrower is charged for (57+3) 60 days, .01 of \$50000 or \$500, and he receives as the *proceeds* of the note \$50000 −\$500=\$49500. Thus he pays interest on \$49500 and on \$500 for the time, while interest for 60 days compounded in advance would be only the interest on \$49500 and on .01 of *it*, or \$495, that is .01 of \$49995=\$499.95. The *true discount* would be \$50000− (\$50000÷1.01)=\$50000−\$49504.95=\$495.05. If this note were dated June 1st, and the full term of 60 days allowed in each case, the following distinctions might be made:

Simple interest on \$50000 for 60 days			=\$500	due Aug. 30.
Bank discount	"	"	=\$500	" June 1.
True discount	"	"	=\$495.05	" June 1.
Principal to be used 60 days			=\$50000.	
Proceeds	"	"	=\$49500.	
Present worth	"	"	=\$49504.95.	

512. GENERAL RULES.

1. To compute **Simple Interest.**—*Multiply the principal by the product of the rate and time.*

2. To compute **Bank Discount.**—*Multiply the face of the note or draft by the product of the rate and time, allowing three days grace. Subtract this product from the face of the note to find the proceeds.*

3. To compute **True Discount.** *Divide the amount by 1.00 plus the product of the rate and time; the quotient will be the present worth; subtract this from the amount, and the remainder will be the true discount.*

NOTE.—*Mercantile or nominal discount* is any amount deducted from a bill or an account for cash payment or any other consideration, and is generally some percentage of the amount, without exact reference to time, called also *per cent off*. The remainder is sometimes called the *nominal present worth* or *net amount* of the bill.

CONTRACTIONS.

513. 1. Bankers and others who are frequently required to compute interest use Interest Tables, but others in practice use various simple contracted methods, chiefly for partial or entire mental computations, depending on the relation of the rate to the time or on the use of aliquots and reciprocals.

2. INTEREST TABLES.—Bryant, Stratton & Packard's Interest Tables, used extensively by bankers and others, reduce the labor of interest computations very materially and insure the correctness of the work. Computations are made at one, five, six and seven per cent., and from these rates any per cent. can readily be obtained.

3. No one method of contraction is the best for all cases, but it is well to be familiar with several and apply the most appropriate in each particular case, or any one that seems most simple may be adopted for general use.

4. The following are believed to be the best methods for practical use. The year is computed at 360 days, or 12 months of 30 days each, unless otherwise indicated.

514. 1. **Year Rule.**—*Multiply the principal by the decimal rate; of the product take such multiples or aliquot parts as*

the given time is of 1 year, or of any other time for which the interest is known; if there be more than one multiple or aliquot, find their sum.

NOTE.—If the *rate* be required, *take multiples or aliquots of the principal for the given time and divide the given interest by their sum.*

2. ***10% Rule.****—Remove the decimal point in the principal one place towards the left to show the interest for 360 days, and two places for 36 days; then take multiples or aliquots for the given time. In the sum of the results remove the decimal point one place towards the left, to show the interest at 1 per cent., and multiply the result by the given rate not expressed decimally.*

3. ***12% Rule.****—Reduce the time to months and days; call the months hundredths and one-third the number of days thousandths; use the sum of these decimals for the multiplier, or decimal per cent., contracting the multiplications; and take aliquants or multiples of the product for interest at the given rate.*

NOTES.—1. Observe that at 12% the multiple for 33 days is .11; for 63 days, .21; for 93 days, .31, etc, (Art. 111,) and that these are the most common periods of time for which discount is computed at banks.

2. At 6%, call one-half the number of months hundredths, and one-sixth the number of days thousandths.

4. ***Day Rule.****—Remove the decimal point in the principal two places towards the left, and compute for the given time according to the following*

TABLE.

Pointing off two places gives the interest

at	1%	for	360	days	at	360	days	in	1 year.
"	2	"	182	"	"	364	"		"
"	3	"	122	"	"	366	"		"
"	4	"	90	"	"	360	"		"
"	5	"	72	"	"	360	"		"
"	6	"	60	"	"	360	"		"
"	7	"	52	"	"	364	"		"
"	$7\frac{3}{10}$	"	50	"	"	365	"		"
"	8	"	45	"	"	360	"		"
"	9	"	40	"	"	360	"		"
"	10	"	36	"	"	360	"		"

at	11%	for	33	days	at	363	days in	1 year.
"	12	"	30	"	"	360	"	"
"	13	"	28	"	"	364	"	"
"	14	"	26	"	"	364	"	"
"	15	"	24	"	"	360	"	"
"	18	"	20	"	"	360	"	"
"	24	"	15	"	"	360	"	"

Notes.—1. Observe that at $7\frac{3}{10}$%, the interest on $100 is 2¢ for each day.

2. Compute at the most convenient rate at first.

3. For *accurate interest*, according to the exact number of days in a year, when it has been computed on a basis of 360 days, subtract $\frac{1}{73}$ (about $1\frac{1}{3}$%) of it, for a year of 365 days, and $\frac{1}{61}$ of it for a leap year; if computed on a basis of 363 days, subtract $\frac{2}{365}$ for a common year; if on a basis of 364 days, subtract $\frac{1}{365}$; if on a basis of 366 days, add $\frac{1}{365}$.

515. The four methods compared.

Ex. I. What is the interest on $3245 for 3 y. 5 m. 27 d., at 7%?

1st. Year Rule.

$3245.	(×.07)
$227.15	Int. for 1 y.
454.30	" 2 y.
75.716	" 4 m.
18.929	" 1 m.
17.036	" 27 d. ($\frac{9}{10}$ of 1 m.)
$793.131	Entire Int. at 7%.

2d. 10% Rule.

$324.50	Int. for 360 days at 10%.
32.45	" 36 " "
$973.50	" 3 y.
108.166	" 4 m. ($\frac{1}{9}$ of 3 y.)
27.041	" 1 m.
24.336	" 27 d. ($\frac{9}{10}$ of 1 m.)
$113.3043	Entire Int. at 1%.
7	
$793.1301	Entire Int. at 7%.

20

3D. 12% RULE.

$$\begin{array}{rll}
\$3245 & & \\
\underline{.419} & \text{(41 m. and } \tfrac{1}{3} \text{ of 27 d.)} & \\
29205 & & \\
3245 & & \\
\underline{12980} & & \\
\underline{\$1359.655} & \text{Int. at} & \underline{12\%} \\
679.827 & \text{"} & 6\% \\
\underline{113.304} & \text{"} & \underline{1\%} \\
\$793.131 & \text{"} & 7\%
\end{array}$$

4TH. DAY RULE.

$$\begin{array}{rll}
\underline{\$32.45} & \text{Int. for 60 d. at 6\%.} & \\
\$584.10 & \text{"} & \text{3 y. (60 d.} \times 6 \times 3). \\
64.90 & \text{"} & \text{4 m. (60 d.} \times 2). \\
16.225 & \text{"} & \text{1 m.} \\
\underline{14.602} & \text{"} & \text{27 d. (} \tfrac{9}{10} \text{ of 1 m.)} \\
679.827 & & \\
\underline{113.304} & \text{(Add } \tfrac{1}{6}). & \\
\$793.131 & \text{Int. at 7\%.} &
\end{array}$$

(Art. 514, 3, Note 2. $\$3245 \times .2095 = \$793.131$.)

2. What is the bank discount on $5250 for 90 d. at 6%? (Art. 511.)

1ST. YEAR RULE.

$$\begin{array}{rll}
\underline{\$5250.} & (\times .06.) & \\
\underline{315.} & \text{Int. for 1 y. or 360 d.} & \\
78.75 & \text{"} & \text{90 d.} \\
\underline{2.625} & \text{"} & \text{3 d.} \\
\$81.375 & \text{Entire discount.} &
\end{array}$$

2D. 10% RULE.

$$\begin{array}{rll}
\underline{\$525.} & \text{Int. for 360 d. at 10\%.} & \\
131.25 & \text{"} & \text{90 d.} \\
\underline{4.375} & \text{"} & \text{3 d.} \\
\underline{13.5625} & \text{"} & \text{93 d. at 1\%.} \\
\$81.375 & \text{Int. at 6\%.} &
\end{array}$$

3D. 12% RULE.

$5250	×.031.
15750	(Art. 111.)
162.750	Int. at 12% for 93 d.
$81.375	" 6% "

4TH. DAY RULE.

$52.50	Int. for 60 d.
26.25	" 30 d.
2.625	" 3 d.
$81.375	" 93 d.

(Art. 514, Note 2. $5250 × .0155 = $81.375.)

COMPUTATION OF TIME IN INTEREST.

516. 1. While most of the States have enacted rigid laws against taking usurious interest, they have left the *mode of computing legal interest* very indeterminate. Nearly all the rules in common use in this country are inaccurate and illegal, and have only been sustained by decisions based upon custom; but custom varies, and the legal decisions have not been uniform.

2. The difficulties attending this question, which has occasioned so much litigation and jeopardized so much capital, can be briefly stated.

517. 1. The fundamental principle upon which lawful simple interest is computed is that the rate should be exactly proportionate to the term for which interest is paid. The time usually assumed for fixing the rate is one year, *e. g.*, 6 per cent. per annum; that is, when the time is one year, the interest should be $\frac{6}{100}$ of the principal; and when the time varies from one year, the proportion of interest should vary in exactly the same ratio. If, then, we assume that the year consists of 365 days (as that is regarded by law a civil year), it must be admissible, in computing the interest on a note running from Jan. 1, 1868, to Jan. 1, 1869, to add one day's interest to the interest for one year; for in the case proposed, February of a leap year intervening, the time is 366 days instead of 365, the legal civil year.

2. One year being the standard of reference in expressing the

rate, all time in computing interest must be expressed in years or aliquot parts of the year. But the year has no *exact* natural or artificial subdivisions except the day, and the day is an aliquot part only as we *assume* the year to consist of a definite number of days. The number 360 being a multiple of more whole numbers than 365, for convenience in reckoning it would have been better to assume 360 days for the *nominal* year in fixing the rate, rather than 365. The *time* in expressing the rate is *arbitrary*, and as neither 360, 365, nor 366 is the exact number of days in all years, either civil or astronomical, would not the increased facility in computation, and the perfect accuracy in the result, warrant the change?

3. The division of the year into twelfths, called months, is purely imaginary; for no month, either lunar or calendar, was ever known which occupied just one-twelfth of a year. Manifestly, if we assume a year of 365 days as the standard for reference in expressing the rate, we never can introduce the denominations of months in any form whatsoever without inaccuracy, unless we involve in the calculation fractional parts of days, which would be as absurd as it would be difficult.

4. If, however, we assume a year of 360 days, we may have assumed months of 30 days. Then 6 per cent. per annum of 360 days would be 1 per cent. for 60 days, and all time being reduced to days or months of 30 days each, or years of 360 days each, the computation would be simple, rapid, and perfectly accurate. As it is, the law having accurately determined when a paper matures, however the time may be expressed in the paper, the only accurate rule for computing interest is to ascertain the actual number of days, and make each day's interest $\frac{1}{365}$ of the annual interest. Some banks are restricted by their charters in their discounts to "6% per annum," but are allowed to compute by Rowlett's Tables. But Rowlett's rule "To find bank interest," makes all time reducible to days, and the interest for each $\frac{1}{365}$ of the year's interest, so that when the time in the note to be discounted reads "two months," the interest for $\frac{2}{12}$ of the year should never be taken except when February 29th of a leap year is included in the term, for in that case only will the "two months" contain just 60 days and no more. In all other cases, the interest should be 59, 61, or 62-360ths of the year's interest, according to the actual number

-ys contained in the time of the note. In Massachusetts and other States interest computed on the supposition that 360 make the year is regarded valid. But in New York each interest must be only $\frac{1}{365}$ of the year's interest.

-18. RULES FOR COMPUTING THE DIFFERENCE OF TIME -WEEN DATES.—Besides counting the exact number of days -ferred to above, two rules are in common use.

-ULE I.—*By compound subtraction, reckoning 30 days for a -th.*

-ULE II.—*By finding the number of entire calendar months -the first date, and counting the actual number of days left.*

-OTE.—By "calendar month" is meant the time from any day of one - to the corresponding day of the next month. If the days of the first - is a higher number than the greatest number of days in the last -th, the calendar month ends with the last day. Thus, from Oct. 31 to - 30 is a calendar month.

-x. 1. From Aug. 20, 1854, to March 10, 1857, -d be,

according to the 1st Rule, 2 y. 6 mo. 20 d.;

" " 2d Rule, 2 y. 6 mo. 18 d.

$$\left\{\begin{array}{rrr} 1857 & 3 & 10 \\ 1854 & 8 & 20 \\ \hline & 2 & 6 \quad 20 \end{array}\right.$$

-. From Aug. 31, 1854, to March 10, 1857, would be,

according to the 1st Rule, 2 y. 6 mo. 9 d.;

" " 2d Rule, 2 y. 6 mo. 10 d.

-OTES.—1. It will be observed that in these particular examples, though -ctual difference of time in the two cases is 11 days, the result by the -d rule shows only 8 days. A discrepancy of 2 days *may* also arise in the - the first rule, for by it the time from Feb. 28, 1857, to March 2, 1857, - be 4 days, while the actual time is only 2 days. The first rule also - no difference of time between March 31 and April 1. Each rule will - result sometimes too large and sometimes too small.

-The examples in this work, except those in Bank Discount, and those -se restricted, may be wrought by the second rule.

-e Part Third (Art. 784.) for table of intervals in days.

-. MENTAL PROBLEMS.

-What is the interest on $320 for 2 y. at 6%?

-tion.—For 1 year the interest is six hundredths of the -, for 2 years twelve hundredths, etc.

-0, 3 y. at 10%? $2000, 3 m. at 10%?

-, 1 y. 3 m. at 7%? $10000 at 12%, 33 d.? 63 d.?

4. $750 at 6%, 4 m.? 6 y. 8 m.? 3 m.? 10 m.?

5. $20000 at 5%, 1 y. 6 m.? 9 m.? 2 y. 3 m.?

6. Amount of $1250 at 4%, 1 y. 9 m.? At 6%, 4 m.?

7. Amount of $15 at 10%, 5 y. 6 m.? 10 y. 3 m.?

8. What principal at 8%, in 1 y. 3 m. would afford $13.25 interest? .

9. What principal would afford $280 interest in 2 y. at 7%?

10. What is the face of a note on which the discount for 90 d. at 12% is $6.20? Proceeds?

11. For what amount must a note be given to afford $9790 proceeds, if discounted at 12% for 60 d.?

Solution.—The discount for 60 d. is the interest for 63 d., or .021 of the face of the note; hence the proceeds, $9790, must be (1.000−.021) .979 of the face of the note; therefore the face of the note must be $9790÷.979=$10000.

12. Face of note to afford $1990 if discounted for 27 d. at 6%?

13. Face of note to afford $980 if discounted for 1 m. 27 d. at 6%?

14. What is the present worth of $4500 due in 5 y. with interest at 10%?

Solution.—$4500 is the amount of the present worth and the true discount; hence it is 1.50 (150 hundredths) of the present worth; therefore p. w.=$4500÷1.50=$3000.

15. What is the present worth of $214 due in 6 m. at 7%? True discount?

16. What rate will afford $42 interest on $300 in 2 y.?

$300 × 2 × (7)=$42.

17. Rate for $7.10 interest on $284 in 3 m.? (Art. 514, 1, and Note.)

18. Rate for $75 interest on $500 in 1 y. 6 m.?

19. Rate for $24 interest on $3600 in 30 d.?

20. Rate per *month* for $60 interest on $1000 in 90 days?

21. What time will produce $56 interest on $200 at 7%?

22. Time required for $150 interest on $1200 at 10%?

23. Time required for $500 to *amount* to $510 at 6%? (Art. 506, 1.) *Int.* $\div(p \times r)=t=\frac{10}{30}=\frac{1}{3}$ y.=4 m.

24. Time for $750 to amount to $825 at 5%?

25. Time for which a note of $400 may be discounted at 6% and afford $392 proceeds?

520. WRITTEN PROBLEMS.

NOTE.—Let the most convenient rule be used in each particular case.

1. What is the interest on $120 for 1 y. 2 m. 12 d. at 6%?
2. What is the interest on $340.50 for 2 y. 3 m. 15 d. at 9%?
3. What is the interest on $1000.25 for 1 y. 9 m. 3 d. at 10%?
4. What is the interest on $25 for 3 m. 3 d. at 12%?
5. What is the interest on $145.20 for 1 y. 11 m. 29 d. at 7%?
6. What is the interest on $450 for 3 y. 2 m. 21 d. at 8%?
7. If a man borrows $10000 at 6% interest, and loans it at 10%, what will he gain in 2 y. 3 d.?
8. A merchant bought 400 yards of cloth at $4 per yard, payable in 6 months, and immediately sold it at $4.10, giving a credit of 3 months, at the expiration of which term he anticipated the payment of his own paper, getting a discount off of 10% per annum. What did he gain by the transaction?
9. A merchant bought 400 yards of cloth at $4 per yard, payable in 3 months, and after holding it for 15 days sold it at $4.25 per yard, receiving therefor a note payable in 4 months. When the purchase-money became due, he had this note discounted at the bank to meet it. What did he gain by the transaction?
10. Taking the conditions of the last example, what would he have gained if he had borrowed at 6% interest, until the maturity of the note he had received, sufficient to pay for the cloth, and why should there be any difference in the results?
11. If I invest $1000 in wool, pay 5% for freight, and sell at 15% advance on cost price, giving 4 months credit, get this paper discounted at the bank at 6% interest, and repeat the operation every 15 days, investing all the proceeds each time, what will I gain in 2 months?
12. If a man borrows $1000 at 10% interest, and with it buys a note for $1100, maturing in 5 m., but which not being paid when due runs 1 y. 6 m. beyond maturity, drawing 6% interest, will he gain or lose, and how much?
13. Jan. 1st a man borrowed $10000 at 6% interest. Fifteen days after he lent $4500 for 8 m. 15 d., without grace, at 10%. Feb. 1st, with the balance he purchased a note for $5650, due July 4, which not being paid at maturity was extended until

the loan of \$4500 became due, at the rate of 8% interest. Both notes having been then promptly paid, he immediately purchased a 7% State Bond of \$10000, which, with its semi-annual interest, would mature Jan. 1st following, for which he paid 1% premium upon its par value, at the same time loaning the balance at the rate of $1\frac{1}{2}$% per month. What was his profit for the year?

14. Find the accurate interest of \$1000 from April 1 to Dec. 1.

Solution.—$244 \times \$60 \div 365 =$ *Ans.* \$40.11. (Art. 517, 4.)

15. Find the simple interest of \$125 from April 1 to Dec. 7.

Solution.—246 days = 8 m. 6 d. Then $\$125 \times .041 = \$5.125$, and $\$5.125 - \frac{5.125}{73} = \$5.055$, the interest required. (Art. 514, 4, Note 3.)

By the first rule we have the following equation: $246 \times \$7.50 \div 365 = \$5.055$.

16. Find the interest of \$1250 for 360 days at 6% per annum of 365 days.

Solution.—$\frac{6}{100}$ of $\$1250 = \$75$. Then $\$75 - \frac{5}{365}$ (or $\frac{1}{73}$) of $\$75 = \$73.97$.

17. Find the interest of \$1250 for 365 days at 6% per annum of 360 days.

Solution.—$\frac{6}{100}$ of $\$1250 = \$75$. Then $\$75 + \frac{5}{360}$ (or $\frac{1}{72}$) of $\$75 = \$76.04$.

18. What would be the difference between the accrued interest for 90 days on \$1000000 of 6% State Bonds, computed first in Ohio, counting 360 days for a year, then in New York, counting 365 days for a year?

NOTE.—In New York the interest for years and months is computed in the usual way without reducing to days, but for the odd days the interest is computed as in Ex. 14.

19. A note for \$1000 runs from Jan. 1, 1856, to Jan. 25, 1858, with interest at 6%. What amount is due according to accurate computation? New York computation? Ohio computation?

20. Find the accurate interest on \$5000 from Jan. 1, 1870, to May 1, 1871.

21. What is the rate of interest if I receive \$20.96 for the use of \$126.75 2 y. 24 d.? $\$126.75 \times (r) \times 2\frac{1}{15} = \$20.96$.

NOTE.—If it seem more simple, either to the pupil or the instructor, a unit of the thing required may be assumed and thus any required term may be found, or as more strictly in keeping with principles and formulas already stated, it may be well to use the following:

Solution.—Since the interest for $2\frac{1}{15}$ y. is \$20.96, the interest for one year would be \$20.96 ÷ $\frac{31}{15}$ = \$10.14 and this is .08 of \$126.75, hence the rate is 8%; or we may say that the interest of \$126.75 for $2\frac{1}{15}$ y. equals the interest of (\$126.75 × $2\frac{1}{15}$ =) \$261.95 for one year, and then we find that \$20.96 is .08 of \$261.95; hence the rate is 8%.

22. What sum invested at 10% per annum will secure \$1000 semi-annually? P × .05 = \$1000.

23. In what time will \$512.60 amount to \$538.31 at 7%?

Solution.—\$538.31 − \$512.60 = \$25.71 the interest, and \$512.60 × (   ) × .07 = \$25.71; hence \$25.71 ÷ (\$512.60 × .07) = the *time* required.

24. In what time would any principal be doubled at simple interest, at 5%? What per cent. of the principal would equal the interest in that time?

NOTE.—If the amount be treble the principal, the interest must be 200% of the principal.

25. At what rate will \$145.60 amount to \$188.07 in 5 y. 10 m.?

26. In what time will \$356.50 amount to \$465.429 at 8%?

27. What principal will amount to \$154.17 in 11 m. 9 d. at 6%?

NOTE.—A\$ = 1 + (r × t); or (by 514, 3, Note 2) 11 m. would afford .055 of the principal for interest, and 9 d. would afford .0015 of the principal; hence the A\$ = 1.0565.

28. What principal would afford \$19.65 in 183 d. at 7%, allowing 365 d. to a year?

NOTE.—\$19.65 ÷ (.07 × $\frac{183}{365}$) =

29. At what rate would \$355 in 3 y. 5 m. 20 d. produce \$86.28 interest?

30. In what time would \$1500 amount to \$1595.833 at 10%?

31. What is the true present worth and discount of a debt of \$1000 due in 1 y. 6 mo., the current rate of interest being 6 per cent.? (Art. 512.)

32. What sum must I put at interest at 10 per cent. to liquidate a debt of \$3000 due 3 years hence?

33. A man can sell his farm for \$5000 cash, or for \$6000 payable in 2 years; if he accept the last offer, and receive instead its

present worth at 8% interest, how much better would it be than the first offer? If he accept the first offer, and loan the $5000 at 8% interest, how much less would he receive at the end of the 2 years than if he accepted the latter? What is the present worth of that difference?

34. What is the proceeds of a note of $2500 discounted at a bank for 60 d. at 10%? What is the true present worth?

35. For what time can a note of $261.53 be discounted at 7% to afford $260 proceeds? How long must $260 be on interest at 7% to amount to $261.53?

36. If the amount of $1200 for 12 y. 11 m. 29 d. be $2135.80, what is the rate per cent.?

37. Which is better for a bank, to discount notes for 90 days at 10%, or for 30 days at 8%?

38. How much must be invested in bonds paying 6% interest to secure a semi-annual income of $600?

39. At what rate would $8275 amount to $9500 in 1 y. 11 m. 12 d.?

40. What is the interest on £80 8s. 8d. for 8 m. 24 d. at 4%?

INTEREST NOTES.

521. If interest be not paid when due, sometimes a note is given for the interest due, annual or semi-annual, and such an *interest note* draws simple interest until paid.

Some courts have allowed simple interest on interest due and unpaid, without an interest note being given.

It is quite customary in making loans, especially such as are secured by mortgages or trust deeds, to take a note for the principal without interest, and separate notes for each sum of interest falling due annually or semi-annually, and these interest notes draw interest after maturity.

522. The entire interest due on the original principal and on unpaid interest may be computed by the following

RULE.

To the interest on the principal for the whole time, add the interest on each sum of simple interest for the time elapsed since

due, or on one such sum for the sum of the several intervals; the final sum will be the entire interest due.

Ex. 1. A note for \$5000 was given Sept. 15, 1870, with interest from date at 10% per annum, payable semi-annually. Notes were given for the semi-annual interest due Mch. 15, 1871, Sept. 15, 1871, Mch. 15, 1872, and Sept. 15, 1872. What would be due Nov. 30, 1872, nothing having been paid?

Solution.—Interest on \$5000 at 10% 2 y. 2 m. 15 d. = \$1104.16⅔.

Interest on \$250, due March 15, 1871,	1 y.	8 m.	15 d.		
" " \$250, " Sept. 15, 1871,	1 y.	2 m.	15 d.		
" " \$250, " March 15, 1872,		8 m.	15 d.		
" " \$250, " Sept. 15, 1872,		2 m.	15 d.		
Int. on the int. notes = int. on \$250 for	3 y.	10 m.		=	\$95.83⅓.
			Entire interest due,	. . . .	\$1200.00
			Add the principal,	. . . .	\$5000.00
			Amount due,	. . . .	\$6200.00

2. What amount would be due Oct. 1, 1871, on a note of \$4200 given for two years and six months, with notes for quarterly interest after April 1, 1869, no payments having been made, at 6%?

3. Find amount due Jun. 1, 1871, on a note of \$6,000, dated July 1, 1868, due in two years, notes for semi-annual interest from date at 6% having been given and no payments having been made?

NOTE.—Observe that the principal note also draws interest after maturity.

COMPOUND INTEREST.

523. 1. In the computation of *Compound Interest*, the *amount* due at the end of each successive term of interest, is regarded as a new principal. The interest is generally compounded with the original principal annually or semi-annually, but sometimes more or less frequently.

2. The difference between the final amount and the original principal indicates the Compound Interest.

524. The successive amounts are readily found by using the amount per cent. as a multiplier. (Art. 272.)

Ex. What is the compound interest of \$5000 for 3 y. 6 m. at 6%?

Solution.

P. =	\$5000
A% =	1.06
Am't due at end of 1 y., used as a new Principal =	\$5300
	1.06
" " 2 " " "	\$5618
	1.06
	33708
	5618
" " 3 " " "	\$5955.08
A% for 6 m. =	1.03
	1786524
	594508
Final am't for 3 y. 6 m.	\$6133.7324
Deduct original Principal	5000.
Compound Interest for 3 y. 6 m. =	\$1123.43

NOTE.—*Compound Percentage* may be computed on any number, as of population, etc.

525. 1. The A% may be taken to represent the amount of the unit of money for the unit of time to which the rate refers.

Thus, if interest be compounded annually at 5%, the am't of \$1 for *one year* would be \$1.05, and for £1 it would be £1.05.

If interest be compounded at 3% semi-annually, the respective amounts for *six months* would be \$1.03 and £1.03.

2. The amount of a unit of money for the required time may be found by using the A% as a factor as many times as there are years or other units of time. Thus the amount of \$1 for 3 y. at 5% would be \$1.05 × 1.05 × 1.05 = \$1.157 +.

3. *Compound Interest Tables* showing the amount of \$1 for various rates and times may be found in Part Third. The amount diminished by 1 will give the Compound Interest.

NOTE.—The *number* of years may be regarded as any intervals of time. Thus the amount of \$1 for 10 months at 2% *a month* would be the same as of \$1 for 10 y. at 2% *per annum*, or \$1.218 +.

526. To find the amount of any principal at compound interest.

RULE.

1. *Of the amount per cent. for 1 year or other unit of time for compounding the interest, find that power whose index equals the number of the same units of time concerned, and multiply this power by the principal.* Or,

2. *Multiply the amount of $1 as given in the table, by the principal.*

NOTE.—For short methods of finding exact higher powers, see Art. 350.

527. If the time include fractions of the principal unit of time, the amount per cent., or the amount of the principal for the integral units should first be found, and the amount of *this* for the remaining time should be found. Thus if the amount of $2500 for 3 y. 5 m. 18 d. at 6% be required, the third power of 1.06 may first be found, $1.06^3 = 1.191016$, and this may be multiplied by the A% for 5 m. 18 d.; $1.19016 \times 1.028 = 1.223484$, which may be regarded as the amount of $1 for the given time, and $\$1.223484 \times 2500 = \$3058.71$ as the required amount. The compound interest is $\$3058.71 - 2500$ or $\$.223484 \times 2500 = \$558.71$.

528. Simple interest varies in the same ratio as the principal, rate, and time, while compound interest varies as the principal, but not exactly as the rate and time.

Doubling the principal doubles the compound interest.

Doubling the *rate* more than doubles the compound interest.

Doubling the *time* more than doubles the compound interest.

Ex. The compound interest of $200 for 4 y. at 5% is $43.10.

The compound interest of	$400	for 4 y.	at	5%	is	$86.20.
" " "	$100	" 2	"	5%	"	10.25.
" " "	"	" 2	"	10%	"	21.00.
" " "	"	" 2	"	6%	"	12.36.
" " "	"	" 4	"	6%	"	26.247.

529. 1. The four terms, the principal, rate, time, and compound interest or amount are so related, that when any three of them are given the other may be found.

2. If we let P = principal, r = rate per cent., and n = exact intervals of time, the rule given above may be expressed by the Formula:

$$\text{I. } P \times (1 + r)^n = A, \text{ and } A - P = I.$$

NOTE.—The tables give $(1+r)^n$. When n does not equal the whole time, see Art. 523.

530. 1. Observe that the formula given corresponds in general form nearly with that for the last term in a geometrical series. (Art. 392.)

2. If the amount of \$1 for one year be regarded as the first term of a series, and the A% as the ratio, then the formula $a\,r^{n-1}=l$ would give the amount of \$1 for n years.

3. By forming a series with any given principal as the first term the formula $a\,r^n=l$ would give the amount of the principal for any number of years.

4. These relations enable us to solve all simple problems involving compound interest, according to principles already explained in Geometrical Progression and Simple Interest.

5. The following formulas indicate the various processes.

I. $P \times (1+r)^n = A$.

II. $A \div (1+r)^n = P$ or $P.\ W.$ (Art. 508.)

III. $A \div P = (1+r)^n$.

531. *Compound interest* or *amount* required.

EXAMPLES.

1. What is the compound interest and amount of \$1000 for 5 y. at 6% per annum, payable annually?

2. What is the compound amount of \$2200 for 3 y. 2 m. 12 d. at 6% per annum, payable annually?

NOTE.—After having computed the compound amount for the number of *entire* intervals at the end of which the interest is payable or to be computed, compute the amount of that amount for any remaining time before the settlement.

3. What is the compound interest of \$1400 for 10 y. 8 m. at 8% per annum, payable quarterly?

Solution.—10 y. 6m. $=\frac{42}{4}$ y.; $(1.02)^{42}=2.29724447$ and \$1400 $\times$ 2.29724447 + \$3216.1423 the compound amount for 10 y. 6 m., and \$3216.1423 $\times$ $1.01\frac{1}{3}$ $-$ \$1400 $=$ the compound interest for 10 y. 8 m. $=$ \$1859.024.

4. If the population of a city containing 10,000 inhabitants should increase 10% annually, what would it amount to in 10 years?

5. If a farmer beginning with one bushel of wheat should sow his entire crop each successive year, and the increase each year should be 1900%, what would he have at the end of 5 years?

6. If a banker's rate in loaning money is 12% per annum, and he reloans all his capital every two months, what must have been the rate at simple interest to realize the same amount at the end of one year?

What, at the end of two years?

What, at the end of eight years?

What, at the end of fifteen years?

What, at the end of twenty-five years?

532. The *principal* at compound interest required.

Formula.—$A \div (1+r)^n = P$ or $P.\ W.$

RULE.

Divide the amount by the amount per cent. for the time, (Art. 442, 1,) *that is, by the amount of $1 for the time.*

NOTES.—1. The *present worth* (P. W.) really corresponds to the *principal*, while the sum of money of which the P. W. may be required is really a future *amount*.

2. The amount per cent. will indicate the amount of $1 or other unit of money for the given time. (Art. 782.)

EXAMPLES.

1. What sum, in 17 y., at 6%, payable annually at compound interest, will amount to $1009.79?

2. What sum, in 14 y., at 8%, payable semi-annually at compound interest, will amount to $10795.34?

3. What principal will yield $3251.50 compound interest in 6 y. 2 m. at 7%, payable semi-annually?

4. How much must a father, at the birth of his son, set apart for his benefit, so that with the interest at 7%, compounded semi-annually, it may amount to $10,000, when his son shall become 21 years of age?

5. What sum at 10%, payable quarterly, will produce $7197.22, compound interest, in 3 y. 6 m. 9 d.?

6. What is the present worth of $50,000, due 50 y. hence, at 9 per cent. payable annually?

How much greater would be the present worth at simple interest?

533. The *rate* of compound interest required.

Formula.—$A \div P = (1+r)^n$, and $\sqrt[n]{(1+r)^n} = 1+r$.

RULE.

1. *Divide the amount by the principal; the quotient will be the amount per cent. for the time.*

2. *Of this amount per cent. extract that root whose index equals the intervals of time, and subtract 1 from the root; the remainder will be the rate expressed decimally.* Or,

3. *After finding the amount per cent. for the time, look for this amount in the table, opposite the time; the rate at the top of the column in which this is found will be the rate required.*

NOTE.—If the time include months or days besides the exact intervals, look in the table for the amount next less than the amount per cent. found.

EXAMPLES.

1. At what rate per cent. will \$13000 amount to \$14835.159 in 3 years at compound interest?

2. At what rate per cent. will the compound interest on \$15000 amount to \$6038.2758 in 5 years?

3. At what rate per cent. will \$5248 amount to \$27157.31 in 33 years at compound interest?

4. At what rates will any sum of money double itself at compound interest in 8, 10, and 15 years respectively, interest payable semi-annually?

5. At what rate will \$7200 yield \$12665.02 compound interest in 15 years?

534. The *time* of compound interest required.

RULE.

1. *Divide the amount by the principal; the quotient will be the amount per cent. for the time required.*

2. *Of the amount per cent. for one interval of time find a power equal to the amount per cent. for the whole time; the index of this power will equal the whole number of intervals of time.* Or,

3. *Find the amount per cent. for the whole time, and in the table, under the given rate, look for a corresponding amount; the time for which this is the amount will be the time required.*

NOTE.—If there be no amount in the table exactly corresponding with the amount per cent. found, the number of years opposite the next less

amount will be the whole intervals of time required, and the difference between the tabular amount and the real amount, divided by the rate per cent., will give the fractional interval to be added to the tabular time.

EXAMPLES.

1. In what time will \$1000 amount to \$1338.22 at 6% compound interest?

2. In what time will \$50000 amount to \$56275.4405 at 6% per annum, interest compounded semi-annually?

3. In what time will \$5428 amount to \$27157.31 at 5% compound interest?

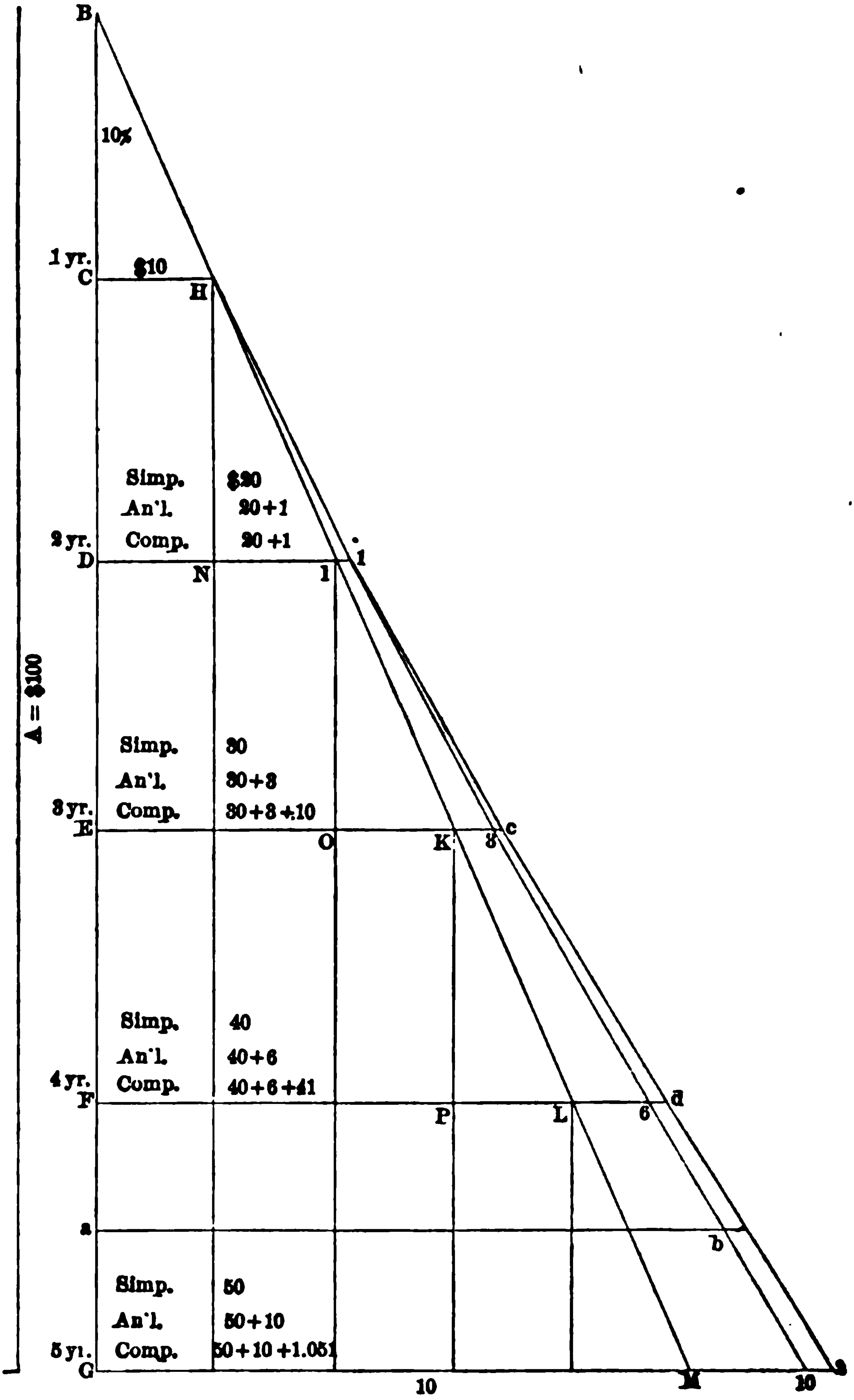
4. In what time will any sum of money triple itself by compound interest at 4%, 7%, 8%, and 10% respectively, interest payable quarterly?

5. In what time will \$17280 produce \$7197.22 compound interest at an annual rate of 10%, interest payable quarterly?

COMPARISON OF SIMPLE, ANNUAL, AND COMPOUND INTEREST.

535. ***Simple Interest.*** The relations between the principal, rate, time, and interest may be represented by the diagram on the following page. Let the single line A (which for convenience is separated from the diagram, but which should be considered as extending horizontally to the left from B, C, D, etc., respectively.) represent the principal; let the perpendicular line BG represent time with its divisions into years, at the points C, D, E, and F; and the horizontal lines CH, DI, EK, FL, and GM the accrued simple interest at the expiration of each successive year. As no ratio can be expressed between time and money, area can represent nothing in the diagram. As *rate per cent.* is nothing but the ratio between the principal and interest, *it* can only be represented by the degree of divergence of the lines BC and BH, by which the lines CH, DI, EK, and FL shall bear a proper relation to the line A. If the rate be 10% per annum, the line CH must be $\frac{10}{100}$ or $\frac{1}{10}$ of the line A, DI $\frac{2}{10}$, EK $\frac{3}{10}$, and so on. The line BM represents nothing but a limit to the lines representing interest for any time on the line BG.

$A + CH$, $A + DI$, $A + EK$, etc., represent the *amount* due each successive year at simple interest.



536. Annual Interest. Simple and annual interest are the same for the first year. At this time in "annual interest," the accrued simple interest on the principal forms a new principal

to draw simple interest till maturity. The same is true at the end of each following year. This increase of interest will be represented by the lines I_1, K_3, L_6, and M_{10}. The rate being 10%, I_1 must be $\frac{1}{10}$ of CH its principal; $K_3=\frac{2}{10}$ of $CH+\frac{1}{10}$ of $NI=\frac{3}{10}$ of CH; $L_6=\frac{3}{10}$ of $CH+\frac{2}{10}$ of $NI+\frac{1}{10}$ of $OK=\frac{6}{10}$ of CH; and $M_{10}=\frac{4}{10}$ of $CH+\frac{3}{10}$ of $NI+\frac{2}{10}$ of $OK+\frac{1}{10}$ of $PL=\frac{10}{10}$ of $CH=CH$. It will be observed that the line B_{10} is not a straight line, but composed of straight lines, the *degree* of divergence from the line BG being increased at the end of each year; also that the numbers $_1$, $_3$, $_6$, $_{10}$, etc., are the sums of the several series $_1$; $_1+_2$; $_1+_2+_3$; $_1+_2+_3+_4$, etc.; and also that they express the number of years that the simple yearly interest of the principal must draw interest to equal the interest on all the several amounts of annual interest. The line *ab*, though limited by the straight line $_6$, $_{10}$, is still a correct representation of the interest due at the end of $4\frac{1}{2}$ years with annual interest.

537. *Compound Interest.* Annual and compound interest are the same for two years. Then the interest which has accrued on the first annual interest becomes a part of the principal. In like manner all the interest at the end of each year becomes a part of the principal for the next year. The line $_1$, *c*, *d*, $_2$, limits the horizontal lines representing compound interest after the first two years. It should be separated from the line B_{10} at the point $_3$, a distance equal to $\frac{1}{10}$ of I_1; at the point $_6$, a distance equal to $_3c+\frac{1}{10}$ of Kc; at the point $_{10}$, equal to $_6d+\frac{1}{10}$ of Ld.

Or, comparing simple interest with compound, the line B_2 must begin to diverge from the line BM at the point H, and be separated from BM at the point I, a distance equal to $\frac{1}{10}$ of CH; at the point K, equal to the $I_1+\frac{1}{10}$ of D_1; at the point L, equa to $Kc+\frac{1}{10}$ of Ec; and at the point M, equal to $Ld+\frac{1}{10}$ of Fd.

538. 1. Observe that simple interest consists simply of interest on the principal for the given rate and time; as $GM=\$50$.

2. Annual interest consist of the simple interest on the principal and simple interest on the interest due each year; as $GM+M_{10}=\$50+\$10=\$60$.

3. Compound interest consists of the simple interest and interest on all interest accrued at regular intervals; as $GM+M_2=\$50+\$10+\$1.051=\$61.051$.

539. EXAMPLES.

Compute the amount due at simple, annual, and compound interest in each of the following:

1. Principal $500, rate 10% annually, time 4 years.

Solution.—$P \times \overline{1+(r \times t)}$ = $700 = amount at simple interest. $500 + $200 + simple interest on $50 for (3 + 2 + 1) y. = $730 = amount at annual interest. $500 × $(1.10)^4$ = $500 × 1.61051 = $805.255 = compound amount.

2. $1000 from Jan. 1, 1863, to May 7, 1868, at 6%?

Solution.—Principal . . . . . = $1000
Simple interest for 5 y. 4 m. 6 d. = 321
Amount, at simple interest . = $1321
Interest for 1 y. = $60.

Simple interest on $60 for	4 y. 4 m. 6 d.		
" "	3 " 4 " 6 "		
" "	2 " 4 " 6 "		
" "	1 " 4 " 6 "		
" "	4 " 6 "		
" "	11 y. 9 m. 0 d.	=	42.30
Total amount, at annual interest .		=	$1363.30
Compound amount for 5 y. . . .		=	$1338.225
Interest for 4 m. 6 d. . . .		=	28.102
Compound amount for 5 y. 4 m. 6 d. .		=	$1366.327

3. $1200, for 3 y. 3 m. 12 d., at 6%?
4. $900, for 4 y. 7 m. 9 d., at 5%?
5. $2000 at 6% from Mch. 15, 1863, to Jan. 1, 1868? Also what was due if the interest was promptly paid each year?

PARTIAL PAYMENTS.

540. 1. When ***Partial Payments*** are made on account of notes, mortgages or other interest-bearing certificates of indebtedness, allowance is made for the same, in computing the amount due at the time of settlement.

2. The method of computing interest when partial payments have been made is a subject that has given rise to much litigation. In many States the only law relating to it consists of *decisions* in

particular cases, which, from the peculiar circumstances, do not always clearly indicate a principle that may be applied justly to other cases. The aim in legislative enactments appears to have been twofold, to avoid usury and the taking of compound interest. All interest is in effect compounded when it is paid, since it allows the lender to loan again and draw interest on interest, while, if not paid, the debtor has the use of the interest money without paying interest. No court ever objected to a man's paying interest as often as he chose, and the statutes generally allow a collection of legal interest as often as was agreed upon by the parties in the original contract. They also allow a collection of simple interest upon any interest money after it becomes due, if not paid. They also allow compounding at the legal rate as often as the debtor chooses, provided that the old obligation be cancelled and a new one given. Compound interest, then, is not of itself *illegal;* it is only certain forms of it.

3. The difficulty attending partial payments is in deciding whether they shall be applied to the debt of interest or principal. If applied to the debt of principal, there is only simple interest; if applied to the debt of interest, the practical effect is that of compound interest.

541. When the time of the note or obligation is more than one year, the following rule has been adopted by the courts of most of the States, and by the Supreme Court of the United States, and may therefore be called the

UNITED STATES RULE.

1. *Compute the amount of the principal at the earliest time when any payment or the sum of any payments equals or exceeds the interest due.*

2. *From this amount subtract the sum paid, up to the time of computation, and use the remainder as a new principal, with which proceed as before.*

NOTES.—1. This rule is based upon the principle that in all cases the payment should be applied first to interest due, then to the principal, and that the principal remains unchanged until the sum paid exceeds the accrued interest, the principal alone drawing interest.

2. When the computed interest exceeds the payment, such interest may be added to the interest for the next interval of time, or if estimated men-

tally to be less, the amount may be computed at once up to the time when the sum paid is not less than the accrued interest.

3. This rule is generally used when the note is not paid within one year from its date.

EXAMPLES.

1. $1650. CLEVELAND, O., May 20, 1866.

For value received, I promise to pay to the order of H. D. Stratton sixteen hundred and fifty dollars, on demand, with interest at 7%?

J. O. SNYDER.

Sept. 8, 1866, paid $45.
Dec. 14, 1866, " $30.
Feb. 26, 1867, " $50.
July 5, 1867, " $80.
Nov. 14, 1867, " $250.

What amount was due May 2, 1868?

NOTE.—Compute at 30 days for a calendar month.

Solution.

y.	m.	d.			
1866	9	8	$45.	$1650	
1866	5	20		.021	
	3	18	afford .021 of the principal as interest.	1650	
				3300	
				$34.650	Int. Sept. 8.
				1650.	
				$1684.65	Am't Sept. 8.
				45.	Deduct payment.
1866	12	14	$30.	$1639.65	New principal.
1866	9	8		.018⅔	
	3	6	afford .018⅔ of the principal.	109310	
				1311720	
				163965	
				$30.60680	Int. exceeds paym't, Dec. 14.
1867	2	26	$50.	$1639.65	Same princ. as before.
1866	12	14		.014	
	2	12	afford .014 of the principal.	655860	
				163965	
				$22.95510	Int. Feb. 26.
				30.6068	Int. Dec. 14.
				1639.65	Principal.
				$1693.2119	Amount Feb. 26.
			$30 + $50 =	$80.	Deduct sum of payments.
				$1613.211	New principal.
				.050⅙	
				272201	
				8066055	
				$81.932751	Int. Nov. 14.
				$1613.211	
1867	11	14	$30 + $250.	$1694.14	Amount Nov. 14.
1867	2	26		$280.	Deduct sum of payments.
	8	18	afford .050⅙ of the principal.	$1435.143	New principal.
				.032⅔	
				956762	
1868	5	2		2870286	
1867	11	14		4305429	
	5	18	afford .032⅔ of the principal.	$46.881338	Interest May 2, 1868.
				$1435.143	
				$1460.335	*Amount due May 2, 1868.*

NOTE.—Consider mentally that the next interval of about 4½ m. would afford about .02½ of the principal or about $41, which exceeds the payment made July 5, 1867; hence compute the interest from Feb. 26, 1867 to Nov. 14, 1867.

2. A note of $1200 is dated June 10, 1864, on which,

Aug. 16, 1865, there was paid,	. .	$100
Dec. 28, 1865, " "	. .	200
June 2, 1866, " "	. .	25
Dec. 29, 1866, " "	. .	25
June 1, 1867, " "	. .	25
Oct. 28, 1867, " "	. .	500

What is the amount due Dec. 10, 1867, the interest being 6%?

3. $1000. BUFFALO, N. Y., April 10, 1862.

One year after date, I promise to pay to the order of James Johnson one thousand dollars, with interest, value received.

THEODORE LELAND.

On the note were the following indorsements:

Nov. 10, 1363,	received	$80.50.
July 5, 1864,	"	100.
Jan. 10, 1865,	"	450.80.
Oct. 1, 1867,	"	500.

What remained due Jan 1, 1868?

NOTE.—Compute interest at the legal rate.

542. When partial payments are made on mercantile accounts which are past due, and on notes running *only for a year or less*, it is customary to use the

VERMONT RULE.

Compute the interest on the whole debt or obligation from the time it began to draw interest, and on each payment from the time it was made until the time of settlement, and deduct the amount of all the payments, including interest, from the amount of the debt and interest.

NOTE.—When a partial payment is made on a note or obligation *before* it is due, no part is applied to the discharge of the interest, but the whole is used to reduce the principal in accordance with the above rule.

$600. CLEVELAND, Nov 18, 1856.

Ninety days after date, I promise to pay to the order of William Penn six hundred dollars, with interest, value received?

WALTER JOHNSON.

Indorsements.—Nov. 30, $100; Dec. 10, $250; Dec. 20, $100; Jan. 2, $80.

Solution.—What was due at maturity?

$600,	with interest for	93 days,	amounts to			$609.30
$100,	" "	81	"	"	$101.35	
$250,	" "	71	"	"	252.96	
$100,	" "	61	"	"	101.02	
$80,	" "	48	"	"	80.64	
Sum of payments, with their interest,						$535.97
Amount due at maturity, Feb. 19, 1857,						$73.33

543. The same result can be obtained more easily by the use of the following

RULE.

Multiply the amount due at first, and the balance of the principal due after deducting each payment, by the number of days that elapse between the several payments, add all the products, and divide the sum by 6000. The quotient will be the interest at 6 per cent. (Art. 514, 3, Note 2.)

Taking the same example as above.

$600	multiplied by	12	= 7200
$500	" "	10	= 5000
$250	" "	10	= 2500
$150	" "	13	= 1950
$70	" "	48	= 3360
	Sum		= 20010

Which, divided by 6000, gives for the interest due . $3.33

This added to the balance of principal, gives . . $73.33

When the principal does not draw interest, the last rule cannot be used without some modification.

544. Another rule for applying partial payments is in use among many business men, and has received the sanction of several legal decisions. This rule, because it is used by merchants, has been styled

THE MERCANTILE RULE.

Compute the interest on the principal or original debt for one year, and add it to the principal. Find the interest also on the payments made during the year, if any, from the time they were made to the end of the year. Deduct the sum of payments and interest from the amount of principal and interest for a new

principal. Do the same for each succeeding year till the final settlement.

NOTE.—It will be observed that this is applying the Vermont Rule to each separate year, beginning with the date of the note, and making yearly *rests.* Sometimes these rests, or times of making a new principal in mercantile accounts, are made to come at the end of each civil year, sometimes once in six months, depending upon the custom of merchants in balancing their accounts. Bankers for the same reason have been allowed to make quarterly rests, carrying forward a new principal *every quarter*, at the time of balancing the ledger.

EX. A note of $2000 is dated Feb. 1, 1860, on which were the following

Indorsements.—March 1, 1860, $200; July 1, 1860, $300; Oct. 1, 1860, $500; July 1, 1861, $100; Oct. 1, 1862, $200; Jan. 1, 1863, $600.

What was due July 1, 1863, the interest being 6%?

$2000 will amount, Feb. 1, 1861, to . . .		$2120
200 " " " to .	$211	
300 " " " to .	310.50	
500 " " " to .	510	
Sum of payments and interest, ...		1031.50
New principal, ...		1088.50
$1088.50 will amount, Feb. 1, 1862, to . . .		1153 81
100 " " " to . . .		103.50
New principal, ...		1050.31
$1050.31 will amount, Feb. 1, 1863, to . . .		1113.33
200 " " " to .	$204	
600 " " " to .	603 .	807
New principal, ...		306.33
$306.33 will amount, July 1, 1863, to . . .		313.99 *Ans.*

545. When partial payments are made on account of several obligations, they may be applied, at the option of the creditor, to either of them, excepting that those coming due first must be first cancelled.

EX. $650. CHICAGO, July 15, 1854.

Two years from date, for value received, I promise to pay to the order of Peter Finney, six hundred and fifty dollars.

SILAS WARREN.

Mr. Warren paid on the above note, Sept. 15, 1856, $105;

May 9, 1857, $250. What amount was due Sept. 24, 1858, annual interest notes having been given, at 10%? (Art. 521.)

NOTE.—In cases like this, the payments should be applied first to the discharge of the interest *on the annual interest*, then the *annual interest*, and finally the principal. The interest on the principal, which has not yet become *annual interest*, not being due, should not be cancelled by payments except it be at the final settlement of the note.

SOLUTION.

First annual interest, due July 15, 1855, ...		$65.
Interest on same from July 15, 1855, to Sept. 15, 1856, .		7.583
Second annual interest, due July 15, 1856, ...		65.
Interest on same from July 15, 1856, to Sept. 15, 1856, .		1.083
		$138.666
First payment, Sept. 15, 1856, ...		105.
Balance due on % of *interest*, ...		33.666
Interest on $33.666 from Sept. 15, 1856, to May 9, 1857, .		2.188
Original principal, ...		650.
		685.854
Second payment, ...		250.
New principal, ...		$435.854
Interest on $650 from July 15, 1856, to May 9, 1857, not due at time of payment, ...	$53.083	
Interest on $435.854 from May 9 to July 15, . .	7.990	
Third annual interest, ...		61.073
Interest on same from July 15, 1857, to Sept. 24, 1858, .		7.278
Fourth annual interest, to July 15, 1858, ...		43.585
Interest on same from July 15, 1858, to Sept. 24, 1858, .		.835
Fifth annual interest, due at settlement, ...		8.354
Amount due Sept. 24, 1858, ...		$556.979

COMPARISON OF PARTIAL PAYMENT RULES.

546. 1. The Vermont Rule is the only one involving no compound interest. The objection to that rule, when the time is more than one year, may be seen in the fact that the payments may be no greater than the interest due at the time of the payment, and still if the payments are sufficiently frequent, and the note run sufficiently long, the entire debt of principal and interest may be discharged, and the holder of the note become indebted to the debtor. (See Ex. 1, p. 331.) This rule is usually more favorable than the others, for by it there is no compound interest, and all the payments draw interest.

2. Both the Mercantile and the United States Rules involve compound interest, the former compounding it once a year, the latter as often as a payment is made which equals or exceeds the interest then due. When the payments occur at intervals of just one year, commencing with the date of the note, both rules give the same result. When they occur oftener than once a year, the Mercantile Rule is the more favorable to the debtor; when more than a year intervenes, the United States Rule is the more favorable. By the use of the United States Rule an inducement is offered to defer payment as long as possible, and the longer payment be deferred, the greater the inducement to continue it. Strict justice to all parties, in all cases, would be to have the interest on the whole debt, whether of principal or interest, compounded instantaneously. This method, though desirable, cannot at present be made practicable.

NOTE.—Compound interest supposes *all* interest, whether upon principal or interest, to be due at the end of equal successive intervals of time, generally of one year or six months. When the interest is considered due the *instant* it has accrued, and *all interest* is made to draw interest, it is called *instantaneous compound interest*. The actual difference between even *instantaneous* compound interest and simple interest is not so great as at first might be supposed. For 6% simple interest for one year will amount to more than $5\frac{1}{2}\%$ instantaneous compound interest.

EXAMPLES.

1. A holds an obligation against B for $1000, which has run 25 years at 6% interest. At the expiration of each year a payment of $60 was made. What is the amount due, as computed by each of the rules given above?

By the United States Rule, B owes A $1000.
By the Vermont Rule, A owes B $80.
By the Mercantile Rule, B owes A $1000.

2. A note of $10000 runs 4 years at 8% interest, on which were made quarterly payments of $500. What was the amount due at the time of settlement?

By the United States Rule, $4408.21.
By the Vermont Rule, $4000.
By the Mercantile Rule, $4322.30.

NOTE.—It will be observed that generally the result obtained by the Mercantile Rule will be intermediate between those obtained by the other two.

3. A note for $1000 was on interest 4 years at 6%.

A payment of		$50	was made		1 y.		from date.
"	"	$250	"	"	1 y. 6 m.	"	"
"	"	$224	"	"	2 y.	"	"
"	"	$20	"	"	2 y. 8 m.	"	"
"	"	$110	"	"	2 y. 10 m.	"	"

What was due at maturity according to the different rules?

4. What amount would be due April 1, 1871, according to the different rules, on a mortgage for $3580, dated May 10, 1868, and bearing 7% interest from date, the following payments having been made on it: Sept. 1, 1868, $200; Jan. 10, 1869, $850; July 28, 1869, $725; March 5, 1871, $1180?

5. $2500. CHICAGO, ILL., April 20, 1867.

For value received, I promise to pay on demand, to the order of Samuel Brown, twenty-five hundred dollars, with interest at ten per cent. JAMES FERGUSON.

Endorsed $125, Sept. 3, 1867; $200, April 20, 1868; $1200, July 1, 1868; $85, Feb. 10, 1869, $245, June 18, 1869, $300, May 15, 1870.

What was the amount due Oct. 20, 1870, by United States Rule?

NEGOTIABLE PAPER.

547. 1. ***Negotiable Paper*** commonly includes all orders, promises, and certificates for the payment of money, the property interest in which may be negotiated or transferred by indorsement and delivery, or by either of them.

2. The laws concerning negotiable paper are extensive and intricate, and they are not uniform in all respects throughout the United States. It is their design to secure the holder especially against loss, and to allow drafts, commonly known as "bankers' exchange," to perform the functions of money.

548. 1. Promissory notes, bills of exchange, bank notes, checks, and certificates of deposit (see Banking, Art. 563), when drawn payable to "bearer" or to the "order" of the payee are negotiable.

2. If neither the word "bearer" nor "order," but simply the name of the payee appear in the instrument, it is not negotiable, and the payee cannot give full title in it to a third party.

NOTE.—The laws of Illinois, and perhaps of some other States, are an exception to this.

549. 1. In the negotiation of paper payable "to the order of" the payee, or to the payee "or order," the transfer is by indorsement and delivery; if payable "to bearer," or to the payee "or bearer," as are bank notes and most checks, the transfer is by delivery.

NOTE.—For exceptions to this, see laws of Illinois.

2. If the payee simply writes his name across the back of the paper it is an *indorsement in blank*, and is afterward negotiable by delivery. But if above this indorsement it be made payable to the order of another person, called an *indorsee*, it is an *indorsement in full*, and is then negotiable only by the indorsement of the indorsee. By repeating this kind of indorsement there may be several indorsees. When the indorsement is in blank, any legal holder is allowed to write that above it, which will make it an indorsement in full. A *qualified* indorsement is one that affects the liability of the indorser, but not the negotiability of the paper, as when made "*without recourse*."

550. 1. In a Bill of Exchange the acceptor stands in the same relation to the holder as the maker of a promissory note to the holder of it, the drawer in that of the first indorser of a note payable to his order, and the payee in that of first indorsee.

2. Bills of exchange and promissory notes are governed by the law of the State where they are made payable, so far as their terms are subject to general law, as fixing the rate of legal interest and day of maturity, for example. If a note is not paid at maturity, it continues to draw the same interest as before; but if no interest be mentioned, it draws simple interest at the legal rate after maturity till paid.

3. If the interest expressed be illegal, both where the note or bill is made and where payable, the law of the place where the contract is made governs as to the legal consequences of usury.

LIABILITY OF PARTIES CONNECTED WITH NEGOTIABLE PAPER.

551. 1. Bank notes designed to circulate as money, checks, and other paper negotiable by delivery, may be legally retained by an *innocent holder*, who receives them in good faith for a valuable consideration, though the party from whom they were received obtained them fraudulently.

2. Bank notes are a good tender if not objected to when offered, unless it should appear afterward that when offered they were worthless, or of less value than represented, as when counterfeit, altered, spurious, broken, or uncurrent.

3. If a person receives a check on a bank, it is his duty to present it for payment at the bank during *the same or the next day at the furthest;* otherwise he holds it at his own risk, the loss being his if the bank fails meantime, provided that the funds were there to meet the check before the failure. If he lives at a distance from the bank he must send it for collection by mail, or otherwise, during the same or next day. If the check passes through the hands of several persons, each one is allowed one day, and his liability, so far as above described, ceases with the succeeding day. Bank drafts, or "bankers' exchange," from their service in making remittances to distant points, may be used to fulfill that mission, but should not be allowed to lie still or circulate as money beyond the reasonable expectation of the drawer.

552. 1. When the holder of a check gets it certified as good by a bank on which it is drawn, the drawer is released though the bank failed to pay; but if the drawer gets it certified, he is not released if it be presented for payment in due time, the same as if it were not certified.

2. As between the maker and payee of a note, the maker is allowed any defense that would be allowed in any other debt due from him to the payee. But as between the maker and indorser, or other holder, no defense can be set up, except it be shown that at the time of the note's coming into his possession the holder had knowledge of a just ground of defense between the maker and payee. If, however, the note came into the possession of the holder, after it became due, the claim of the holder would be subject to all the equities in favor of the maker that existed at maturity, or that had arisen after maturity.

3. On a promissory note the maker is directly responsible to any *bona fide* holder. The indorsers are responsible in the order of their indorsements, that is, each one to all those who follow, on condition of their being duly notified of non-payment, as explained hereafter. The liability of those who indorse as *guarantors* is *not* discharged by a failure to give prompt notice of non-payment.

553. 1. A bill of exchange involves no direct liability until presented for acceptance. If acceptance of a time draft or payment of a sight draft be refused by the drawee, the drawer immediately becomes principal, and is bound to redeem the draft from the holder without delay, though it be a time draft, and the time not yet expired. If the bill be accepted, the acceptor becomes principal, the same as the maker of a promissory note, in which case the drawer sustains practically the position of first indorser, in case of non-payment on the part of the acceptor. The liability of indorsers on bills is the same as of those on promissory notes. That liability, however, may be avoided in both cases by their writing over their indorsements "without recourse," or other words of equivalent signification, except so far as to warrant that the bill or note is genuine, that is, not forged or fictitious, a liability which attaches not only to all indorsers, but to all who negotiate the paper by delivery, as owners, or even as agents, unless that agency, with the name of the principal, be known at the time of the transfer.

2. Indorsers may be released from liability, if they are not duly notified of non-acceptance or non-payment, the paper having been duly presented; but this rule varies in different States.

554. If a man lends his name and credit by making a note or accepting a bill of exchange for the accommodation of another party, it is called *accommodation paper.* He thereby becomes liable to any *bona fide* holder, to the same extent as if he had received a full consideration, except to the person for whose accommodation the credit was given. But for his indemnity for payment he has a valid claim on the party accommodated.

PRESENTMENT, PROTEST, AND NOTICE.

555. The limits of this work will not allow the detail of all the particulars necessary to be observed by the holder of a bill or note, in making a proper demand for payment, and, in case of non-payment, in properly notifying the indorsers, so that they may not be released from liability. The importance of the subject demands the careful study of those who deal in negotiable paper, or who undertake the collection of it for others. Business men, unless thoroughly posted, had better intrust their collections with some responsible banker. A few brief rules only will be given.

556. 1. There should be no unnecessary delay in presenting for payment any paper payable on presentation, and for acceptance all time drafts (unless drawn "acceptance waived"), especially if the time of maturity is to be determined by the time of sight or presentment.

2. When the time is definitely fixed by the date of the instrument or of the acceptance, it must be presented for payment on the *exact day of maturity*, as regulated by the law of the State where it is made payable, if the indorser is to be held. A protest on any other day would be of no avail.

3. The paper itself must be presented by the holder to the acceptor or maker personally, or to his authorized agent, or at the place where it is made payable, during reasonable business hours. If no such person or agent is found with funds to meet it, the paper may be treated as dishonored. In case of non-acceptance or non-payment the paper should be protested, and the drawer and indorsers notified.

557. 1. "A *protest* is a solemn declaration on behalf of the holder, drawn up by an official person, against any loss to be sustained by the non-acceptance or non-payment of a bill."

2. The protest should be made by a notary public, who should also *personally* make due presentment or demand, and should *on the same day*, or, at furthest, *the next day*, send written notices of protest to the parties to be notified. If the residence of all the indorsers be not known, and all the notices be sent under one cover to the last indorser, he is allowed only one day to forward the notices to antecedent indorsers. So also for each of the others. Sundays and legally recognized holidays are excepted. Notices to parties residing in the same town must be delivered in person or by a messenger. Notices to all others must be sent by mail.

3. If an indorser writes over his name "waiving demand and notice," a protest is not necessary to retain his liability.

DAYS OF GRACE AND TIME OF MATURITY.

558. 1. It may be observed here that each of the United States makes its own laws in regard to negotiable paper, and probably the laws of no two States agree in all respects. The laws of that State are applied in which the paper is made payable, though it be drawn in another. For a valuable compend upon

this whole subject the student is referred to a "Manual for Notaries Public," published by I. Smith Homans, New York.

2. As a general law in the United States, the day of maturity for all negotiable time-paper does not come till three days after the expiration of the time mentioned in the instrument, except when the time is limited by the expression "without grace." These days are called *days of grace;* but they give the maker no special advantage, for interest is allowed on those days the same as others, and no presentment need be made till the last day of grace.

3. If the last day of grace falls on Sunday, or any legally recognized holiday, the paper is payable on the preceding day.

559. 1. Bills drawn at sight are sometimes allowed grace and sometimes not. The statutes of different States, so far as they exist, do not agree, and in the absence of special statutes the custom is not uniform. In New York, commercial bills, drawn at sight, are payable *without grace*, and all paper in which either the maker, drawer, or drawee is a bank or banker, is also payable without grace.

2. If the time be expressed in months, calendar months are generally understood. For example, three months from January 31, without grace, would be April 30; including grace, May 3. This is different in some States.

560. To find the time of maturity if the time be expressed in days.

RULE.

1. *Taking the remaining number of days in the month of the date, and as many days of the following months separately as will equal the given number of days plus three. The number of days in the last month will be the date of the month on which the paper matures.*

For example, a note dated August 20, 1858, payable ninety days from date, would mature November 21, 1858.

Solution.—$11+30+31+21=93$.

2. Or, *to the day of the date add the time of the note plus three days, from which subtract consecutively the number of days of each following month, beginning with the month of the date, until the remainder be smaller than the number of days in the next month. The remainder will be the date of maturity.*

Solution.—$20+93=113$, and $113-31-30-31=21$.

3. Or, if the time be 30, 60, or 90 days, ***call each 30 days a calendar month, and correct by subtracting 1 for each month passed over containing 31 days, and adding 1 or 2, according as it is a leap year or not, if the last day of February be included.***

Thus, 90 days from January 10, 1856, would be, counting three calendar months, April 13, including grace.

Now, from 13 subtract 1 for January and 1 for March, and add 1 for February, and we have April 12, for the result. The last rule is convenient for bank paper, which usually runs 30, 60, or 90 days.

4. It is evident from the above rules that the day of the date should be excluded from the calculation.

5. The following fact may be worth remembering by those who get "accommodations" at bank.

A paper having 60 days to run will mature on the same day of the week as that on which it was made. Having 30 days to run, it will mature 2 days *earlier* in the week, and having 90 days to run will mature 2 days ***later in the week.***

PROOF.

$$33 = 7 \times 5 - 2$$
$$63 = 7 \times 9$$
$$93 = 7 \times 13 + 2$$

PROMISSORY NOTES.

561. 1. A ***Promissory Note*** is a written or printed agreement by one party to pay to another a specified sum at a specified time or on demand. The one signing the note is called the *maker* or *principal.* The person to whom the amount is payable is called the *payee,* and the owner of the note is called the *holder.* The *principal* is directly responsible for the payment of a bill or note at maturity.

NOTE.—For different forms of notes, see examples under the subject of *Interest.*

2. A *joint and several note* is one signed by two or more distinct parties, in which case each one becomes liable as *maker* or *principal,* either jointly with the other or separately as if no others signed with him.

562. Some of the features of a valid promissory note are the following:

1. A full consideration is implied from the nature of the instrument, but a want or failure of consideration would be a valid

defense on the part of the maker as against the payee, but not as against any other holder, into whose possession it may have come before maturity without a knowledge of such want of consideration, in which case he would be called an *innocent holder*.

The words "value received" should properly be inserted.

2. A note is commonly dated when made, but it may be dated at an earlier or later time, the date itself being only *prima facie* evidence of the time when made. If dated in advance and any of the parties die before that date, such death will not affect the rights of a *bona fide* holder.

3. It should be an unqualified promise to pay in money, definite in amount, and independent of all contingencies. The amount should be expressed in the body of the note, in words which should be relied on for accuracy rather than the figures in the margin. If it contain a promise to do any act other than pay money it is not negotiable. In England this rule is so strictly held that a promise to pay a sum "in cash or Bank of England notes" is not negotiable, although Bank of England notes are a legal tender except by the bank itself.

4. The payee must be a specified person, firm, or corporation. If no payee be designated, or if it be made payable to A *or* B, the note is incomplete and invalid.

5. If no time of payment be specified it is payable on demand.

6. If no place of payment be specified it is payable at the place of business or residence of the maker.

7. A note may be either written or printed. The signature must be the usual business mark or name of the maker written or printed by himself or by his authorized agent.

BANKS AND BANKING.

563. 1. A *Bank* is an association of persons authorized by law to *issue bank notes or bills* for circulation, to be used as money (Art. 404), *receive deposits, discount notes, buy and sell exchange* (Art. 598), and *buy and sell gold and silver coin, bullion, and uncurrent money.*

2. Banking properly includes all these functions, but there are many banking associations and especially private bankers that do not issue bank notes.

3. A partial banking business is done by most Savings Banks, Loan and Trust Companies, Real Estate and Building Associations, Transportation and Express Companies, and private bankers. Those who deal exclusively in buying and selling gold, silver and bank notes are called "*brokers*" or "*money brokers*."

564. A regular bank has a corporate name, is managed by a Board of Directors, a President, and a Cashier, can sue and be sued in its corporate capacity, and perform such other acts as may be specified by law.

565. The advantages to a business man in keeping a bank account are the following:

1st. If he has an honest, prudent banker, his surplus funds are ordinarily safer than if kept by himself.

2d. The settlement of bills with checks drawn upon bankers is not only more convenient, but there is less liability of error, and if errors do occur, the vouchers, which should always be preserved, will aid in detecting them.

3d. He will lose less from counterfeit, broken, and uncurrent money, and will be relieved from frequent charges of paying out the same by throwing the responsibility upon his banker.

4th. By depositing his Drafts and *Bills Receivable*, he avoids much trouble and risk attending their collection. If by mistake, oversight, or neglect, drawers and indorsers are released from liability, the banker, by assuming the collection, becomes responsible for the consequences.

5th. It aids him in establishing his own credit, and learning the credit and responsibility of others with whom he wishes to do business.

NATIONAL BANKS.

566. 1. Most of the banks now in the United States are organized and operating under the National Bank Act passed by Congress in February, 1863, entitled "*An Act to provide a National Currency, secured by a pledge of United States Stocks, and to provide for the circulation and redemption thereof.*"

2. By the provisions of this Act, the National Banks operate under the direct limitation and supervision of the general government, thus affording ample security to the bill-holder who is an involuntary creditor.

3. The chief officer of the department of the United State Treasury, having charge of the national currency, is the Comptroller of the Currency. He is directly responsible to the Secretary of the Treasury. He has general supervision over all National Banks, attends to the printing, registering and issuing to them, of their circulating notes, and orders and receives their periodical reports.

4. Quarterly reports are required by the Comptroller, showing the liabilities and assets of the bank, which reports are published in some local newspaper.

567. 1. Any number of persons, not less than five, may form articles of association for Banking, which must be filed with the Comptroller.

2. These banks are empowered to issue circulating notes of the national currency, discount bills, notes and other evidences of debt; receive deposits; buy and sell gold and silver bullion, foreign coins and bills of exchange, and loan money on real and personal security.

3. No association can be formed with a less capital than one hundred thousand dollars—except that, with the consent of the Secretary of the Treasury, a Bank having fifty thousand dollars may be organized in a place whose population does not exceed six thousand.

4. One half the capital shall be paid in, at the time of filing the articles of association, and ten per cent. of the whole capital each month after being authorized to commence the business of Banking.

5. Stockholders are liable, in case of failure of the Bank, to *twice the amount* of capital stock held by them.

568. 1. Registered United States gold-bearing bonds shall be deposited by the Bank with the Treasurer of the United States, as security for its circulating notes.

2. These notes shall be issued to the Bank by the Comptroller at the rate of *ninety per cent.* of the par value of the bonds deposited, if bearing interest at five per cent. or more per annum. If the bonds should depreciate *below par* in the market value, the amount represented by their par value must be kept good by the deposit of additional bonds.

569. All National Bank notes are receivable *at par,* in all

parts of the United States, in payment of *all* debts due to or from the General Government, except duties on imports, and interest on the public debt. By common consent they are also received at par in payment of individual debts and in settling balances between banks, being regarded as a legal tender.

570. 1. No loan may be made to an individual or firm, or the individual members of a firm, to an amount greater than ten per cent. of the bank capital. This limit does not include business paper, or time bills drawn against existing values, as represented by invoices, bills of lading, and merchandise receipts.

2. The rate of interest authorized to be taken, is the same as that allowed by the laws of the State where the bank is located.

571. 1. All banks are required to keep constantly on hand, an amount of lawful money of the United States equal to twenty-five per cent. of *all liabilities*, to insure prompt payment of any obligation.

2. Three-fifths of the amount required, in the case of all banks except those in certain specified cities, may consist of balances due from the banks in those cities, and available to redeem the circulating notes of the banks to which the balances are due.

572. If any bank fails to redeem its notes on presentation, they may be protested, and notice of protest sent to the Comptroller. The bills will then be redeemed at the Treasury of the United States, on presentation after protest, and the Comptroller is authorized to cancel or convert into money a sufficient amount of the bonds deposited by the bank to redeem its bills, and to appoint a receiver to wind up its business.

BANK NOTES.

573. 1. Banks are authorized to issue Bank Notes by depositing with the United States Treasurer Government Bonds for their security. The bills are printed and countersigned by the Department at Washington and forwarded to the bank, where they are signed by the president and cashier. They are now ready for circulation, and it is evident that the bank receives ninety per cent. of the original cost of the bonds, and, as long as it can keep these bills in circulation, it is receiving the interest on so much of the bonds deposited with the Treasurer as clear profit.

While this is profitable to the bank, it is safer to the bill-holder than any other system yet devised in this country; it is based on the *credit* and *faith* of the *Nation.*

2. The paper currency is economical, and specie is thus left to perform its other functions, lie accumulated in the vaults at the great centres of commerce, or to be converted into bullion for use in the arts. The value of the bank note does not depreciate, because it is receivable at par in all parts of the United States, and will be redeemed promptly at the bank, or, on failure of this, at the United States Treasury.

574. Were banks to retain in their vaults sufficient gold or silver to redeem all their circulating notes at once, there would be no profit to them from the circulation except so far as the notes should be lost or destroyed, and never presented for redemption, which has been found to amount, *extraordinary losses* excepted, to about one-tenth of one per cent. per annum. If, on the other hand, they were loaned as money, and no actual capital kept idle to redeem them, the banker would receive the same revenue, until their redemption, as he would from an equivalent amount of capital furnished him in gold and silver. In short, his credit would at all times afford him as much working capital as his notes in circulation amount to.

575. The value of bank notes as currency depends upon the ease and certainty with which they may be converted into gold or silver coin. Hence the importance of rigid restrictions being imposed by government to insure a prompt and certain redemption. Without these the field is open to frauds, limited only by the intelligence and forbearance of the community.

576. 1. The paper currency of our country formerly furnished by the different States, was under somewhat different laws and regulations in each. In general, they might be classified under three different systems, *a specie basis, a safety fund, and "free banking."*

2. The *specie basis* requires a part, or all its capital, to be paid in coin, limits the amount of circulation in proportion to its capital paid in, and makes the assets of the bank, with perhaps the individual liability of the stockholders, furnish the means to redeem the circulating notes.

3. The "*safety fund*" system requires each of several banks to

deposit, with a State officer or Board of Control, a certain percentage of its capital or circulation, which shall be safely invested as a "bank fund" to redeem the notes of any insolvent bank that may have contributed its due proportion for this purpose.

4. In *"free banking"* the circulating notes are secured by State stocks, to at least an equivalent amount at their marketable value. The stocks are deposited with an officer of state, for which he issues registered blank notes. These, when signed, are used as money by the banker, while he receives at the same time, the interest on the stock deposited. If the bank fails to redeem, the stocks are sold, and the proceeds applied to the redemption. The National Banks are organized on the general principle of the free banking system.

BANK DEPOSITS.

577. 1. *Deposits* are received by banks for safe keeping.

2. A *special deposit* is made when the identical money is to be returned to the depositor, the bank being responsible only for the safe keeping; the loss, for instance, attending the failure of the banks whose notes are deposited being sustained by the depositor. In other cases, the *bank* or *banker* becomes indebted to the depositor, the banker being allowed to use the money as he pleases, but obligating himself to pay the depositor the whole or any part of the amount due him whenever it is demanded, if demanded during business hours. The improbability that all the depositors of a bank will call for the entire balance of their account at the same time, renders it safe for the banker to use a portion of the funds thus intrusted to him, in loaning to those who need the money but for a short time, and may therefore be relied upon for prompt payment. The interest money thus received is the banker's compensation for keeping the accounts of his depositors. Sometimes interest is paid by the banker for the deposit; but, as a general rule, that this interest may be refunded, there is a strong temptation to loan too large an amount "on call," or to seek largely paying investments, with doubtful securities, which is against the interest of both banker and depositor.

3. When the depositor usually has a large balance with his banker, there is an implied obligation with the banker to giv

such a customer or dealer the preference in "bank accommodations," if he offers equally good security.

578. The Bank of Hamburg is exclusively a ***bank of deposit,*** the silver in the vault always being equal to the amount of the deposits. This may be withdrawn at pleasure by the depositors, but the business is mostly done by checks, which have the effect merely of transferring the credits from one account to another. The expenses of the bank are met by a small percentage charged the depositors on the amount of business done. The currency of Hamburg being almost exclusively silver, exchanges are greatly facilitated through the means of this institution.

BANK DISCOUNTS.

579. 1. The prime objects in discounting are to aid business men by short loans, and, in seasons of activity, to furnish capital for the shipment of produce and other merchandise to points where they may be in demand.

2. In discounting, banks act as a balance-wheel to trade, aiding and encouraging the honest merchant by timely help, and, in periods of undue excitement and speculation, withholding discounts, and thus tending to save the community from extravagance, loss, and ruin.

580. 1. Paper offered for discount is of two classes. Home or *Domestic* paper is that which is made payable at the bank where it is discounted. Such generally have one or more indorsers. *Foreign paper* is that made payable at some place other than the place where it is discounted. This latter class constitutes the bulk of bank discounts, and consists largely of time drafts on some person or firm, at some commercial centre, to whom grain or other merchandise has been shipped.

2. Such paper is generally accompanied by a "bill of lading" of the property shipped, and is held by the bank or its agent, as security for the payment of the draft discounted. Such discounted bills are the most desirable to the bank, as well on account of their security, as that they furnish the bank with exchange free of cost, by means of their payment at the commercial centre where the bank's funds are most available. The security usually taken at bank is good indorsed names, stocks, bonds, other "business paper," and bills of lading.

581. It may be taken for granted, that although banks, railroad companies, etc., may have been established for "the accommodation of the people," yet so long as they are controlled by *human nature,* and the profits go into the pockets of *individuals,* corporations cannot be expected to furnish "accommodations" without compensation. As a general rule, a business man may expect accommodations from a bank only so far as he makes it for the interest of the bank to grant them.

582. The interest which is charged on notes discounted at a bank is generally paid in advance, and is computed on the amount due on the note at maturity. The difference between the interest and face of the note is the *proceeds,* which is received by the customer.

Thus the proceeds of a note for $2000, having 63 days to run, including grace, would be, at 6% interest, $2000 − $21 = $1979.

583. If "business paper," drawing interest, is discounted, the amount due at maturity, *including interest,* is taken as the face of the note upon which the *bank discount* is computed. It will be observed that *bank discount* exceeds the "*true discount,*" as heretofore explained; for while the latter is the interest on the *present worth,* or principal, the former is the interest on the *amount* of principal and interest, and the excess is equal to the interest on the *true discount* for the given time. The *ratio* of this excess will also increase as the time is lengthened, so that, other considerations remaining the same, the longer the time the more profit to the bank. If the note run 16⅔ years, the bank discount, at 6%, would absorb the whole note, and the proceeds would be nothing. Frequent renewals, so far as the matter of interest is concerned, are unfavorable to the bank.

584. The reason for the custom among banks of discounting only "short paper," as it is called, is threefold;

1st. A large portion of the capital invested in discounts is based upon deposits, which are subject to "call," and their own "circulation," which must be redeemed on presentation. In case of unusual demands for redemption, or withdrawal of deposits, the early maturity of Bills Discounted is their main reliance.

2d. The risk arising from the varying circumstances of the makers and indorsers is lessened by shortening the time.

3d. If, however, bills of exchange are discounted, payable in a

better currency than that used in the discount, or for which a charge is made for collection, the shorter the time the greater the pecuniary profit.

585. 1. In considering the percentage of profit in "bank discount," with frequent renewals, there is a partial offset in favor of the banker by his being able to compound the interest at each renewal. But this advantage is very small if we consider its effect for one year only, at which time *simple* interest, if paid, may also be compounded by re-loaning.

2. Comparing simple interest with "bank discount," including the advantage from compounding the interest, we obtain the following result:

Bank discount at 6% on paper,

Renewed	once in	12	m.,	is equivalent to	6.383%	simple interest.
"	"	6	"	"	6.281%	"
"	"	4	"	"	6.248%	"
"	"	3	"	"	6.232%	"
"	"	2	"	"	6.216%	"
"	"	1	"	"	6.200%	"
"	every instant		"	"	6.182%	"

From the above we see that the excess of bank discount over true discount, as affecting the *rate* of interest received, when the time is less than a year, *can be* but trifling, being for 6% always less than $\frac{4}{10}$%.

586. In negotiating promissory notes and time-bills of exchange, their estimated value depends upon three considerations, viz.:

1st. The responsibility and promptness of the maker.

2d. The relative value of the currency, used in the purchase, compared with that of the payment of the obligation at maturity.

3d. The market rate of interest.

587. The range of the first consideration is from A No. 1 to worthless.

A man may make a bad bargain in buying a note having sixty days to run, if he pay for it but 10 cents on a dollar. The United States may perhaps borrow money at 4% per annum, when individual States would have to pay 5 or 6%, and railroad companies 10 or 15%. A corresponding difference is found in promissory notes made by individuals and business firms.

588. The range of the second is governed by the currency price of gold, and when there is *no* depreciation of paper money this element of variation is removed. The purchase of a gold draft when gold is quoted at 112, and the same purchase when gold is at 100 would illustrate this point.

589. 1. The range of the third may be said to be between 4 and 24% per annum.

2. The *market*, or *ruling rate of interest*, depends mainly upon the *rate of profit* with which capital can otherwise be employed. New countries, rapidly developing, furnish profitable investments, and therefore sustain a high rate of interest. Sudden expansions and contractions of currency *temporarily* affect the rate, causing it to fall with the expansion and rise with the contraction, but a *continued* increase in the supply of money stimulates prices, awakens enterprise, and increases the profits in business and speculation, thereby *raising* the rate of interest proportionably.

590. 1. The *rate of interest* does not express the *value of money*, but only the value of the *use* of it for a limited time, or rather, it expresses the value of the use of the *capital* or *credit* measured by money.

2. *Money*, from its nature, is always *cheap* when *prices* are *dear*, and *vice versa;* for as money measures the value of other commodities, so the comparative price of the standard articles of commerce measures the relative value of money. Generally, when money is cheap, interest is high. For many years money has been cheaper in the United States than in England, but during the whole time the rate of interest has ruled higher. In the early history of California money was exceedingly cheap, but the rate of interest remarkably high.

3. The current rate of interest is also made higher from the effect of unwise usury laws, and laws under which the collection of valid claims can be enforced only after a protracted, uncertain, and expensive prosecution.

4. There are many other causes that occasion remarkable fluctuations in the market rate of interest, as war, commercial revulsions, etc. Unlimited confidence in business encourages a high rate of interest, while excessive caution and distrust cause it to decline.

5. As a general rule, the market rate of interest, like the price of exchange, is not subject to arbitrary control, but is the resultant of sundry contributing causes; and whatever legislation is necessary should be expended on the *cause* rather than on the *effect.*

BANKERS' ACCOUNT CURRENT.

591. Bankers frequently receive and pay interest on the account current with their correspondents and depositors, paying interest on the deposits and receiving interest on the over-drafts. A settlement occurs once in 3, 6, or 12 months, as custom or special agreement may dictate, at which time the balance of interest is entered to the debit or credit of the account, as the case may be, after which *it* draws interest the same as other items in the account. The principle involved in this kind of interest account forms the basis of the "MERCANTILE RULE" in Partial Payments, as given in this work.

592. The process of computing the interest on such account, is made easy by the use of the following

RULE.

Divide the sum of all the daily balances by 6, and the quotient, after pointing three places for decimals, will be the interest required.

NOTES.—1. It is evident that each daily balance draws interest one day. The interest, then, of the sum of daily balances for one day is all that is required.

2. If the daily balance remains the same for several days, instead of setting down the amount as many times as there are days, use the product of the balance into the number of days.

3. If the balances are sometimes debit and sometimes credit, take the difference between their sums before dividing.

4. The above rule gives the interest at 6%. To find the interest at 4% divide by 9 instead of 6. For 3%, divide by 12. In general, the divisor for any rate may be found by dividing 36 by the rate. Or, having found the interest at 6%, the interest for any other rate may be found by aliquot parts.

5. If a different rate of interest is to be charged on the over-drafts or debit entries, the footings of the daily balances should be divided by their appropriate divisors before subtraction.

593. The following abbreviated form will serve to illustrate the foregoing rule:

Account Current.			Daily Balances.		Total Daily Balances.	
1859.	Dr.	Cr.	Dr.	Cr.	Dr.	Cr.
July 1		$500		500		500
2	$200	100		400		400
3	75			325 × 17=		5525
20		500		825 × 10=		8250
30	1000		175	× 5=	875	
Aug. 4		375		200 × 10=		2000
14	125	250		325 × 30=		9750
Sept. 13		125		450 × 10=		4500
23		1000		1450 × 8=		11600
		Int. 4.63				42525
Bal.	1454.63					875
	2854.63	2854.63			9)	41.650
Oct. 1	By Bal.	$1454.63			Int. at 4%	$4.63

BANK CLEARING HOUSE.

594. The "Clearing House" is a voluntary association of such banks as may become members of it, for the convenience of making daily settlements of circulating notes, checks, drafts, and other items due among themselves. This was formerly done, at great expense, loss of time, and much risk, by messengers, who were obliged to go from bank to bank, with the certificates of debt in their hands, often occupying a whole day in large cities, where banks were numerous. The present system was first adopted in New York, and is now deemed indispensable among its seventy banks.

595. 1. The plan of clearing is as follows: A manager is elected, who superintends the clearing. A desk is assigned to each bank, at which, at the hour of clearing, the bank is represented by its clerk and messenger. The messenger has in his possession all evidences of debt against each bank, strapped up and marked with the amount due, ready for delivery. Each bank clerk, on entering the clearing-house, hands to the manager a memorandum of the amount he brings to the clearing-house against all the other banks. This amount is placed to the credit of the account of his bank with the clearing-house.

2. At a given signal from the manager, the messenger from each bank delivers to the clerk of every other bank in due order, the package of checks and other certificates of indebtedness which

he may have against that bank, with a memorandum of the total amount.

3. When this delivery is completed by all the banks, each clerk foots up the amount so delivered to him by the messengers of all the other banks. This amount is to the debit of his bank, and if it is greater than the amount brought by him to the clearing-house, his bank is a debtor bank to the clearing-house, and if less, then it is a credit bank.

4. To avoid any errors in the footings, each clerk now hands the manager a memorandum of the amount which he has received from the other banks, which amount is placed by the manager to his debit. It is evident that the total of these debit balances must just equal the amount of credit balances handed in at the commencement. If they do not balance, there is an error, which must be corrected.

5. The amount due from the debtor banks will now just satisfy the claims of the creditor banks. At a stipulated hour these balances must be paid into the clearing-house, and the creditor banks must be there to receive their balances.

6. Thus, in ten minutes time, can be cleared many millions in balances, by means of a resulting difference of but a few thousand, and without any risk, loss of time or trouble. The daily clearances at the New York Association range from seventy to one hundred millions, and are often settled by less than two millions.

7. The Clearing House Association also serves the purpose of establishing rules for the general guidance of the banks, and furnishes a common meeting ground for discussing the business and financial questions of the day, that so intimately concern their prosperity, and the welfare of the whole business community.

596. PROBLEMS IN BANKING.

1. A national bank organized with a capital of \$150000, and, within four months after commencing business, deposited with the Comptroller of the Currency United States bonds required by law of the following amounts and current market value, viz.: \$15000 of 1862 quoted @ 108, \$20000 of 1864 @ $107\frac{3}{4}$, \$15000 of 1867 @ $110\frac{1}{8}$, \$30000 of 1868 @ 111. What per cent. of the whole capital was deposited? What amount of notes should the bank

receive to issue for circulation, on the basis of 90% of the current value of the bonds?

2. What must be the capital of a bank to entitle it to issue circulating notes to the amount of \$69984 if 60% of its capital be paid in and the current value of its securities deposited be at 8% premium?

3. The number of national banks in operation in the United States at the close of 1870 was 1627, with an aggregate capital of \$436,478,000, and a circulation of \$299,729,000. In 16 specified cities there were 215 of these banks with a capital of \$185,641,000. What was the average capital of each of the others? What was the entire circulation of the 215 banks, and the entire circulation of the others, apportioning the circulation in the same ratio as the capital?

4. In January, 1869, there were 110 savings banks in the State of New York; these had received during the previous year \$110,148,050 from 588,566 depositors, and had allowed \$8,666,374 interest. In January, 1870, there were 133 savings banks in the same State that had received \$133,389,700 from 651,474 depositors, and allowed \$10,320,207 interest in the previous year. What was the average credit of each depositor in each year? What was the per cent. of increase of the aggregate deposits in the second year?

5. The aggregate amount *due* the 651,474 depositors in the New York State savings banks in 1870 was \$194,360,000, and the savings banks of the New England States owed \$218,378,685 to their 790,058 depositors. What was the average credit of each depositor in New York? In New England? What per cent. greater was the credit of each depositor in New York than in the New England States?

6. The savings deposits in the State of New York were, in 1859, \$48,194,847; 1860, \$58,178,160; 1864, \$93,786,384; 1865, \$111,793,424; 1869, \$169,808,678; 1870, \$194,360,217. Allowing $61\frac{67}{100}$ per cent. of the aggregate deposits for 1870 to have been in New York city, what was the amount of the same? What was the per cent. of increase from 1859 to 1860? 1864 to 1865? 1869 to 1870?

NOTE.—Find the time of *maturity*, the *discount* and *proceeds* of the following notes:

7. \$750. BOSTON, Aug. 10, 1869.

Three months after date I promise to pay to Samuel Brown, or order, at the Second National Bank, seven hundred and fifty dollars, value received.

Discounted Sept. 1. GEO. P. RICE.

8. \$1200. LANCASTER, PENN., May 11, 1868.

Sixty days after date we jointly and severally promise to pay to Myron H. Strong, or order, twelve hundred dollars, without defalcation, with interest after one month; value received.

WM. T. GRAY.

Discounted May 24. CHAS. R. FOX.

NOTE.—The *amount* of the note at maturity should be taken as the basis for discounting.

9. \$3000. LA FAYETTE, IND., July 18, 1871.

Ninety days after date, for value received, I promise to pay Benjamin Burling three thousand dollars at the First National Bank of Indianapolis, without relief from valuation or appraisement laws. JOSIAH BIDDLE.

Discounted July 27.

10. \$2765$\frac{48}{100}$. TRENTON, N. J., Aug 23, 1870.

Four months after date, for value received, I promise to pay Buell Anderson, or bearer, two thousand seven hundred sixty-five and $\frac{48}{100}$ dollars, with interest after two months, without defalcation or discount. LEMUEL TRUMAN.

Discounted Sept. 5.

11. What would be the proceeds of a note for \$3250 due in 120 days at 6% interest, dated Nov. 10, 1870, and discounted Jan. 12, 1871, at 10%?

NOTE. The amount at maturity (123 days at 6%)=\$3316.62, which discounted at 10% for 60 days (Jan. 12 to March 13), affords \$3261.35 proceeds.

12. A note of \$4800, dated May 16, 1871, and payable in 6 months, with interest at 7%, was discounted July 10, 1871, at 12%. What was the discount?

NOTE.—Compute the bank discount for 132 days on the amount of the note for 6 m. 3 d.

13. A note of \$1650, dated Feb. 7, 1871, due in 4 months, with interest at 6%, was discounted April 14 at 10%. What were the proceeds?

14. For what amount must a note be made for ninety days to afford \$7520, if discounted at 8%? What would be the present worth of the note without grace? (Art. 512, 3.)

15. What rate of *interest* is paid in getting discount for 117 days at 6%?

NOTE.—The per cent. of discount for the time is to be regarded as the interest on the per cent. of proceeds as a principal, and as $I \div P = r \times T$ (Art. 506), hence (*rate of discount* × T)÷*Proceeds* % = rate of interest × T, and dividing the dividend by T (Art. 146), the equation becomes *rate of discount* ÷ *Proceeds* % = rate of interest. Thus, discount at 6% for (117 + 3) 120 days would afford 98% proceeds, and .06 ÷ .98 = .06$\frac{6}{49}$, *rate of interest.*

Proof.—The interest on 98% of any principal for 120 days at 6$\frac{6}{49}$% equals the interest on the whole principal for the same time at 6%.

RULE.

Divide the rate of discount by the per cent. of proceeds for the time, to find an equivalent rate of interest.

16. What rates of interest are equivalent to 4, 6, 7 and 10 per per cent. discount for 60 days?

17. What rate of bank discount for 117 days is equivalent to 6% *interest?*

NOTE.—Discount being included in the amount on which it is computed, the amount per cent. of any principal at *interest* for any given time may be taken as the base of discount, and (*rate of interest* × T) ÷ *Amount* % = *rate of discount* × T, and (Art. 146) *rate of interest* ÷ *Amount* % = *rate of discount.* Thus, interest at 6% for 120 days would afford as the amount of the principal 102% of it, and .06 ÷ 1.02 = .05$\frac{15}{17}$, *rate of discount.*

Proof.—The interest on 102% of any proceeds for 120 days at 5$\frac{15}{17}$% equals the interest on 100% of the same proceeds for the same time at 6%.

RULE.

Divide the rate of interest by the amount per cent. of the principal for the given time, to find an equivalent rate of discount.

18. What rates of bank discount for 87 days are equivalent to 5, 6, 7 and 8 per cent. interest?

19. How much would be gained or lost by paying 2% interest per month for 6 months on \$2000, rather than getting the same discounted for 5 m. 27 d. at the rate of 24% per annum?

20. What does a bank make on \$250000, if it pays 6% interest on it for one year, and discounts it all at 10%, on an average, every 60 days? How many days in the last term of discount in the year, if it be the year 1870?

EXCHANGE.

597. When a purchase is made, a satisfactory equivalent is rendered by the purchaser in various ways. It may be by labor or services, or he may give other commodities in exchange, which last transaction is called *barter*. He may give gold and silver, which are also commodities of an equivalent value, but called money, because they are serviceable mainly in making other purchases, thereby facilitating several transactions in barter. Frequently, however, no equivalent is rendered; but an obligation merely on the part of the purchaser for a fixed amount is recognized by both purchaser and seller. This constitutes *debt* on the part of the purchaser, and *credit* on the part of the seller, and is expressed in the denominations of the "money of account." If now the debtor gives *a written obligation to pay*, in the form of a *due bill* or *promissory note*, this evidence of credit with the holder may be transferred as other property, and another become the creditor. In bookkeeping, the account with the seller is closed, and "Bills Payable" receives the credit. Instead of giving his own promissory note, he may use those which he himself has received in the same way; as for example, bank-notes which were issued expressly for this kind of circulation. When bank-notes, or certificates of deposit, are held as evidence of debt against a bank, the debt is collected by the return of these to the bank. If it be an account current, and kept by a *pass-book*, it is subject to *drafts* or *checks*.

598. The facility with which business is transacted by means of *drafts* or other paper substitutes for money, has given to the term *Exchange* a technical use, and now signifies *the method of making payments in distant places by the use of Drafts or Bills of Exchange, without the transmission of money.* The business is usually transacted through bankers, who buy the credits payable in distant places, and sell to those having payments to make in those places.

599. To illustrate, suppose the pork dealers of Cincinnati to

send their pork to New York for sale, and receive for it gold, which is returned to them by express. Suppose also the dry-goods merchants of New York to send their goods to Cincinnati for sale, and receive for them gold, which is returned to them by express. If the pork purchasers in New York had paid the dry-goods merchants there, and the dry-goods purchasers in Cincinnati had paid the pork dealers there, the whole business might have been closed without the risk and expense of transmitting gold either way. This would be done by the pork sellers drawing drafts or orders on the pork buyers, in favor of the dry-goods buyers, who, having paid for these drafts, would forward them to the dry-goods sellers in payment of their purchase. These drafts being presented to the pork buyers would be cashed, and thereby the debts arising in both cities liquidated without the transmission of any money. In making this system general, to include all kinds of trade in many different places, it would frequently be very difficult for those having bills of exchange to sell to find buyers, and *vice versa.*

An exchange broker, or bank of exchange, will obviate this difficulty. They bring the buyers and sellers together, by buying bills with their own capital, and sending them forward for credit, then selling their own drafts drawn against this credit, in amounts to suit purchasers. If between any two places the amount of bills bought equals those sold, then no gold need be transmitted, and the difference between the buying and selling rate would be the commission charged by the broker for his services, use of his capital, and risk in buying such drafts as would not be honored.

600. A ***Bill of Exchange*** or a ***Draft*** is a written or printed order of one person upon a second to pay a certain sum of money to a third person, to his order, or to the bearer. Bills of Exchange afford a convenient method of paying amounts due in distant places without the transmission of money, by setting off a debt due in one place against a debt due in another place. (See Art. 599.)

601. 1. [*Common form of a draft.*]

$1000. CLEVELAND, O., Nov. 6, 1871.

Sixty days after date, pay to the order of J. F. Whitelaw one thousand dollars, and place to the account of

To MESSRS. SMITH & BROWN, New York. ALBERT CLARK.

2. The person making a draft is called the *drawer*; the person to whom the order is addressed is called the *drawee*; and the one to whom the amount is payable is called the *payee*.

3. If the drawee accepts, by writing his name across the face of the bill, under the word "accepted," he then becomes an *acceptor*, and the instrument is then called an *acceptance*.

4. If the payee writes his name upon the back of a note or bill of exchange he becomes an *indorser*. The person to whom it is transferred by indorsement is called an *indorsee*.

The words "or order" need not be repeated in an indorsement.

602. 1. A *Foreign Bill* is a draft made in one country but payable in another.

In Commercial Law each of the United States is regarded as a separate country, so that bills drawn in one State and payable in another are *foreign bills*.

2. A *Domestic or Inland Bill* is a draft payable in the country where drawn.

3. *Time Bills* are those requiring payment at a certain specified time after sight or after date. All others are payable on demand or at sight. When time bills are drawn "acceptance waived," they may be held till maturity before being presented to the drawee; otherwise, they should be presented immediately for acceptance.

4. The essentials of a promissory note belong also to a bill of exchange, and the *drawee* must be specified in the latter.

PAR OF EXCHANGE.

603. To understand the quotations of premium or discount in exchange, it is necessary to consider the currencies of the different places. Supposing gold, as a metal, to be so distributed as to have in all places a *uniform intrinsic value*, and gold coin to be the only currency, the *true par of exchange* between two countries *is the exact equivalent of gold in the standard coin of one country compared with the gold in the coin of the other*. If, however, gold is the standard of currency in one country, and silver in the other, the relative intrinsic values of the two must be compared. This need be computed only when the coins and money of account in the two countries are different.

604. By comparing French coin with that of the United States, we find 20 francs Louis Napoleon equal to \$3.84, or one dollar in gold equal to 5 francs and 21 centimes nearly, or $5\frac{21}{100}$ francs. The quotations of Paris exchange are usually made in this way, without involving percentage. If a bill of exchange for more than 521 francs can be bought for \$100, Paris exchange is at a discount; if less, it is at a premium, and the quotations express the number of francs that can be thus bought.

605. The sum mentioned in a bill of exchange on a foreign country is usually expressed in the denominations of the money of account in the place where it is made payable. Computations in foreign exchange, therefore, require the use of tables of foreign money, including the comparative values of the coins or currencies in those countries. (See Part Third for these tables.)

STERLING EXCHANGE.

606. 1. Comparing the sovereign of England with the half eagle of America, for instance, we find the sovereign to weigh 123.3 grains, of which $916\frac{2}{3}$ thousandths of it is pure gold. The half eagle weighs 129 grains and 900 thousandths pure gold. If we reduce the fineness of the sovereign to that of the half eagle, without changing its value, it must weigh $125\frac{583}{1000}$ grains. In this estimate the alloy is reckoned of no value. To ascertain the true equivalent we have this simple proportion, 129 grains : 125.583 grains : : \$5 : \$4.8675.

2. As the weight and fineness of the sovereigns coined previously to the present reign were somewhat less than the value, as derived above, the average value, as fixed by our mint, is \$4.84. A new Victoria sovereign, however, is worth $\$4.86\frac{3}{4}$. A pound sterling (£) is a denomination in the *money of account* only; the sovereign is a *coin* of an equivalent value. It follows from the above that exchange on London is *par* when a bill for £100 can be bought for \$486.75 in American gold.

3. The *common quotations* are based upon a *purely nominal value* of the pound sterling, viz.: $\$4.44\frac{4}{9}$, for that is not now its value in any other sense.

True value of the pound sterling, . . .	\$4.8675
Nominal " " . . .	4.4444 +
Difference $= 9\frac{1}{2}\%$ (nearly) of the nominal par, .	.4230

4. When the *quotations* are $109\frac{1}{2}\%$, sterling exchange is really at par; when 110%, it is at a premium; when 109%, it is at a discount. Quotations are generally made on sterling bills drawn at 60 days' sight. As the cost of transmitting gold, including insurance, is about equal to the interest on the bill for 60 days, the time for the passage of both being the same, remittances often are made in these time drafts, for which the same is paid as for sovereigns of equivalent amount.

607. RULE FOR STERLING EXCHANGE.

To \$40 add the premium on \$40, at the quoted rate. By this sum multiply the amount of sterling exchange expressed in pounds, and divide the product by 9. The quotient will be the value in dollars. (For another Rule, see Art. 428.)

EXAMPLES.

1. What will a bill for £224 5s. 6d. cost in New York when sterling exchange is par, quoted at $109\frac{1}{2}\%$ or $9\frac{1}{2}\%$ premium?

£224 5s. 6d.	40.
43.8	$9\frac{1}{2}\% =$ 3.80
1792	43.80
672	
896	
10.95	
1.095	
9)9823.245	
\$1091.47 *Ans.*	

2. What will a bill for £175 12s. 8d. cost in New York when sterling exchange is quoted at $110\frac{1}{4}$?

3. What amount of sterling exchange @ 111 can be purchased for \$2500?

4. What would be the cost of the following bill @ $9\frac{3}{4}$ premium?

£125. NEW YORK, Aug. 22, 1871.

Sixty days after sight of this *Second of Exchange* (First and Third of same tenor and date unpaid) pay to the order of Thomas Waite one hundred twenty-five pounds, value received, and charge to account of SAMUEL HATCH & Co.

To BROWN, SHIPLEY & Co.,
London.

NOMINAL EXCHANGE.

608. Inasmuch as *increasing the supply* of even a metallic currency depreciates its relative value, the *nominal* exchange between two places, using the same kind of currency, with the same mint standard, will be in favor of the place having the smallest amount of currency in proportion to its business wants, and therefore having the least depreciation. *The nominal exchange is then measured by the excess of the market price of bullion above the mint price, and is, so far, unfavorable.* A depreciation of metallic currency which affects the nominal exchange may also be occasioned by abrasion or wear of circulation, or by making only one of two metals legal tender where the other is in general circulation. It will be observed that, although this "exchange," which is merely nominal, almost universally enters into the quotations of exchange between different countries, it belongs rather to the exchange of currencies in the same country, and expresses the difference between the current and the standard moneys of that place.

609. *Agio,* meaning "difference," is the proper term to express this nominal exchange when considered alone. In the United States the expense of sending coin to and from New York, by the modern express companies, being so trifling, the premium on New York exchange must always be very nearly the same as on coin. The fluctuations in the *nominal rate* of exchange, or agio, where a depreciated paper currency is used, will be much greater than if the currency were coin or its equivalent, for the reason that the depreciation will be more variable. Sometimes the *scarcity* of such currency, compared with business wants, raises its current value temporarily to nearly par with coin. Just so far the nominal exchange disappears. Bank-notes that can be converted into coin at less expense than the usual local currency are, for that place, at a premium. Those convertible at a greater expense are at a discount, and are called *uncurrent.* Indeed, notes of local currency, when removed from their native habitation, resemble bills of exchange on the places where they are redeemed, and are bought and sold at nearly the same rates as exchange. The "National Currency" tends to bring nominal exchange to par.

COURSE OF EXCHANGE.

610. Having ascertained the *par of exchange*, we have a basis for computation. The *nominal exchange* modifies that computation, by showing the relative value of the metallic currency affected by scarcity and abundance or abrasion, and also the depreciation arising from the use of a paper currency not equivalent to coin, though bearing the same denomination in the money of account.

611. *The course of exchange* relates to the relative supply and demand for bills, or the relative amount of indebtedness between different countries or cities. If the debts and credits between two countries are equal, the *real exchange* is at par; if unequal, it will fluctuate with the inequality. If New York owes London more than London owes New York, bills on London will be at a premium. The range of this course of exchange will be limited by the expense of transmitting coin or bullion, and the premium cannot for a long time exceed that expense.

612. Although the direct commerce between two separate nations may be very unequal, yet the total amount of importations to any country are for the most part paid for by its exportations, through the agency of bills of exchange, drawn against the latter, and transmitted to other countries in payment of the former. Sometimes it is effected by a succession of bills drawn by bankers through intermediate points, or a more circuitous route, which gives rise to Circular Exchange and an Arbitration of Exchange. For example, a merchant in New York may remit to Hamburg by buying first a bill on Paris, and then by his agent another on London, and there a bill on Hamburg. Remittances to remote points are more frequently made by bankers' bills drawn on some commercial center, where other bankers are accustomed to keep an account, so that they may be easily negotiated, making the place thereby a kind of clearing-house. Thus, London has been styled "the clearing-house of the world." Nearly all our foreign trade is *settled* through England and France. In like manner, remittances between inland towns in the United States are made in drafts on New York. The course of exchange between London and New York does not arise alone from the commerce between the two cities, but from all that commerce that is settled

for through those places. Thus, if we pay for our importations of tea with bills on London, our balance of payments with London is affected the same as if the tea came directly from London. (See Art. 617, etc.)

613. The *current*, or computed rate of exchange, includes both the *real* and *nominal* exchange, taking the *true par* for a basis. Within the United States it is reckoned by percentage. Between the United States and England it is reckoned also by percentage, but the *true par* is at a premium above an *assumed fictitious par*, so that an advance in quotation from 109 to 110 is not really 1%, that is, one on a hundred, but less, it being only 1 on 109½.

614. With other countries the current exchange is generally expressed by equivalents, thus \$1=5 francs 15 centimes, 1 marc banco=35½ cts. If the depreciation of currency in any place were 1%, and the real exchange on New York ¼% premium, the *current* rate would be the *sum* of the nominal and real, viz., 1¼% premium. If, however, the real exchange be ¼% in favor of the other place, the current rate would be equal to the difference, or ¾% premium.

615. Equilibrium in the course of exchange is restored only to a small extent by a shipment of coin or bullion, for the reason that almost always other articles of merchandise can be shipped with more profit, gold and silver bearing a nearly uniform value among all civilized nations. When, however, new productive mines are opened and worked, the metals depreciate in value in the mining country, in which case they become profitable articles of export to non-producing countries, until the depreciation becomes general. The unequal depreciation occasions a variation in the *nominal exchange* before the coin or bullion is shipped. The *transfer* of the metal would affect the *real exchange*, because it either pays or creates a debt.

616. 1. The *fluctuations* in the rate of exchange depend upon a variety of conditions, only a few of which can be noticed in this work. They cannot, to any great extent, be controlled by an arbitrary decree of bankers or merchants. Excepting when disturbed by a panic, or an unusual distrust in the credit of those who draw or accept bills of exchange, which give it a fictitious value, the current rate represents the actual resultant of all the

movements in trade and currency, whether traceable or not, and is, therefore, if properly analyzed, a better test of the condition of accounts between different countries and cities than any estimate that can be made, independent of it, based upon exports and imports and other Custom House data.

2. To understand the *current rate*, however, requires a thorough knowledge both of the *par of exchange* and the *nominal rate*, for frequently the fluctuations in the current rate are wholly due to the fluctuations in the nominal rate, which latter depends entirely upon the relative condition of the currency.

"BALANCE OF TRADE."

617. If a country, in her trade with other nations, buys more than she sells, so as to incur a debt, the payments of which, in bullion or coin, would reduce the amount of metallic currency below her proper proportion, as compared with the supply in other nations, she is said to "over-trade," and the "balance of trade" is against her. If the reverse be true, the balance of trade is in her favor. Some restrict the term "balance of trade" to the exchange of commodities other than gold or silver. But why should not gold be considered a staple article of export from California and Australia, as iron is from Sweden or lumber from Maine? Indeed, it is now conceded by the best authorities that the precious metals are only commodities subject to the same general laws of commercial demand that govern other imports and exports. It is not proposed here to discuss this subject in its bearing upon the *prosperity* of a country, but merely to offer a few suggestions to the student, in its relation to the subject of exchange. It is rather the *balance of payments* between separate countries, and the mode of estimating the amount, the direction, and means of liquidating it, that are to be considered here.

618. 1. So far as the commerce of any country is carried on by its own capital and labor, a large share of the excess of imports over the exports arises from the *profit* of the trade, which does not increase the *balance* of payments. If, for example, an American vessel leaves New York for Liverpool, with a cargo of wheat, valued at $10,000, which is sold there for $12,000, and that amount invested in manufactured goods, and taken to China and sold for

$15,000, and that amount, with $5,000 cash invested in tea, which is brought home to New York, it is evident that, from that transaction, the importations exceed the exportations $10,000, one-half of which represents the gross profit for the round trip, not including the enhanced value of the tea arising from being transported from China to New York.

2. So far as *foreign* vessels, sustained by foreign capital, and labor, transport our exports and imports, the difference between the two, as valued at our own ports, will show the balance of payments.

3. Goods lost at sea have been entered at the Custom House whence they cleared as *exports*. But if the loss is sustained by the exporting country, they pay for nothing abroad, and foreign exchange is affected no more than if destroyed before shipment. If the loss be sustained by the country whither they were bound, exchange is affected the same as if they had reached their destination.

4. When capitalists emigrate from one country to another, so far as they carry their capital, either in coin or goods, *with them*, the real exchange is not materially affected; but if they remove their capital through the agency of certificates of deposit, letters of credit, or their own bills of exchange, it becomes a debt of one country to the other, which, in the end, is generally paid in merchandise rather than money. This fact often affects sensibly the course of exchange between the east and west of the United States.

5. The negotiation of bonds, stocks, and other loans in a foreign country creates a debt against that country, which, though nominally for money, is generally paid in merchandise. After this debt is paid, though the bonds are truly the evidence of debt against the country that issued them, yet, with the exception of the payment of the interest, the balance of payments and course of exchange are not affected till the maturity of the bonds.

6. An excess of imports over exports, as shown by the Custom House returns, by no means prove that a country is in debt. Indeed, it is clear from what has been stated, that with every nation engaged in the carrying trade the imports will generally exceed the exports, and, so far as the latter pay for the former, the greater the excess the more profitable the commerce.

619. The following statistics will illustrate the truth of the foregoing principles;

Total imports to the United States, including bullion and specie, from 1790 to 1857, inclusive, ...	$7,658,722,496
Total exports for the same time, . . .	6,860,004,549
Excess of imports for 68 years, ending 1857, .	798,717,947
" " " 7 " " " .	36,363,971
" " " 30 " " 1850, .	250,438,055
" " " 31 " " 1820, .	511,915,921

The valuation of imports, as obtained from Custom House returns, owing to the *ad valorem* system of tariff, is generally estimated to average even 10% below their cost. It will be observed that allowing an undervaluation of 1% will increase the *excess* of imports mentioned about 10%.

Excess of *imports* of bullion and specie for 30 years ending 1850, before the supply of gold from California, ...	$69,995,789
Excess of *exports* of bullion and specie for 7 years ending 1857, ...	269,797,168

From 1790 to 1820 the imports, including bullion and specie, exceeded the exports each year, except in 1811 and 1813. From 1821 to 1857 the imports exceeded the exports each year, except in 1821–5–7, 1830, 1840–2–3–4–7, 1851–5–6–7.

Total amount of public and corporation debt in 1857, held in foreign countries against the United States in the form of bonds, stocks, etc., is generally estimated at . .	$300,000,000
On which there was probably paid an annual dividend of about	20,000,000

% of total exports and imports in American vessels in 1850, . .	75
" " " " " " 1855, . .	74
" " " " " " 1860, . .	66
" " " " " " 1865, . .	40
" " " " " " 1869, . .	34

620. The average current rate of exchange on England at

New York, for first-class bankers' bills, as quoted on the first of each month was, for

1822 . . 12	1831 . . 8¾	1840 . 8	1849 . . 9
1823 . . 7½	1832 . . 9	1841 . . 8½	1850 . . 9¾
1824 . . 9	1833 . . 8	1842 . . 7⅛	1851 . . 10⅓
1825 . . 8¼	1834 . . 3½	1843 . . 7⅛	1852 . . 10⅛
1826 . . 10	1835 . . 9¾	1844 . . 9	1853 . . 9⅔
1827 . . 10¾	1836 . . 8	1845 . . 9½	1854 . . 9¼
1828 . . 10½	1837 . . 13⅛	1846 . . 8½	1855 . . 9½
1829 . . 9	1838 . . 8¼	1847 . . 7	1856 . . 9½
1830 . . 7¼	1839 . . 9¼	1848 . . 9¾	1857 . . 9

Average for the 9 years ending			1830, . .	9⅓%
"	" 10	"	1840, . .	8½%
"	" 10	"	1850, . .	8⅛%
"	" 7	"	1867, . .	9⅔%

It will be perceived that the average rate of sterling exchange at New York, for the twenty years ending 1850, was 1% below par, or 1% in favor of New York; while for the seven years following, it was above par, or in favor of England.

Of the $300,000,000 of gold deposited at the Mint and branches, and Assay Office at New York, for the six years ending 1855, about 94% was produced by California.

In San Francisco sight exchange on New York averaged about 3% premium, the currencies of both places having a metallic basis.

621. If we put 900 new sovereigns and 900 new shillings into average ordinary circulation, in 12 months time the former will be worth about 899 and the latter about 894.

In London, previous to the re-coinage in 1774, exchange was uniformly about 2% in favor of Paris, owing to the fact that the old coinage, by wear, had sunk below its standard weight about 2%, while the coinage of France was not thus degraded. As soon as the new coinage took the place of the old, exchange became par. Before the re-coinage, in the reign of William III, owing to the wear and clipping of the silver coins, the nominal exchange between England and Holland was 25% against England, while at the same time the *real* exchange was in her favor, as was shown upon the issue of the new coins.

622. PROBLEMS RELATING TO EXCHANGE.

1. What is the cost of a draft on New York for \$1250, the rate of exchange being $1\frac{1}{4}\%$ premium?

2. What must be the face of a draft, to cost \$1000, at $\frac{5}{8}$ per cent. premium?

NOTE.—For a strictly accurate solution assume, say \$1, for the face, and find *its* cost, then by it divide the given cost. Custom, however, allows, for small sums, the percentage to be computed on the cost instead of the face. By that rule the answer to the last question would be \$993.75. The approximation may be brought nearer by adding the premium on the premium, which, in this case, is $\frac{5}{8}\%$ of \$6.25 = \$0.04 nearly.

3. What would be the proceeds of \$4000 invested in exchange on New Orleans, at a premium of $\frac{1}{2}\%$?

$$\tfrac{1}{2}\% \text{ of } \$4000 = \$20, \text{ and } \tfrac{1}{2}\% \text{ of } \$20 = \$0.10.$$
$$\$4000 - \$20 + \$0.10 = \textit{Ans.}$$

NOTE.—If the rate had been $\frac{1}{2}\%$ discount we should have had \$4000 + \$20 + \$0.10 = \$4020.10.

4. What must be paid in New York for a draft on London for £1374 5s. 9d., at 10% premium?

5. What amount of sterling exchange can be bought for \$3122.25, the premium being $9\frac{3}{4}\%$?

NOTE.—Find by the rule the cost of £1, by which divide the given cost.

6. What will a draft on Paris for 12144.5 fr. cost if \$1 = 5.35 fr.?

7. What was formerly the cost, at Milwaukee, of a bill on London for £1500, when the quotation at New York was 110, the agio of Milwaukee currency being 2% discount compared with that of New York, and the real exchange, or course of exchange, being $\frac{1}{4}\%$ in favor of New York?

8. New York quotations of Paris exchange being 5.18 fr., and the agio of Cincinnati current funds being $\frac{3}{4}\%$ discount compared with United States coin, what would a bill of 1000 fr. have cost at Cincinnati, if the purchaser bought coin and sent by express, at a charge of \$1.50 per thousand dollars, and bought the exchange in New York through a broker whose charges were $\frac{1}{2}\%$ for commission?

9. The money of account in Hamburg is of two kinds, each reckoned in marcs or marks, viz.: *marks banco* and *marks current*. The former is the account kept at the bank where specie

or bullion is deposited, and is generally the standard of reference in quotations of exchange. The latter is current in business, and is much depreciated, the agio of the two accounts being subject to slight variations. If the par of exchange between Hamburg and London is 1 mark banco=1s. 5½d., assuming £1=$4.86⅔, what is the par of exchange between Hamburg and New York?

10. Assuming the quotations of 109½ on London and 35½ on Hamburg to represent the par of exchange, how much per cent higher is Hamburg exchange than sterling exchange, when the quotations are 110 and 36?

11. Assuming the mark current at 1s. 2d. sterling, what is the agio between the two moneys of account at Hamburg?

NOTE.—It usually varies from 20 to 26% premium.

12. What would be the cost, at Chicago, of a bill on Hamburg for 10,000 marks banco, the banker in Chicago drawing direct, at New York quotations (37 cts. per mark), adding the current rate of exchange on New York (1½% premium), and 1% commission?

13. If the agio between the former New England paper currency and coin was ¼%, and between former Illinois currency and coin 2%, what would it have been if both had circulated in equal proportions?

14. If the currency formerly in circulation in Cincinnati had an agio of ¾% compared with United States coin, what would have been the ultimate effect of making Illinois currency "bankable" if its agio was 2%?

NOTE.—It would drive from circulation everything but Illinois currency or its equivalent, and depreciate the money of account what per cent.?

15. A banker in New York sends 1000 eagles to London, at a cost for freight and insurance of ¾%, which is paid in New York, and receives credit at the rate of £3 16s. 2d. per oz., and 3% per annum interest on the account. At the same time he sells a sixty days sight bill drawn against the proceeds of the coin and the accrued interest, at the rate of 110½%. Suppose the bill to be accepted on the day of the credit, and payable without grace, what profit does the banker receive in the transaction?

16. At one time the laws of Spain rigidly restrained the exportation of the precious metals from that country; still they were secretly exported at a risk of about 2%. What, then, was the

nominal exchange between that and other countries having a free-trade in bullion, arising from the depreciation occasioned by relative excess?

17. If from the large increase of California gold, or excessive paper issues in the United States, the nominal exchange between England and the United States should be 2% in favor of England, what should be the quotations of sterling exchange, other things being equal, to represent the balance of payments in equilibrium?

18. If the nominal exchange, at London, on Hamburg, be $16\frac{2}{3}$% discount, what would a London merchant make for his net profit, the cost of transportation, insurance, etc., being 5% on the purchase price, and payable at London, if he sells in Hamburg for £12,000 what cost in London £10,000?

19. If the currencies of England and the United States were in due proportion in amount compared with business wants, what would be the effect upon the "movement" of the precious metal between the two countries, if the United States should add to its currency a large issue of paper money or gold coinage, thereby raising prices and depreciating the relative value of money?

20. Why is any country better able to sustain an increase of importations compared with the exportations, when it arises from an excess of *specie* currency, than when it arises from an excess of *paper* currency?

Ans. Because nothing but metal will pay the balance, and in the one case we can afford to part with it, while in the other we cannot.

21. Suppose the circulating medium in San Francisco to be depreciated below the currency of New York $\frac{1}{2}$% in consequence of imperfect coinage, and the expense of transportation, including risk, be $\frac{1}{2}$% more, and the broker's commission in New York be $\frac{1}{4}$%, what does an exchange broker or gold exporter in San Francisco make, if he sells sight drafts on New York for 3% premium, and to make his exchange is obliged to ship gold?

22. If a wheat merchant in Toledo bought wheat at \$1.00 per bushel, and sent it to Buffalo for sale at \$1.02$\frac{1}{2}$ per bushel, the cost for transportation, insurance, and commission being $1\frac{1}{4}$%, what per cent. profit would he make, if, in view of the rate of exchange between the two places, he is able to negotiate at $\frac{1}{2}$% premium the drafts drawn against the proceeds of the sale?

NOTE.—In the last example the rates were made to correspond with those of the 21st, to show more clearly to the pupil that in general the same laws govern the movement of gold in large quantities as regulate the movements of wheat.

23. During the year ending June 30, 1857, our exports, including specie, to England, exceeded our imports from England $54,216,623; but in our trade with Cuba, Brazil, China, and France, our imports exceeded our exports as follows: Cuba, $30,319,658; Brazil, $15,915,526; China, $3,961,802; France, $9,553,840. During the same time our total excess of exports of specie was $56,675,123, of which $46,821,211 went to England, and we will suppose, for this example and the one following, that the balance of excess went in equal amounts to the other four countries. Why did the specie go to England, when we were not in debt to her, and how was our debt to the other countries probably settled?

24. Suppose the last example to represent our entire foreign commerce and trade for that year, after a full settlement, and to include nothing else, and our due proportion of specie for currency to have been preserved by supply from California, and the Custom House value to be the exact exchangeable values of both importations and exportations, what was the balance of net profit as shown by the excess of imports?

25. What per cent. would that profit be on the entire exports to those countries which, for that year, specie included, were about $240,000,000?

26. If the exports, as entered at the Custom House, not including specie, were $170,000,000, and the imports, as received, were entered $231,000,000, what was the balance of payments in specie, if the exports, being carried by American vessels, brought in the foreign market 10% advance on their Custom House valuation, and the imports were entered 5% below their cost?

27. If our due proportion of currency required no increase of specie for the year 1857, and California, with other American mines, furnished for the market $49,000,000, how was our *balance of trade* for that year?

28. Suppose we had redeemed, during that year, of our foreign indebtedness in stocks and bonds, $10,000,000, what would then have been our balance of trade?

STOCKS AND BONDS.

623. 1. *A business corporation* consists of several persons who are authorized by a general or special law to engage in a specified business and enjoy certain privileges on certain conditions, in the capacity of a single individual. The general purpose of such corporations is the combination of the influence and capital of a large number of individuals for the prosecution of some enterprise or business which could not otherwise be undertaken with safety and profit. Thus great works of internal improvement, material development, and commercial importance are undertaken and completed by a combination of men and capital, far beyond the ability of any single individual.

2. The legal instrument or certificate of the incorporation of a company is called its *Charter*, and this is made perpetual, or limited, its perpetuity or the time of its expiration being fixed when granted.

624. 1. The interested parties in a stock company are the officers and the stockholders.

2. The capital is furnished by the stockholders and is called the capital stock. This is divided into shares of twenty-five, fifty, or more generally of one hundred dollars each, and each stockholder (also called a shareholder) is entitled to one vote in the management of the company for each share of stock owned by him, and also to any dividends made on account of such shares. Stocks of $50 shares are sometimes called "half-stocks," and those of $25 shares "quarter-stocks."

625. 1. *Stock Certificates*, duly signed by the proper officers, are issued by the company to each stockholder in his name, showing the number of shares owned by him and the par or face value of each share.

2. These certificates are commonly known as *Stocks*, and may be bought and sold like other property, being transferable on the books of the company and by indorsement by the owner in blank or in full.

3. A corporation sometimes creates two or more classes of stocks, called "*common*" and "*preferred*," the holders of the latter taking precedence in the dividends, while the holders of the former receive any dividend left after paying the others.

4. A "*Guaranteed Stock*" is entitled to its stipulated divi-

dend before all other classes, whether it is earned in any one year or not, as its right to an annual dividend is carried over from year to year until it is earned.

5. A *Scrip certificate* is given when a dividend is declared but not paid in cash, and certifies the amount to which the holder is entitled, with interest thereon after interest due on the capital stock has been paid. Such scrip is regarded as stock in market, and is convertible into capital stock on certain conditions.

6. All stock, including also scrip, is liable to assessment to meet the liabilities of the company if the profits are not sufficient to meet them.

626. "Watering Stock" is a merely nominal increase of the amount of capital stock by an extra issue of stock to the stockholders without requiring the payment of a corresponding amount of money to the company. When the capital stock is thus increased, the same earnings as before must afford a smaller rate of dividends, and both the intrinsic and market values of the stock are diminished.

627. 1. If more money is needed by the company than is represented by the capital stock, the company is authorized to borrow money by means of a *mortgage* on its real and personal property, made to a third party as *trustee,* to secure a series of *Bonds,* duly numbered, for a specified amount, rate of interest and time. If the interest on a Mortgage Bond be not paid when due, the mortgage may be foreclosed by the trustee and the property sold for the benefit of the class of bondholders represented in the mortgage.

2. Bonds issued under a first mortgage are called *First Mortgage Bonds,* and those issued under subsequent mortgages are denominated *Second, Third,* etc., *Mortgage Bonds.*

3. The certainty of the payment of interest due on bonds, and the uncertainty of a dividend on stocks, gives to the former the greater intrinsic value.

628. Government Securities. 1. *Government Stocks* or *Bonds* are certificates of indebtedness issued by the General Government, or by a State government, as security for loans made to them. Similar issues are made by many cities, counties and towns, and for the payment of all these the public faith is pledged. In a strict sense such government certificates are not *Bonds* but only *Stocks.*

2. In England it is provided that so long as the interest is paid regularly by the government the principal is not to be called for. In the United States provision is made for the regular semi-annual payment of the interest and for the payment of the principal after a certain time.

629. A *Registered Stock* is one that is duly recorded in the name of the owner, in a book kept for that purpose, is not negotiable without the indorsement of the owner, and the interest is only collected on his written order, there being no coupons attached; hence it cannot be used by any one but the real owner, even if lost or stolen.

630. 1. *Consols* is a term commonly applied to the 3% *consolidated annuities* of the public debt of Great Britain, but more recently it has been used by the Treasurer of the United States to indicate bonds issued in 1865, 1867 and 1868, by which a portion of the United States public debt was consolidated.

2. By Act of Congress in July, 1870, and certain additional amendatory acts of a later date, provision has been made for refunding our national debt by the issue of bonds bearing 5 per cent. interest, payable ten years from date, bonds bearing 4½ per cent. interest, payable fifteen years from date, and bonds bearing 4 per cent. interest, payable thirty years from date.

631. 1. The bonds formerly issued known as *five-twenties*, (5-20s) are those redeemable at the option of the government in from five to twenty years after date, and bearing 5% gold interest.

2. *Sixes of* 1881 (6s '81) are bonds issued mostly in 1861 payable in 1881, with 6% gold interest.

3. *Ten-forties* (10-40s) of 1864 are redeemable in from ten to forty years, and bear 5% in gold.

4. *Seven-thirties* (7-30s) are bonds bearing seven and three-tenths per cent. interest in gold.

5. *U. S. Pacific R. R. Currency Sixes* are Government bonds issued to aid in the construction of railroads to the Pacific coast, on the completion of successive portions of the track, bearing 6 per cent. interest payable in currency, and are redeemable in thirty years.

6. See Part Third for a statement of the Public Debt December 31, 1871. The interest on all the bonds mentioned is payable semi-annually.

STOCK EXCHANGE.

632. The par value of Government and Railroad Stock alone in this country is estimated to be over ***$5,000,000,000***, and these, together with the Stocks of Banks, Insurance Companies, Mining Companies, Manufacturing companies, and many others of less importance largely constitute the basis of all financial operations, and their value is taken as an index of the financial *status* of what they represent in the commercial world. The better class of securities form the basis of bank circulation, the investments of Savings Banks, Insurance and Trust Companies, and fluctuations in their value affect the financial interests of the whole country as well as international and foreign transactions.

633. 1. At the great commercial centres where money, stocks, and the various products of a country are offered and demanded in large amounts, an association to facilitate exchange, and by general intelligence determine values, becomes of great importance. This has given rise to the Parisian *Bourse*, the German *Guild*, the London *Exchange*, and the various Chambers of Commerce, Boards of Trade, Produce Exchanges, Stock Exchanges and Brokers' Boards in this country.

2. If there were no stock exchange, the market price would be dependent on the chance dealings of a buyer and seller, and would be the subject of constant dispute or doubt, attended with great inequality of quotation and consequent confusion.

3. The chief securities dealt in at the London Stock Exchange are "Consols" (Art. 630, 1). They bear an interest of 3 per cent. per annum, and amount to £801,477,741, or about $4,000,000,000. The fluctuations of consols is a good indication of the condition of the London money market, inasmuch as they form a staple credit and are a standard of reference.

4. The great Paris Exchange is called "the Bourse," from the belief that the first gathering for the purpose of dealing in stocks was in the house of a family by the name of Van De Beurse. The dealings here are chiefly in French "Rentes," which represents the national debt, now amounting to about $4,400,000,000.

634. 1. The ***New York Stock Exchange*** was incorporated by the Legislature of that State for the purpose of dealing in stocks and bonds, recording transactions in the various secu-

rities, and publishing the current value of the same. Their transactions determine the market value and establish daily quotations of nearly all stocks and bonds in this country.

2. The business is now so extensive that it is divided between two Boards, as follows:

The *Government Stock Board* deals exclusively in Government, State, and City Stocks, and in Railroad Bonds.

The *New York Stock Board* deals in Railroad, Bank and Insurance Stocks.

3. Each Board has two regular sessions daily at 10 A. M. and 1 P. M., but members engage in transactions among themselves from 9½ A. M. till 6 P. M.

4. At the regular sessions, stocks are called by the president, according to a regular list, and the members buy or sell, and each makes a record of his transactions. This business is transacted by brokers, who report to their respective offices, and after the sessions, comparisons are made to rectify errors.

635. The following customs and rules are observed by the New York Stock Exchange, and similar rules prevail in other cities, and in the New York Gold and Produce Exchanges.

1. Stocks bought for "cash" are paid for the same day; when "regular," they must be paid for the next day.

2. "*Options*" include both privileges and obligations within a specified time.

3. "*Buyer's Option*," called "buyer 3," 5, 10, or 30 days (or other time), is the purchase of stocks at a fixed price with the privilege of demanding them within the time specified, and with the obligation to take them at the expiration of that time, that is, on the last day.

4. "*Seller's Option*" called "seller 3," 5, 10, or 30 days (or other time), is the sale of stocks at a fixed price with the privilege of delivery within the time, and the obligation to deliver them at the expiration of the time specified.

636. 1. "Puts" and "Calls" are simply privileges bought without any obligation to use them, the buyer losing the price paid for them.

2. A "put" is the privilege of delivering a stock at a stipulated price within a specified time, and becomes valuable only when the stock declines in value below the price named. Thus, if A

buys of B the privilege of "putting" to him 100 shares Rock Island within 30 days at 109, and pays $100 for the "put." if the stock does not decline below 109 he loses the $100, but if it declines to 105 he may buy the stock and put it to B at 109, making the difference less the cost of his "put," or $300.

3. "Calls" are the opposite of "puts," being valuable only when the stock advances above the specified price, when the seller may be called on to deliver the stock to the buyer within the time of the "call." "Puts" and "calls" are usually bought at the rate of 1% for every 30 days.

637. 1. All time contracts in stocks, as "options," bear interest at 6 per cent., to be paid by the buyer. A "call" bears interest, but a "put" being a kind of "short sale," is not chargeable with interest.

2. At present a Government tax of $\frac{1}{100}$ of 1% on all sales is paid by the seller. This may be modified or repealed by Congress.

3. Interest and the tax are computed on the actual amount of sales, while commissions are computed on the par value of the stocks. The usual commissions are $\frac{1}{8}$% on Government Stocks and Bonds, and $\frac{1}{4}$% on all other stocks, but special rates are made for some rare transactions.

4. All dividends, whether cash or scrip, on real purchases of stocks or contracts for them, as also on puts and calls, belong to the buyer or holder of the real stock or option at the date of closing the stock books for transfer.

638. Selling "*short*" is selling before buying, or selling what one has not. If A thinks a certain stock is too high in price, and must decline, he sells it to B, who thinks, on the contrary, that it is worth more money. A is then "short" of that stock, and must look for his profit to a decline in the price.

In such a case, A, who is short, is also called a "*Bear,*" because he is "bearing" the price down, and B is a "*Bull,*" as he is elevating or "tossing up" the price.

639. "A Corner" is made in stocks, when nearly the entire capital stock of a company is bought up by one or more parties. Then the price is rapidly advanced, and those who have sold "short" are obliged to purchase at a loss, to fill their contracts.

640. 1. The members of the Board are called "*Brokers,*" who act as agents for those who are not members, and who execute orders to buy or sell. They require from their customers a deposit

of 10 per cent. on the face or par value of each stock bought or sold. This is called a "Margin," and this margin must be kept good to save the broker from loss.

Margins are kept good in case of a decline in the market, by depositing an additional amount equal at least to the decline.

2. *Legal interest* is allowed and charged on all *accounts* of stock transactions between brokers and their customers.

641. 1. The large fluctuations in market value are due to heavy purchases, which produce a rise, or to heavy selling, which causes a decline, or, in other words, to the usual law of supply and demand.

2. The orders to buy or sell are caused to some extent by the condition of the money market, and hence the changes in the market value of stocks is some indication of the value of money.

3. Some stocks, being looked upon as sure in the prompt payment of their dividends, fluctuate but little in their market value, and are dealt in only by those who buy them as an investment, and are called "*investment stocks.*"

4. Other stocks, less sure of paying dividends, are dealt in as a speculation, and consequently vary much and sometimes rapidly in their market value. These are called "*speculative stocks,*" and are dealt in by those who are willing to take their chance of making a loss for the chance of making a profit.

5. The changes in the market price of Government, State, and some railroad stocks are very small, which, with their reliability, make them a good security for "call loans," investments of estates, charitable funds, trust funds, etc.

642. PROBLEMS IN STOCK EXCHANGE.

NOTE.—The following problems may be readily solved according to the general principles of Percentage and the customs of the Stock Exchange already explained. Commissions and taxes are to be deducted unless otherwise stated.

1. A bought 100 sh. (shares) Mich. So. @ 88, and sold the same @ $90\frac{1}{4}$. What was his net profit, making allowance for the usual brokerage of $\frac{1}{4}\%$ and the Government tax of $\frac{1}{100}\%$?

2. If I buy 200 sh. Pacific Mail @ $44\frac{3}{4}$ (a "half stock") and 200 sh. St. Paul @ $62\frac{1}{4}$ and sell the former @ $45\frac{3}{8}$ and the latter @ 63[illegible], what is my net profit, allowing usual expenses?

3. A broker sold for me 200 sh. Wabush @ $65\frac{1}{4}$ and 300 sh.

N. J. Central @ $111\frac{7}{8}$. The sale of Wabash was "covered" @ $63\frac{1}{4}$ and of N. J. Central @ $113\frac{5}{8}$. Did I gain or lose, and how much?

4. If I sell 200 sh. Erie @ $33\frac{1}{2}$ and 300 sh. Chi. & N. W. @ $65\frac{1}{4}$, and cover my short sales on Erie @ $29\frac{5}{8}$ and Chi. & N. W. @ $57\frac{1}{2}$, what is the amount of my profits?

5. If I buy 200 N. Y. C. @ $91\frac{1}{4}$ and sell 200 C. & N. W. short @ $83\frac{3}{8}$, and later sell my N. Y. C. @ $94\frac{5}{8}$ and cover my C. & N. W. @ $76\frac{7}{8}$, what is the result of the two operations?

6. July 27,1870, bo't 200 Rock Island @ $108\frac{1}{4}$ and on Aug. 30 sold @ $111\frac{7}{8}$. What was my net profit, allowing for commission, tax and interest?

7. Bo't 300 Pacific Mail @ $41\frac{1}{4}$ Oct. 12, 1870, and 200 Ft. Wayne @ $96\frac{1}{8}$, and Nov. 10 sold the former @ $41\frac{7}{8}$ and the latter @ $96\frac{3}{4}$; what was my gain?

8. Sept 2, 1871, sold 300 Harlem (half-stock) @ $139\frac{3}{4}$ and 200 Mich. Central @ $122\frac{1}{4}$, and Oct. 19 covered my "short" Harlem @ $139\frac{1}{2}$ and Mich. C. @ $122\frac{1}{4}$; what was my profit or loss?

NOTE.—Observe that the gross profit was equal to the commission; hence the net loss equals the tax. See Note under Ex. 10.

9. What amounts should be entered to complete the following stock %, and what is the true balance?

DR. J. D. BROWN.

1871.			Ds.	Interest.	
Jan. 2	Bo't 100 sh. N. Y. C. $88\frac{1}{2}$ . .	$ 8850			
	200 Lake Shore $91\frac{1}{4}$ . .	18250			
	400 O. & Miss. $29\frac{3}{8}$ . .	11750			
	300 W. Un. Tel. $84\frac{1}{4}$ . .	10275			$49125.00
	Int. on purchases		67	$659.99	659.99
	$\frac{1}{4}$% commission				250.00
	Gov't tax on sales				5.73
	Balance to Cr.				
					67480.27

J. D. BROWN. CR.

1871.		Deposits.	Interest.	
Jan. 2	Cash Deposit	$10000		
	Interest 67 days		$130.27	$10130 27
Mch. 20	Sold 100 N. Y. C. $96\frac{3}{4}$			9675.00
	200 Lake Shore $102\frac{1}{8}$ . . .			
	400 O. & Miss. $35\frac{3}{4}$			
	300 W. Un. Tel. 43			
				67480 27
Mch. 20	By Balance			

10. C. L. Woodworth deposited with his broker $5000 July 12, and gave him orders to sell short on his account the stocks mentioned below on Cr. side of the %. What was the true balance of the % ?

DR. C. L. WOODWORTH.

1871.				
Aug. 21.	Bo't 100 Erie 27⅝			$2762.50
31	100 St. Paul 82¼ . . .			
Sept. 9	200 Quicksilver 11¼ . .			
20	100 St. Paul 75¾ . . .			
	Commission 500 shares . .			
	Gov't tax on gross sales . .			

C. L. WOODWORTH. CR.

1871.				
July 12	By Deposit	$5000		
	Int. on deposit, ...			
"	Sold 100 Erie 34¼			$3425.00
19	200 St. Paul 87½ . . .			
27	200 Quicksilver 17½ .			

NOTE.—Brokers generally allow interest only on actual deposits and not on short sales, and they charge interest only on stocks purchased and carried.

11. April 3, 1871, I bo't 500 N. Y. C. @ 96¼. Apr. 15 the books of the N. Y. C. R. R. were closed to pay a dividend of 4%. Sold my stocks May 4 @ 94⅛. What was my profit ?

12. Sept. 28, 1871, bought 200 Rock Island @ 97⅝, on which a dividend of 3½% had been declared and books closed for the same Oct. 8. Sold all this stock Nov. 25 @ 101⅞. What was my gain ?

NOTE.—$20875 – $19525 + $700 + Int. on div. for 47 da. – (Int. on $19525 for 58 da. + com. of ¼% on $20000 + Gov't tax of $\frac{1}{100}$ of 1% on final sales) = Gain.

13. May 4, 1868, bo't 500 N. Y. C. @ 98¼ and sold the same July 3, 1870 @ 98¼. In the meantime four cash dividends of 4% each had been made and a stock dividend of 84%. I sold my new stock dividend in scrip @ 93. How much did I gain, allowing no interest on *dividends* received ?

NOTE.—The stock dividend = 84% of 500 shares = 420 shares.

14. March 3, 1871, bo't 300 Lake Shore @ 91⅜. A cash divi-

dend of 4% was paid May 4, and a stock dividend of 42% was made Sept. 13, the latter being declared full paid stock on the payment of an assessment of 20% on it. Oct. 28, sold my entire interest in the stock @ 93¼. What are my profits, allowing interest on the cash dividend to date of sale, with usual commission on 300 shares and ⅛% on scrip?

15. I deposited $5000 with my broker Sept. 20, 1871, for the following stocks purchased the same day on my order, viz. 400 sh. Reading (half stock) @ 114½, 200 sh. N. J. Central @ 113⅜, 100 sh. St. Paul @ 73¼. On Oct. 9 (the time of the great fire in Chicago), these same stocks were quoted as follows: Reading @ 101¾, N. J. Central @ 102¼, and St. Paul @ 63⅝. How much cash should my broker have required or called from me to make my margin of 10% good, and cover commission?

16. June 10, 1871, sold 200 sh. Rock Island @ 108¼ s. 30 (seller, 30 days), and bo't the same July 10 @ 106⅜ for delivery. What was my profit?

17. Bo't a put April 10, 1871, on 200 Erie @ 33¼, for 30 days, for which I paid $200. On April 30 I bo't 200 Erie @ 30⅝, which was delivered on my put. What was my profit?

18. Jan. 15, 1871, bo't a put on 300 sh. Mariposa Preferred @ 24½, for 30 days, for $300. I made my delivery on the put @ 19¼ for 100 sh., @ 17⅜ for 100 sh., and @ 14⅝ for 100 sh. at different times during the contract. What was my profit?

19. Dec. 10, 1870, bo't a call on 200 sh. N. Y. Central @ 92⅛, for 30 days, for $200. Sold the stock Jan. 10, 1871, @ 97¼, and "called" it for my delivery. What were my profits?

20. June 8, 1868, A bought a call on B for 1000 shares Hudson River R. R. @ 116½, for the balance of the year for $2500. A cash dividend of 4% was paid Oct. 15, and about Nov. 10 a stock dividend of 100% was made, subject to a payment of 15%. A sold the stock and scrip @ 125½, and called it from B for delivery before the contract expired. What was A's profit?

NOTE.—See the similar case of Curry & Martin *vs.* C. G. White in New York Court of Appeals.

INVESTMENTS.

643. *Investments* properly include all outlays of money the design of which is to secure some chance of profit, establish a source of income, or purchase some permanent possession.

644. *Temporary investments* are those made for a comparatively short time and include all speculative investments. These are generally attended with considerable risk, and the profits are somewhat uncertain though sometimes very large, being governed more by accidental and artificial circumstances than by the well-known laws of business and trade. Most of the operations in real estate, short loans, stocks, gold and grain come under this head, operators in these looking to fluctuations in the markets for profits proportionate to the risk assumed.

645. 1. *Permanent investments* are those made for a comparatively long time, and consist mostly of outlays of money for real estate property and for improvements thereon, and of loans to the Government, to corporations or to individuals, for the reasonably certain income to be derived from rents, interest, or dividends.

2. Bonds and mortgages, promissory notes, and stocks are the subjects of permanent investments, or are used as pledges for the payment of loans. These are all known as *securities*. Their desirability and price depend on the nature and degree of the security furnished, and the rate or amount of income to be realized.

646. 1. ***Real Estate,*** embracing lands and permanent improvements on them, is regarded as one of the most secure forms of investment; and yet when it is the subject of speculation no source of profit is much more uncertain, and the profits or losses realized are frequently very great.

2 It is believed that wealth is most surely perpetuated and retained in families by investing mostly in real estate property. By a careful selection, judicious improvements and gradual additions by the investment of rents, this may become not only the permanent wealth of a person or family, but the very best security and basis for using and directing outside capital, for making family endowments, and for worthy and munificent bequests.

647. Many real estate investments have for their design sim-

ply the profits to be realized from the natural increase of its value. For such purposes large tracts or plats of land in favorable localities are purchased of the Government or of early owners at low rates, and after improvements have been made on them or in their vicinity, or to secure such improvements, they are divided and subdivided into farms or town lots convenient for property investments, and gradually sold at a considerable, yet just, advance, to persons of small means desiring homesteads.

648. 1. When real estate is not paid for in full at the time of purchase, the purchaser generally gives his promissory notes for the balance due, or a bond for a larger amount, and these are secured by a Mortgage on the property or a Trust Deed of it. This gives to the mortgagee a claim on it until the balance due is paid, and in case of default, he may, under certain legal provisions, foreclose the mortgage, or more properly the mortgagor, and sell the property at a fair price to satisfy his claim.

2. The purchaser of real estate assumes all existing mortgages on the same. These are regarded as incumbrances, and if none exist the property is said to be unincumbered.

649. 1. ***Bonds*** and ***Mortgages*** on real estate and also on personal property are extensively used, not only as security for purchase-money but as security for long loans to persons or corporations.

2. A bond is given for a definite amount, the whole payable at one time with interest, generally semi-annually, and to this bond is attached a mortgage on the property of the borrower, executed for this particular bond. Such bonds are always at par, and the holder of such securities never needs to consult the daily price lists of stocks and bonds to know if he is growing richer or poorer.

3. It is thought by some of the best authorities that wealth may *certainly* be accumulated more rapidly by investing surplus capital in real estate or in good bonds and mortgages than in any other way. Many business men wisely invest enough in real estate to secure themselves and their families against want, even if all their other sources of income be cut off.

650. Government Securities afford perhaps the most certain, regular and definite income of any, on money invested in them. These are sought largely by corporations, persons holding trust or endowment funds, and by individuals who desire a certain

support for themselves or others. The rate of income is not large but is without change, and on most United States bonds is payable in gold.

651. The bonds of the General Government are generally regarded as the most desirable. Bonds issued by different States, counties and municipalities, require some investigation as to their legality and certainty of payment. Certain conditions are generally attached to the issue of such bonds, by the legislature of the State, to render them secure. The State constitution usually provides for the creation of such securities.

652. The value of railroad bonds secured by trust deeds depends chiefly on the management of the road and the amount of receipts.

653. 1. The chief considerations with reference to all bonds and mortgages are first, *the aggregate amount of indebtedness evidenced by them* as compared with *the total amount of security;* and second, *the rate per cent. which they draw.*

2. In the first consideration may be included the special one as to priority of issue, as every issue for security on any property tends to diminish the value of all subsequent issues on account of the same property.

654. One desirable feature of bonds is that they have a long time to run, and other things being equal, the longer the time the more desirable the bond for the capitalist. Such an investment saves the trouble and risk of frequently reinvesting the money, and also has the advantage of furnishing the means for obtaining temporary loans by depositing the bonds as collateral security, thereby avoiding the unpleasant necessity of getting indorsers. Such securities always have a market price, and are readily convertible into money if the holder desires to purchase others that promise a larger income, immediate or remote.

655. 1. ***Notes*** given either for purchase-money of land or for borrowed money, and secured by mortgages or trust deeds, are sometimes offered to the capitalist on good terms, and are bought, although they are not the most desirable except on account of the special terms secured.

2. Purchase-money notes secured by the land purchased, are usually payable at different times, so as to make annual instalments. This divides the amounts into such small sums that they

can answer only for short investments, and are sought only by those in active business who wish to turn their capital as frequently as possible.

3. Notes given for borrowed money with real estate security partake more of the nature of bonds, which have been mentioned before.

4. It is more profitable, generally, to purchase notes having but a short time to run; for in any transaction that affords more than ordinary interest, the more frequently the same money is turned the better. Thus, it is customary to purchase a sixty days note of say $500 for $450, and at the end of each sixty days the same amount may be invested again in like manner. In a single year the profits on such a series of investments would amount to $(50 \times 6) = \$300$ nominal discount, and the interest received on each note when due, besides the use of the discount and interest allowed and received. This illustrates what is done not only six times in a year, but much more frequently by many. *Turning money frequently in business* is one real secret of prosperity at which active business men aim, while silent partners are quite willing to furnish capital and receive their share of profits arising from such a use of it.

656. Stocks embrace a large class of securities, including stocks of Banks, Railroads, Insurance, Mining, Manufacturing, Transportation and other companies. Bank stock is perhaps the best of these for investments, as it is least liable to fluctuation in value.

657. 1. Assuming unimpaired capital in a corporation, the desirability of the stock may fairly be determined by the *dividends earned*, not by those *paid*.

2. Permanent investments may be made in stock, as one is never obliged to change it, and is not likely to do so, so long as it pays well.

3. Among the stocks generally preferred for investments are first-class railroad bonds, secured by a first mortgage on the road and on real estate worth at least twice the amount thus secured. The bonds of the Northern Pacific Railroad, bearing $7\frac{3}{10}\%$ gold interest, being a first lien on the railroad and its traffic, and in addition, secured by a first mortgage on adjacent lands at the rate of 50 acres for every $100 of indebtedness, or 23,000 acres of land for each mile of track, furnish a good illustration of the profitableness and stability of such securities.

INVESTMENT COMPUTATIONS.

658. ***Real Estate transactions*** are so varied, and frequently so complicated, that all the cases cannot well be treated in detail, but some customs may be noticed and some general principles may be stated that shall be of practical value in the solution of problems arising in such business.

659. Full cash payments for real estate at the time of purchase are very rare. The buyer invests in the property and looks to its rent or rise in value for his profits, the seller invests in some note, bond or mortgage security instead of money given by the buyer, and receives regular interest and the final payment of the principal. It is quite common to sell for one-fourth or one-third cash with the balance secured by a mortgage on the property, to be paid in annual instalments in one, two, or three years. In some cases the seller requires only semi-annual interest in advance and all the principal at the end of five or ten years.

NOTE.—Farms and plats of land are sold at a certain price per acre, town and city lots at a certain price per front foot, except in Boston and some other places, where the price is a certain rate per square foot.

660. The computations required in most investments in *stocks* and *bonds* are readily made according to the general principles of percentage, but in transactions requiring the present value or equitable price of securities affording one rate of interest or dividend to make the income a certain per cent. on the investment or equivalent to the current rate of interest, the computations are somewhat more complicated, but are capable of plain analysis.

661. Properly, all computations of value depending on time as an element of production, should be made on the basis of compound use or interest, as value may be used, reinvested, or transformed at regular intervals so as to be reproductive. In business transactions, however, it is quite common to disregard this principle and use short, simple methods of computation more for convenience than equity. Thus, a note of \$5000 bearing 8% interest and having 3 years to run, if sold so as to afford 10% to the buyer, would commonly be discounted at 2% a year, making the price equal to \$5000 × .94 = \$4700. The inaccuracy of this will be seen when it is considered that each year's interest may be used so as to produce 10% per annum during the time the note runs. The correct method of computation is shown in Art. 667.

DIVIDEND OR SIMPLE INTEREST INVESTMENTS.

NOTE.—The student should analyze each of the following cases.

662. ***To find the amount of income on any investment.***

RULE.

Multiply the par value of the stocks, etc., owned, by the rate per cent. of interest or dividend, expressed decimally. This will give the annual income. One-half will be the semi-annual.

NOTE.—Gold value, as interest on United States bonds, may be reduced to currency value by multiplying by the quoted rate of gold regarded as hundredths.

Ex. The semi-annual income on \$5500 invested in U. S. 10–40s bought at 110 and bearing 5% gold equals $5500 \div 1.10 \times .05 \div 2 =$ \$125, and if gold be quoted at 110, this will equal $\$125 \times 1.10 =$ \$137.50 in currency.

663. ***To find what must be invested to produce a required income.***

RULE.

Divide the required income by the rate of income to be realized; this will give the par value of securities required to be purchased, and this multiplied by the price of the securities will give the amount to be invested.

Ex. To produce a semi-annual income of \$600 from 8% bonds quoted at 105, bearing semi-annual interest, the amount to be invested would equal $\$600 \div .04 \times 1.05 = \$15750$.

664. ***To find what price of bonds or stocks will afford a certain per cent. of income on the investment.***

RULE.

Divide the rate of actual income on the securities by the required rate on the investment.

Ex. R. R. bonds bearing 6% to afford $7\frac{1}{2}$% must be purchased at a price equal to $6 \div 7\frac{1}{2} = .80$.

665. ***To find a rate of interest equivalent to a certain rate of income on stocks, purchased at a premium or discount.***

RULE.

Divide the rate of income by the price of the stocks.

Ex. 6% R. R. bonds bought at 115 would afford a rate of income on the investment equal to $.06 : 1.15 = .05\frac{5}{23}$.

NOTE.—A rate of dividend or interest on any stock equivalent to a certain rate on the investment may be found by multiplying the given rate by the price of the stock.

Ex. Any stock bought at 110, to equal a 6% investment, must afford a rate of income equal to $.06 \times 1.10 = .066$.

666. ***To find a rent that shall afford a certain net rate of income on the value of real estate property.***

RULE.

To the net rate required, add any per cent. to be allowed for taxes, collection, etc.; multiply the value of the property by this sum, and to the product add any incidental expenses for repairs, etc.

Ex. A house and lot worth \$8000, requiring \$150 for repairs, paying a tax of $\frac{1}{8}$%, and the agent receiving 1% for collection, to afford a net income of 6% must have a rent equal to $\$8000 \times .07\frac{1}{8} + \$150 = \$720$.

COMPOUND INTEREST INVESTMENTS.

PRESENT WORTH OF BONDS OR OTHER SECURITIES.

667. To find the true present worth of a bond or note, having a number of years to run and bearing any rate of *annual interest*, that would afford a *different rate of interest annually*.

Ex. What is the present worth of a \$2000 bond bearing 6% annual interest and having 4 years to run, when money is worth 10% per annum?

Solution.—The amount to be received on account of the bond is \$120 each year and \$2000 at the end of four years. If money be worth 10% per annum, the several amounts of annual interest received may be put at compound interest till the principal is paid, and the true present worth of the bond will be the present value of the amount of these several interest investments plus the principal at the end of the four years. The interest received at the end of the fourth year will amount to \$120, that received at the end of the third year to $\$120 \times 1.10$, that received at the end

of the second year to $\$120 \times 1.10^2$, that received at the end of the first year to 120×1.10^3 (Arts. 526, 530), and the whole amount of interest and principal will be $\$120 + (\$120 \times 1.10) + (\$120 \times 1.10^2) + (\$120 \times 1.10^3) + \$2000$. The several amounts of interest form a geometrical series whose sum is required. The series may be made more simple by factoring; thus, $\$120 \times (1 + 1.10 + 1.10^2 + 1.10^3)$, giving for the series whose sum is required (Art. 395) 1 for the *first term*, 1.10 for the *ratio*, and 1.10^3 for the *last term*. The sum of this series is equal to $\frac{(1.10^3 \times 1.10) - 1}{1.10 - 1} = \frac{1.10^4 - 1}{.10}$ $= .4641 \div .10 = 4.641$, and this multiplied by the factor \$120 gives \$556.92 as the *amount* of the entire interest value at the end of four years, and this plus the principal gives \$2556.92 as the amount of which the present worth is required. This is to be found by dividing by the compound amount of \$1.00, at the required rate for the whole time. $\$2556.92 \div 1.4641 = \$1746.41$, *the true present worth.* The equation of the problem would be $(1 + 1.10 + 1.10^2 + 1.10^3) \times \$120 + \$2000 = \$1746.41 \times 1.10^4$, and making the changes for the sum of the series it becomes $\overline{(1.10^4 - 1) \div .10} \times \$120 + \$2000 = \$1746.41 \times 1.10^4$. Now observe that 1.10^4 is the compound *amount* of \$1.00, and $1.10^4 - 1$ indicates the compound *interest* on \$1.00 for 4 years at 10%, and that .10 indicates the interest on \$1.00 for one year at the required rate. From this analysis the following rule is derived:

668. RULE.

Divide the compound interest on one dollar at the required rate for the given time, by the required rate; multiply the quotient by the interest on the bond for one year at the given rate; to the product add the face value of the bond and divide the sum by the compound amount of one dollar at the required rate for the given time.

NOTES.—1. This rule may be applied in all cases where the interest *intervals* are alike both for the given and the required rate. Thus it may be used to find the present worth of securities bearing a certain rate of semi-annual interest if money is worth a different rate semi-annually, by substituting a half-year for the year and half the rate for the rate, in the rule.

2. If money be worth only the rate borne by the bond, but a different rate of income be required, the true present worth may be found by

dividing the compound amount of the bond for the whole time at its own rate, by the compound amount of one dollar for the whole time at the required rate.

3. The work may be greatly shortened by the use of the Compound Interest tables found in Part Third.

4. The more common method of dealing with such problems used by business men, is to *divide the amount of the principal at simple interest for the whole time at the rate borne, by the amount of one dollar at simple interest for the whole time at the required rate;* but this is not accurate.

669. To find the present worth of a bond or note having a number of years to run, with *interest payable annually*, that would afford a certain rate of interest *semi-annually* or *quarterly*.

Ex. What must be paid for a note of \$6600 having three years to run at 8 per cent., with interest payable annually, that the investment may afford 10 per cent., with interest payable semi-annually?

Solution.—We must find the present worth of the amount of the 8 per cent. interest at the end of three years, compounded semi-annually, at 10 per cent. plus the principal. The interest received at the end of the third year will amount only to \$528; that received at the end of the second year may be compounded at 10% for two half years, and will amount to $\$528 \times 1.05^2$; that received at the end of the first year will amount to $\$528 \times 1.05^4$. This will give the series $(1 + 1.05^2 + 1.05^4) \times \$528 = (1.05^6 - 1) \div .1025 \times \$528$; or $.3401 \div .1025 \times \$528 = \$1751.93$. Adding the principal \$6600 to this, gives \$8351.93 as the amount of which the present worth is required. This equals $\$8351.93 \div 1.05^6 = \$6232.32$. The equation of the problem is $(1 + 1.05^2 + 1.05^4) \times \$528 + \$6600 = \$6232.32 \times 1.05^6$, or in a changed form (Art. 395), $(1.05^6 \overline{-1}) \div 1.05^2 - 1 \times \$528 + \$6600 = \$6232.32 \times 1.05^6$.

From this analysis the following rule is derived:

670. RULE.

Divide the compound interest on one dollar for the whole time, at the required rate and intervals, by the compound interest of one dollar for one year at the required rate and intervals. Multiply the quotient by the interest on the bond for one year at the given rate, and to the product add the principal.

Divide the last sum by the compound amount of one dollar for the whole time at the required rate and intervals.

671. To find the present worth of a bond or note having a number of years to run with *interest payable semi-annually*, that would afford a certain rate of interest or income *annually*.

Ex. What is the true present value of a bond of $1000 due in 7 years and bearing 6% with interest payable semi-annually, when money is worth 8% annually?

Solution.—At the end of the first half-year $30 interest is receivable, and at the end of the second half-year another $30, and so on. At the end of the first year the value of the interest receipts would be the *amount* of the first $30 for six months at the current rate, 8%, plus the second $30; that is, $30 + $1.20 + $30 = $61.20. This amount at the end of each year may be invested for the remaining time at 8% annually, giving the series $(1 + 1.08 + 1.08^2 + 1.08^3 + 1.04^4 + 1.08^5 + 1.08^6) \times \$61.20 = (1.08^7 - 1) \div .08 \times \$61.20 = .7138 \div .08 \times \$61.20 = \$546.06$. This added to the principal gives $1546.06 as the entire *amount* of value at the end of seven years, and the present worth of this is equal to $\$1546.06 \div 1.08^7 = \$1546.06 \div 1.7138 = \$902.71$.

From this analysis the following rule is derived:

672. RULE.

Divide the compound interest of one dollar for the given time at the required rate by the required rate. Multiply the quotient by the amount of interest receivable in one year plus the interest on half of it for six months. To the product add the principal and divide this sum by the compound amount of one dollar for the whole time at the required rate.

NOTES.—1. The multipliers to be used in the second step of the solution are as follows for a $1000 bond:

Rate of semi-annual interest on the bond.	Multiplier when money is worth 6% per annum.	Multiplier when money is worth 7% per annum.
5%	50.75	50.875
6%	60.90	61.05
7%	71.05	71.225
8%	81.20	81.40
10%	101.50	101.75

2. For the present worth of a $1000 bond bearing 5, 6, 7, 8 or 10 per cent., when money is worth 6 or 7 per cent., see table in Part III, Art. 789.

673. PROBLEMS IN INVESTMENTS.

1. What must be paid for 25 shares Illinois Central Railroad stock at a discount of 12%? What rate of dividend would be equivalent to 10% interest on the investment?

2. What will be the cost of \$15,000 of Ohio State Bonds at a discount of 2½%?

3. Bought 40 shares New York and Erie Railroad stock, at a discount of 3%, and sold the same at a discount of 37½%. How much did I lose in the transaction?

4. If the New York Central Railroad Company declares an annual dividend of 14%, what amount will a stockholder receive who owns 240 shares? What per cent. on his investment, if he bought the stock at 95?

5. How many shares of canal stock, of \$100 each, at 14% discount, can be bought for \$1032? How much would be lost by selling them at 33⅓% discount?

6. If the capital stock of a bank be \$500,000, what amount is necessary to declare a dividend of 5½%? What would a stockholder receive who owned 25 shares?

7. A person owns 20 shares of bank stock, and receives a dividend of \$150; what was the rate of dividend? What per cent. on his investment if bought at 98?

8. A certain stockholder draws \$270 when a dividend of 9% is declared; what is the amount of his stock?

9. Bought stock at 4 per cent. discount, and sold the same at 5% premium, and gained \$450. How many shares were transferred?

10. At a discount of 3%, how many shares bank stock of \$50 each could be purchased for \$9748.50?

11. Which is the better investment, railroad stock paying a semi-annual dividend of 4%, bought at a discount of 25%, or money loaned at 10% interest, payable annually? What per cent. better?

12. Bought bank stock, paying 12% dividend, at a discount of 20%. What per cent. interest did the investment pay?

13. When the annual dividend of railroad stock is 15%, and the interest of money is 10%, at what premium ought the railroad stock to sell?

14. At what per cent. discount must I buy bank stock, paying 6%, that the investment may pay 9%?

15. If the C. & E. R. R. Co. declare a dividend of 15% per annum, what is the value of its stock, money being worth 8%?

16. The free banking law of New York required that the stocks deposited with the superintendent, as security for bank-note circulation, should be made equal to stock producing an interest of 6% per annum. What per cent. of the stock could a bank receive in circulating notes on 5 per cent. securities? What on 7% stock?

17. In March, 1870, the total amount of British consols, bearing 3%, was £399,642,184. What was the amount of interest paid on them semi-annually?

18. The debt of Great Britain and Ireland, in round numbers, is £741,000,000, and the annual revenue £75,000,000. Supposing the annual interest to average 3¼%, what per cent. of the revenue is needed to pay the interest on the debt?

19. In July, 1859, forty-five New York Fire Insurance Companies (out of fifty), on a capital of $8,712,000, divided among the stockholders, as a semi-annual dividend, $679,950. Compared with railroad stock paying 5% semi-annually, which would yield the greater income, railroad stock bought at 65% or insurance stock at par?

20. A man subscribed $20,000 stock in a mining company, the capital stock of which was $500,000, but only 20% paid in. A cash dividend of 2% on the par value was declared and a dividend of 10% was credited to the stockholders as an installment on their unpaid stock. What amount of cash did he receive, and what was the balance due on his subscription?

21. I buy 100 shares of railroad stock, the capital stock being $3,000,000. The first year they declare a cash dividend of 10%. The second year they increase their stock by declaring a stock dividend of 10%. The third year they divide among their stockholders the same amount as in the first year. What would be the per cent. of the last dividend? How much more would they need, to declare a dividend of 10%, the same as in the first year?

22. If the paid up stock in a railroad company be worth 100%, and a stock dividend of 10% be made to the stockholders, what would be the value of the stock after the dividend?

23. If the net earnings of a bank with \$200,000 capital be sufficient to pay an annual dividend of 10% and reserve \$4000 as a surplus to provide for future losses, and it pay 6% on its net earnings to the State in lieu of taxes, what would be the rate of taxation on its capital?

24. What should be paid in New York for a railroad bond of \$1000, bearing 7 per cent. interest, to make it a 10% investment?

NOTE.—It is manifest that, if a corporation sells in New York its bonds, drawing 7% interest, for less than par value, it is borrowing money at a higher rate of interest than the legal rate, and the contract under the general law of that State, regulating interest, becomes tainted with usury. But for the accommodation of corporations, and the security of capitalists investing in such bonds, it was enacted by the Legislature of New York, in 1850, that "no corporation shall hereafter interpose the defense of usury in any action." With this restriction upon them, corporations can negotiate their bonds more readily and at better rates than without such restriction. A large class of individual borrowers desire a similar legal prohibition for a like accommodation.

25. How much must be paid for \$2000 U. S. 5-20s, 6 per cent., @ 104, allowing a brokerage of ⅛%? Gold being 112, what is the semi-annual income on the same in currency?

26. What is the rate of income on U. S. Currency sixes bought at 114½?

27. What rate of interest is equivalent to the income on the new U. S. five per cent. bonds bought at 108⅜?

28. What is the semi-annual interest, in currency, on \$50000 U. S. 10-40s, six per cent. gold, when gold is at 108⅝? What is the per cent. of income, if these bonds are purchased at 109¼?

29. What amount must be invested in U. S. bonds bearing 4½%, bo't at 109⅛, to afford a semi-annual income of \$562.50?

30. At what rate must 6% R. R. bonds be purchased to afford 8% on the investment?

31. If a certain bank stock affords a semi-annual dividend of 10%, at what price would it pay 8% semi-annually on the money invested?

32. What per cent. better are Chicago 7s at 95 than U. S. currency 6s at 114¾? Which is the more desirable investment, and why?

33. What is the yearly value of a rent of \$500 per month in

advance, each month's rent being allowed to draw simple interest for the balance of the year at 6%?

34. What should be the true discount on a 3 yr. mortgage of $5000, bearing 8%, if sold so as to draw 10% on the money paid for it, interest being payable annually?

35. What must be paid for a note of $10000, bearing 8% per annum, due in 2 yr., to make it equivalent to a 10% investment, semi-annually?

36. A person owes, and has outstanding against him, three notes of $2000 each, due in one, two, and three years, drawing 9% interest, payable annually, and secured by mortgage on real estate. He proposed to the holder of the notes to exchange two bonds for them and pay the difference in cash.

NOTES.	BONDS AND CASH.
Note $2000, 1 yr. @ 9% an. int.	Bond $2000, 10 yr. @ 7. semi-an. int.
" $2000, 2 yr. @ 9% "	" $2000, 10 yr. @ 7% " "
" $2000, 3 yr. @ 9% "	Cash balance, $

If 8% annual interest be considered the market value of money, what would be the proper cash balance in the transaction?

37. A capitalist has certain bonds that he thinks of changing for others, if he can make a fair exchange: they are as follows:

2 bonds of $1000 each, drawing 6% semi-annual interest, payable in currency.

3 bonds of $500 each, drawing 7% annual interest, payable in currency.

5 bonds of $100 each, drawing 5% semi-annual interest, in currency.

It is proposed to change these for other bonds, of $500 each, drawing 8% interest, payable semi-annually, in currency. Suppose the security and other things to be equal on the different bonds, and that they all mature at the same time (in 5 years), how many of the 8% bonds should be received at their face, and what difference paid or received in currency, to make an equitable exchange?

38. Money being worth 10% per annum, what per cent. of the face value is the present value of a 7% bond, interest payable semi-annually, running 20 years?

NOTE.—As the *ratio* only is sought, any convenient amount may be assumed for the face of the bond.

39. In 1813 the U. S. Government borrowed $16,000,000, sell-

ing their bonds to run 12 years, at 6% interest, payable semi-annually, at 12% discount. At what discount should the purchasers have taken them, to realize on their investment an average annual interest of 8%?

40. A proposed to purchase through a real estate agent a piece of property for $40,000, one-quarter down, and balance in one, two and three years, at eight per cent. semi-annual interest. The agent, not having been fully authorized to sell, did not know the exact condition of the property. By conferring with the owner, he found there were four mortgages upon the property of $6400, $9000, $7500, and $8500, and that the purchaser would be expected to assume these mortgages and pay $8000 cash. The agent, stimulated by a prospective commission, set himself about a reconciliation of the two propositions, which were, when stated in full, as follows:

PURCHASER'S PROPOSITION.	OWNER'S PROPOSITION.
10,000 cash.	8,000 cash.
10,000 in 1 yr @ 8% semi-an. int.	6,400 in 2 yr @ 7% an int.
10,000 in 2 yr. @ 8% " "	9,000 in 3 yr. @ 8% "
10,000 in 3 yr. @ 8% " "	7,500 in 4 yr. @ 9% "
40,000	8,500 in 5 yr. @ 9% "
	40,000

Starting upon the basis that the market value of money is ten per cent. per annum, the agent endeavors to show how much more, or less, than $8000 should be paid in cash to make the owner's proposition equal, or equivalent to, the purchaser's proposition. A will take the property, assume the four mortgages, and pay enough cash to make this an investment equivalent to the one first proposed by him. What should be the cash payment?

If money be worth 8% interest, payable semi-annually—if the purchaser assume the mortgage incumbrance, how much cash should he pay the owner to make it equal to the purchaser's proposition?

41. A proposed to sell to B some real estate for $20000, $\frac{1}{4}$ cash, and the entire balance in 3 yr. at 10% simple interest. There was a mortgage of $10000 on the property, given three years before, to run 5 years at 8% semi-annual interest. B agreed to assume the mortgage on condition that he is to lose nothing by it. For what amount should B give his note to A?

42. How much more income might be realized in 10 years from Northern Pacific Railroad 7-30 gold bonds (that are bearing $7\frac{3}{10}\%$ interest) bo't at par, than from U. S. 5 per cent. gold bonds bo't at 110, if the interest received on the bonds every half year were reinvested till the end of the ten years at 6%, payable semi-annually, if $55000 be invested in each?

EQUATION OF PAYMENTS.

674. 1. ***Equation of Payments*** is the process of finding the time when the sum of several debts falling due at different times, can be paid without loss to either debtor or creditor. This time is called the *equated time*.

2. The common methods of finding the equated time are based upon the principle that money kept *after* it is due is counterbalanced by an equal sum of money paid the same length of time *before* it is due.

Thus, if A has had the use of $1000 of B's money for one year, to adjust the matter at the end of the year, A gives B the use of a like amount for the same length of time.

NOTE.—The party who has the use of the money first really has the advantage, because the gain from this use may be made productive of farther profit while the second party is using the principal. Thus let A use $10000 of B's money for 5 years and realize 10% on the same annually, and then to repay the *principal debt* let A return the $10000, and to pay for the *use* of it let him lend B another $10000 for 5 years. It will readily be seen that at the end of *ten* years A will have the advantage of the first use of the money, by just the amount of profit he realizes the second five years, arising from the *use* of the $10000 during the first five years. However, as the intervals of credit for which accounts are averaged are generally short, this advantage of first use is but small and is not considered in adjustment by averaging.

675. To find the equated time for the payment of several sums of money with different terms of credit.

Ex. A owes B $1200, of which $300 is due in 4 months, $400 in 6 months and $500 in 12 months. What is the equated time for the payment of the whole sum?

FIRST METHOD.

Explanation.—Of the entire $1200 to be paid, A is entitled to the discount or use of $300 for 4 mos., which is equivalent to

$300 \times 4 = 1200$
$400 \times 6 = 2400$
$500 \times 12 = 6000$
$1200 \quad)9600$
8 mos. *Ans.*

the use of \$1.00 for 1200 mos.; of \$400 for 6 mos. or \$1.00 for 2400 mos.; and of \$500 for 12 mos. or \$1.00 for 6000 mos.; that is, A is entitled to the use of \$1.00 for 9600 mos. or its equivalent, the use of \$1200 for $\frac{1}{1200}$ of 9600 mos. = *8 mos.* Now observe that if the \$1200 be paid in 8 mos. the \$300 is paid 4 months after it is due, the \$400 2 months after it is due, and the \$500 4 months before it is due, so that A gets the use of \$300 4 mos. plus the use of \$400 2 mos., or the use of \$1.00 for 2000 months, and B gets the use of \$500 for 4 mos. or the use of \$1.00 for 2000 mos., making it equitable for both.

676. RULE.

Multiply each payment or debt by its time of credit, and divide the sum of the PRODUCTS *by the sum of the* PAYMENTS.

NOTES.—1. By the term discount, as used above, is meant mercantile discount or *simple interest in advance.*

2. The student should not get the impression that the *law* will substitute the payment of \$1200 in 8 months in the place of the express stipulations as given in the example. The object of finding the mean or average time is to show at what time the whole amount *may* be paid without loss to either, should they chose to make that arrangement.

677. If we suppose all the sums to be paid in 12 months, the time upon which the last debt becomes due, the amount to be paid will be \$300 + its interest for 8 months, 400 + its interest for 6 months, and \$500; or \$1200 + the interest of \$1 for 4800 months. It is plain that the debts will be cancelled by paying \$1200 *four* months *before* the last debt is due; or, which is the same thing, eight months *after* the first debt is due.

For convenience we have commenced at first date and discounted.

SECOND METHOD.

Discount on	\$300 for	4 months,	at	6% =	\$ 6.00
"	400 "	6	"	6% =	12.00
"	500 "	12	"	6% =	30.00
Discount on	\$1200			=	\$48.00

\$6 = Discount of \$1200 for 1 month.

\$48 ÷ 6 = 8. *Ans.* 8 months.

Explanation.—The interest of $1200, or the sum of the payments, being $6 a month, A is entitled to the use of $1200 as many months as $6 is contained times in $48=8. Hence, 8 months is the equated time.

RULE.

Find the interest of each payment, or debt, for its term of credit, and divide the amount of interest thus found by the interest of the sum of payments for one month.

NOTES.—1. As the result will be the same for any rate of interest, take that rate which is most convenient, and compute for months or days as required.

2. In both methods we have assumed the present time for a date of settlement, or as it is sometimes called *focal date*, and have discounted each sum for the time it has to run from that time. The equated date would have been the same if we had assumed as a focal date the time the first sum fell due, in which case there would have been the discount on the last two sums or $24, and it would have required as many months forward from the date of settlement as $6 is contained in $24 or 4 months, which is the same as before, as our focal date is 4 months later than in the preceding cases.

3. Or, suppose we had taken as our date of settlement the time when the last sum fell due. In that case, interest would have been computed on the first sums from the time they fell due up to the focal date, which, on the first is $12, the second the same, on both $24. Now the debtor owes $1200 plus the $24 interest, and in order to give the creditor the use of the $1200 for a sufficient length of time at the same rate to cancel the $24, which is 4 months, the equated date must be 4 months *before* the focal date. It is purely a matter of convenience as to what date be assumed as the focal date; some prefer the first date, or the last day of the month preceding the first date, and work by discount. It is convenient to assume the last day of the month preceding the first date, as that gives the exact months and days of the % as the intervening time. In taking the last date as the focal date the question becomes one of interest, and is perhaps the more easily understood by the student.

678. That the equated time obtained by both of the above methods is correct, will appear from the following proofs:

First Proof.—By paying the $1200 at the close of 8 months A *gains* the use of $300 for 4 months=$6 interest, and $400 for 2 months=$4 interest, and *loses* the use of $500 for 4 months= $10 interest. Hence, A gains $6+$4=$10 interest, and loses $10 interest. On the other hand B *loses* $6+$4=$10 interest, and *gains* $10 interest.

Second Proof.—If neither payment should be made till the

last debt is due, A would then owe B \$300 + its interest for 8 months = \$300 + \$12 = \$312; \$400 + its interest for 6 months = \$400 + \$12 = \$412; and \$500 without interest: that is, A would owe B in 12 months \$312 + \$412 + \$500 = \$1224. Now, the present worth of \$1224, four months before it is due, is \$1200. Hence, A's paying B \$1200 at the close of eight months is the same as his paying him \$1224 in 12 months, or \$300 in 4 months, \$400 in 6 months, and \$500 in 12 months.

Third Proof.—If A should pay each debt when it is due, and B lend to C the money received, at the time A's last payment is due C would owe B \$24 interest. If A should pay the *sum* of the debts, or \$1200, at the equated time (8 months), and B lend as before, he would also receive from C \$24 interest. Hence, the amount of interest is the same in either case, and 8 months is an equitable time for the payment of the debts.

679. 1. The correctness of the above methods is called in question by a number of good authors. This can be accounted for only by the well known fact that a specious error, well supported and often repeated, sometimes passes current among good scholars, without being submitted to the rigid test of examination. The following is the common method of demonstrating the incorrectness of the above methods of finding the equated time:

2. "If I owe a man \$200, \$100 of which is now due, and the other hundred in two years, the equated time is not *one* year. For in deferring the payment of the first \$100 one year I ought to pay the *amount* of \$100 for the time, which is \$106; but for the \$100 which I pay one year before it is due, I ought to pay the *present worth* of \$100, which is $\$94.33\frac{51}{53}$; and $\$106 + \$94.35\frac{51}{53} = \$200.33\frac{51}{53}$; whereas by the mercantile method I only pay \$200."

3. This argument is fallacious. For *if* I ought to pay the present worth ($\$94.33\frac{51}{53}$) of the \$100 I pay one year before it is due, I ought *not* to pay the amount (\$106) of the \$100 I pay one year after it is due. The \$6 interest in this amount is not due until the close of the two years. I ought to pay \$100 + the present worth of \$6 due in one year, which is $\$5.66\frac{2}{53}$; and $\$100 + \$5.66\frac{2}{53} + \$94.33\frac{51}{53} = \$200$.

4. The mistake is in considering the sums of money payable at different times as *separate* from each other; whereas, by the very nature of the problem of finding a *common time* of payment,

they must be regarded as parts of the *same contract*. Suppose, for example, I buy a horse, and agree to pay $100 in one hour, and $100 in two years, without interest. Failing to pay the $100 due in one hour until the close of one year, which I then pay *without interest,* how much must I pay at the close of the second year? Evidently $106 (if the legal rate is 6), since I paid at the close of one year only the principal ($100), leaving the interest ($6) unpaid, which cannot draw interest. Now, in finding the equated time for the payment of several debts due at different dates, the question is to find a time for the payment of the several *principals without interest.* Instead of paying the *amount* of $100 in the problem proposed, the principal alone is paid.

5. The following is given by these authors as the "only accurate rule:"

"*Find the present worth of each of the given amounts due; then find in what time the sum of these present worths will amount to the sum of all the payments.*"

The inaccuracy of this "accurate rule," tested by the logic of its authors, will appear from the following:

The equated time for the payment of $200, $100 of which is now due, and the other $100 due in two years, as found by this rule, is 11.32075 months. Now, the amount of $100 for 11.32075 months at 6 per cent., is $105.660387; the present worth of the other $100, due in 12.67925 months, is $94.03832, and $105.660387 + 94.03832 = $199.698707, whereas it ought to be $200.

It is also evident that the equated time, as found by this "accurate" rule, will not be the same for all rates of interest. At 50 per cent. the equated time of the above example is 8 months, and the error, by the above test, $8.33⅓; at 100 per cent. it is 6 months, with an error of $10.

6. This supposed accurate rule is based upon the principle that the amount to be paid on a debt due at a future date, without interest, *at any time previous to this date,* is the *present worth* of the debt at any prior date, plus the interest of the present worth up to date of payment. The incorrectness of this principle is easily shown. Suppose I owe a man $100, due in two years, without interest; how much ought I to pay in one year?

The present worth of $100, due in two years (at 6 per cent.), is $89.2857, and the interest on this sum for one year is $5.3571;

hence the sum to be paid is $89.2857 + $5.3571 = $94.6428. The true amount to be paid, however, is the present worth of $100, due in one year, which is $94.339.

680. In finding the equated time for the payment of a bill of goods or of an account current, the exact number of days between the different dates is used. The pupil may commence with the first dates or with the last. In commencing with the first dates, each item, except the first, is subject to *discount*; if the last date is taken, each item, except the last, draws *interest*.

681. Ex. A merchant sold goods to one of his customers, at different dates, as by the statement annexed. What is the average time for the payment of the same?

June 16, 1871,	a bill	amounting	to	$500,	on credit.
" 30,	"	"	"	220,	"
July 30,	"	"	"	300,	"
Aug. 15,	"	"	"	250,	"
Sept. 1,	"	"	"	112,	"
Oct. 1,	"	"	"	100,	"

1. OPERATION BY FIRST METHOD.

		Days.	Days.
June 16, 1871,	$500 ×	00	
" 30, "	220 ×	14 =	3080
July 30, "	300 ×	44 =	13200
Aug. 15, "	250 ×	60 =	15000
Sept. 1, "	112 ×	77 =	8624
Oct. 1, "	100 ×	107 =	10700
	1482)		50604 (34 days.
			4446
			6144
			5928

Counting forward 34 days from June 16, the date of the first bill, we have July 20, the equated time for the payment of the above bills.

NOTE.—A little reflection will make it evident that the above example is similar to one requiring the equated time for the payment of $500 June 16, the assumed date of settlement; $220 due in 14 days; $300 due in 44 days; $250 due in 60 days, $112 due in 77 days; and $100 due in 107 days. The average date of *purchase* of several bills is found in the same manner.

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2. OPERATION BY SECOND METHOD.

				Days.	Disc't.
June 16, 1871,	$500	for		00=	
" 30, "	220	"		14=	$ .51
July 30, "	300	"		44=	2.20
Aug. 15, "	250	"		60=	2.50
Sept. 1, "	112	"		77=	1.44
Oct. 1, "	100	"		107=	1.78
	$1482				$8.43

$14.82 dis. for 60 days.
.247 " 1 day.

```
.247 ) 8.430 ( 34     Ans. 34 days.
       7.41
       -----
       1.020
        .988
       -----
         32
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Counting forward 34 days from June 16, we have July 20, the equated time.

NOTES.—1. The result will be the same at any rate of interest or discount, as is illustrated in the foregoing example. If a settlement is assumed June 16, the seller is entitled to $1482 from the buyer, and the buyer to $8.43 discount from the seller. But instead of paying the buyer $8.43 in cash on the assumed date, the seller proposes to give the buyer the use of $1482 a sufficient length of time to cancel this discount. The same rate being taken in estimating the use of the money in both cases, it would make no difference what the rate was. It is generally most convenient to compute the interest at 6 or 12 per cent.

2. When the time is in days, as in the second example, the interest is readily found *by removing the point, or separatrix, two places to the left, and taking such aliquot parts of the result as the given days are of 60 days.*

Suppose, for example, we wish to find the interest of $230.60 for 39 days. Since $39 = 30 + 6 + 3$, the interest for 39 days will be the sum of $\frac{1}{2}$, $\frac{1}{10}$ and $\frac{1}{20}$ of the interest for 60 days. Thus:

Interest for 60 days		=	$2.306
" " 30 "	($\frac{1}{2}$)	=	1.153
" " 6 "	($\frac{1}{10}$)	=	.231
" " 3 "	($\frac{1}{20}$)	=	.115
" " 39 "		=	$1.50

3. If the equated time contains a fraction greater than $\frac{1}{2}$ add 1 to the number of days; if less than $\frac{1}{2}$ disregard it.

682. EXAMPLES.

1. I owe $450, due in 6 months; $300, due in 8 months; $125 due in 10 months; and $100, due in 12 months. What is the equated time for payment?

2. Bought a farm for $3500; $\frac{1}{4}$ of it is to be paid down, $\frac{1}{5}$ of it in 8 months, $\frac{1}{3}$ in 12 months, and the remainder in 15 months, without interest. What is the equated time for the payment of the whole?

3. A merchant owes a bank $1500, of which $300 is due in 30 days, $250 in 45 days, $350 in 60 days, $450 in 80 days, and $150 in 90 days. What is the equated time for the payment of the whole?

4. Bought of Ivison & Phinney the following bill of goods:

June 3, 1871,	a bill	amounting	to	$300
July 1,	"	"	"	220
" 20,	"	"	"	400
Aug. 15,	"	"	"	330
Sept. 13,	"	"	"	240

What is the *average date* of purchase?

5. A merchant has charged on his ledger $120, due May 15, 1871; $90, due July 3, 1871; $75, due Aug. 30, 1871; $60, due Sept. 10, 1871; $160, due Oct. 18, 1871; $150, due Dec. 20, 1871. What is the equated time for the payment of these accounts?

6. L. T. Perry bought of Mansell & Dean:

March 3, 1868,	Mdse.	for	$250
April 15,	"	"	180
June 20,	"	"	325
Aug. 10,	"	"	80
Sept. 1,	"	"	100

What is the equated time for all these bills?

683. To find the equated time for the payment of several sales, made at different dates, and for different terms of credit.

Ex. 1. James Russell bought of Fink, Hall & Co., several bills of goods, as below stated:

April 3, 1868,	a bill of	$220, on 3	months'	credit.
May 1,	"	125, on 5	"	"
" 15,	"	200, on 6	"	"
June 24,	"	140, on 8	"	"
July 1,	"	190, on 9	"	"

What is the equated time of payment?

OPERATION.

				Days.	Days.
Due,	July	3, 1868,	$220 ×	00=	
"	Oct.	1, "	125 ×	90=	11250
"	Nov.	15, "	200 ×	135=	27000
"	Feb.	24, 1869,	140 ×	236=	33040
"	April	1, "	190 ×	272=	51680
			875)		122970 (141 nearly.
					875
					3547
					3500
					470

The equated time for the payment of the above bills is 141 days from July 3, which is Nov. 21.

METHOD BY DISCOUNT.

					Dis.
Due,	July	3, 1868,	$220 for	00=	
"	Oct.	1, "	125 "	90=	$1.88
"	Nov.	15, "	200 "	135=	4.50
"	Feb.	24, 1869,	140 "	236=	5.51
"	April	1, "	190 "	272=	8.61
			$875	.1458)	$20.5000 (141 days.
			8.75		1458
			$(\frac{1}{60})$.1458		5920
					5832
					880

141 days from July 3, is Nov. 21, the equated time as above.

Explanation.—The bill of $220 falls due 3 months from April 3, which is July 3; the bill of $125 falls due 5 months from May 1, which is Oct. 1, and so on: the *time of maturity* of each bill being found by adding its term of credit to its date of purchase. The *average* time of maturity is the equated time for the payment of the bills.

RULE.

First find the MATURITY *of each bill (or the time when it falls due) and then proceed as in the previous case. The equated time is found by counting forward from the date of the first amount falling due if that be taken as a focal date.*

NOTES.—1. The bill having the earliest *date* does not always fall *due* first. It sometimes happens that the term of credit of the first bill is longer than that of the succeeding bills. It is most convenient to arrange the statement of maturity so that the bill *maturing* first shall stand first.

2. The equated time for the payment of several bills may be found by commencing at *the last date and finding how long each bill draws interest.* Thus, the last example may be equated as follows:

				Days.		Days.
Due,	April 1, 1869,	$190	×	00	=	
"	Feb. 24, "	140	×	36	=	5040
"	Nov. 15, 1868,	200	×	137	=	27400
"	Oct. 1, "	125	×	182	=	22750
"	July 2, "	220	×	273	=	60060

875) 115250 (132 nearly.
875
2775
2625
1500

The equated time is 132 days previous to April 1, 1869, which is Nov. 20, 1868. The difference of *one* day between the results of the two methods is due to the fractional parts of days being omitted.

684. EXAMPLES.

1. T. W. Cook & Co. sold to Murray & Co. several bills of goods, as shown in the statement annexed. What is the average time of maturity?

April 15, 1867, a bill amounting to $450, on 5 months' credit.
June 16, " " " 560, on 2 " "
July 31, " " " 180, on 6 " "
Sept. 19, " " " 760, on 5 " "

2. Bought goods of Smith & Moore, at sundry times, and on different terms of credit, as follows:

Dec. 18, 1870, a bill of $375.50, on 6 months' credit.
Jan. 10, 1871, " 290.60, on 6 " "
March 13, " " 800.00, on 8 " "
April 30, " " 650.80, on 7 " "
June 15, " " 460.25, on 4 " "

What is the equated time for the payment of the whole?

3. O. Blake & Co. sold goods to J. B. Foster, at sundry times, and on different terms of credit, as follows:

Sept. 30, 1868,	a bill of	$ 80.75,	on 4	months'	credit.
Nov. 3, "	"	150.00,	on 5	"	"
Jan. 1, 1869,	"	30.80,	on 6	"	"
March 10, "	"	40.50,	on 5	"	"
April 25, "	"	60.30,	on 4	"	"

How much will balance the account June 2, 1869?

NOTE.—The equated time for the payment of the above account is May 5, 1869; hence the several bills above are equivalent to a bill of $362.35 due May 5. It is evident that the $362.35 should draw interest from May 5 to June 2, the time of settlement. When it is required to know the amount due at any date *previous* to the equated time, the *present worth** of the sum of the several bills must be found.

4. A merchant sold to one of his customers several bills of goods, as follows:

May 9, 1871,	a bill of	$340	on 4	months'	credit.
June 6, "	"	400	on 3	"	"
July 8, "	"	345	on 5	"	"
Aug. 30, "	"	130	on 5	"	"
Sept. 30, "	"	240	on 6	"	"

How much money will balance the account Jan. 1, 1872?

5. J. D. Stuart bought of Geo. A. Davis & Co. several bills of goods, as follows:

March 3, 1870,	a bill of	$250,	on 3	months'	credit.
April 15, "	"	180,	on 4	"	"
June 20, "	"	325,	on 3	"	"
Aug. 10, "	"	80,	on 3	"	"
Sept. 1, "	"	100,	on 4	"	"

What is the equated time of payment, and how much money would balance the account July 1, 1870?

6. Purchased goods of a merchant at sundry times and on different terms of credit, as follows:

Nov. 9, 1867,	a bill of	$ 20.00	on 5	months'	credit.
" 30, "	"	50.60	on 3	"	"
Dec. 31, "	"	90.00	on 4	"	"
Feb. 1, 1868,	"	120.00	on 3	"	"

What is the average date of *purchase*, and what the average time of *maturity*?

* The mercantile method of finding the present worth in such cases is to deduct *interest* for the time.

7. A merchant sold goods to one of his customers, as stated below:

April 6, 1867,	a bill of	$450,	on 4 months'	credit.
May 12,	"	600,	"	"
June 20,	"	750,	"	"
Aug. 1,	"	300,	"	"

When must a note for the whole be made payable?

NOTE.—When the sales have the *same* term of credit, as in the above example, it is most convenient to find *first the average date of purchase.* The equated time of payment is then readily found by adding the common term of credit to this average date of purchase. The average date of purchase in the above example is 54 days from April 6, which is May 30; the equated time of payment is 4 months from May 30, which is Sept. 30.

The days of grace generally allowed may be added to the equated time.

8. Sold John Smith, on a credit of 90 days, the following bills of goods:

Jan. 10, 1868,	a bill of	$20.
April 12,	"	45.
May 27,	"	60.
June 30,	"	75.

What is the equated time of payment?

9. Purchased goods of Stratton & Co., at different dates, and on a credit of 6 months, as below stated:

Oct. 12, 1868,	a bill of	$460	on 6 months'	credit.
" 30,	"	95	"	"
Dec. 1,	"	180	"	"
" 25,	"	390	"	"
Jan. 20, 1869,	"	410	"	"

How much money will balance the account July 1, 1869?

685. To find what *extension* should be granted to the balance of a debt, partial payments having been made before the debt was due.

EX. A owed B $1200, due in 6 months, but to accommodate him paid $400 in 2 months. When ought the balance to be paid?

Explanation.—Since A paid B $400 four months *before* it was due, B, at the close of the 6 months, owed A the interest of $1 for 400×4 months $= 1600$ months. To balance this interest due A, he can keep the $800 unpaid $\frac{1}{800}$ of 1600 months $= 2$ months after the debt is due.

RULE.

Multiply each payment by the time it was paid before due, and divide the sum of the products by the balance unpaid.

686. EXAMPLES.

1. Singer & Morton sold Wm. Williams, June 10, 1868, goods to the amount of \$1300, on 6 months credit. Aug. 20, Mr. Williams paid \$200; Sept. 18, \$250; Oct. 30, \$350. When, in equity, ought the balance to be paid?

OPERATION.

		Days.		
200	×	112	=	22400
250	×	83	=	20750
350	×	41	=	14350
\$800				57500

$57500 \div 500 = 115$

The balance ought to be paid 115 days from Dec. 10, 1868, which is April 4, 1869.

2. A sold B, July 1, 1871, goods to the amount of \$1500, on a credit of 90 days. Aug. 5, B paid \$400; Sept. 3, \$600; Sept. 15, \$300. When ought B to pay the balance?

3. A merchant sells a customer to the amount of \$600, $\frac{1}{2}$ of which is to be paid in 3 months, $\frac{1}{3}$ in 4 months, and the balance in 7 months. The customer pays $\frac{1}{2}$ down. How long may he keep, in equity, the remainder?

4. A owes B \$600, payable in 6 months. At the close of 3 months he wishes to make a payment so as to extend the time of the balance to one year. How great a payment must B make?

NOTE.—B wishes to pay such a sum of money three months *before* it is due, as will extend another sum 6 months *after* it is due. It is evident the sum *paid* must be twice as great as the sum *extended*. Divide \$600 into two parts, which shall be to each other as 2 to 1.

5. A owes B \$1000, payable in 6 months. At the close of 2 months A pays B \$1200, and B gives A his note for the balance. When ought the note to be dated?

NOTE.—Since A paid B \$1200 four months before the \$1000 was due, B, at the close of the 6 months, owed A the interest of \$1200 for 4 months, or

\$1 for 4800 months. It is evident that a note for the balance, \$1200 − \$1000 = \$200, must be dated $\frac{1}{200}$ of 4800 months, or 24 months *previous* to the time the \$1000 was due, or must draw interest from that time.

6. July 10, 1858, A paid B \$600; Sept. 12, 1858, B paid A \$800. When ought A to pay the balance?

NOTE.—Sept. 12, B owed A \$600 + its interest for 64 days. He paid A \$600 + \$200. Hence, A is entitled to the use of the balance (\$200) until its interest equals the interest of \$600 for 64 days, or 192 days. 192 days from Sept. 12, 1858, is March 23, 1859.

7. July 10, 1868, A paid B \$800; Sept. 12, 1868, B paid A \$600. What should be the date of a note for the balance?

NOTE.—Sept. 12, B owed A \$800 + its interest for 64 days. He paid A but \$600. Hence, he owes A the balance (\$200) and the interest of \$800 for 64 days, or the interest of \$200 for 256 days. A note for the balance must therefore begin to draw interest 256 days *previous* to Sept. 12, 1868, which is Dec. 30, 1867.

Remark.—The above seven examples, if well understood, will aid the student in equating accounts which contain both *debits* and *credits.*

EQUATION OF ACCOUNTS.

687. 1. Equation of accounts (also called "Averaging of Accounts," and "Compound Equation of Payments") is the process of finding at what time the *balance of an account* can be paid without loss to the debtor or creditor.

2. The *balance* of an account is the difference between the sum of the debits and credits, or what one party owes the other in excess.

3. It is a principle of common law that a debt does not draw interest after it falls due, with the exception of negotiable paper. If the balance of an account is not paid when it is demanded, still it does not draw interest. In consequence of the hardship the common law imposes on the creditor, many of the States have passed statutes giving him the right to compute interest on the balance of his account, after he has submitted a statement of it to the debtor, and obtained his acknowledgment of its correctness. The courts would not enforce the payment of interest on the balance of account from the equated date to the commencement of the suit, nor would they defer the payment of the balance until the equated date, should it fall subsequent to the last transaction. All that

can be claimed is simply the balance without interest, which is due at any time, unless changed by express agreement of the parties, or by a statutory provision giving interest, provided the conditions of the statute have been strictly complied with.

4. Since the debit and credit sides of an account are respectively equivalent to the sum of their several items, due at the *equated time* (Art. 684, Ex. 3, Note), the *first step* in equating accounts is to find the time when each side of the account becomes due.

5. This may be found by equating each side of the account, *without any reference to the other*, commencing either at the *first* or the *last* date of each, or by using the *first* or *last* date of the account as a common *starting-point* for both sides.

The date from which the intervals of time are reckoned is commonly called the *focal date.*

When each side of the account is equated separately, the process may be called that of *double equation;* when both sides are equated from one focal date, the process may be called that of *single equation.*

688. Ex.

Dr. D. B. CLARK & Co. *in account with* G. C. COOK. *Cr.*

1871.			Time of credit.	1870.			
April 3	To Mdse.	$220	3 mo.	July 1	By Cash	$200	
May 1	"	125	5 "	Oct. 3	"	150	
" 15	"	200	6 "	Dec. 20	"	300	
June 24	"	140	8 "				
July 1	"	190	9 "				

1. ***First Method by Double Equation.*** In the following solution we have taken the earliest date when any item became due on each side of the account for the *focal date* of *that* side, and have equated each side by itself.

Debits.

Due.		
July 3, 1870,	$220 × 00=	
Oct. 1, "	125 × 90=	11250
Nov. 15, "	200 × 135=	27000
Feb. 24, 1871,	140 × 236=	33040
April 1, "	190 × 272=	51680
	$875	)122970
		141 d.

Debits are due 141 days from July 3, which is Nov. 21.

Credits.

Due.		
July 1, 1870,	$200 × 00=	
Oct. 3, "	150 × 94=	14100
Dec. 20, "	300 × 172=	51600
	$650	)65700
		101 d.

Credits are due 101 days from July 1, which is Oct. 10.

By thus equating each side, the account will stand as follows:

Dr.		*Cr.*	
Due, Nov. 21, 1870, . .	$875	Due, Oct. 10, 1870, . .	$650.

2. ***Second Method by Double Equation.*** In this operation we use the earliest date when any item of *either* side of the account *becomes due*, as a *focal date*.

Debits.

Due.		
July 3, 1870,	$220 × 2 =	440
Oct. 1, "	125 × 92 =	11500
Nov. 15, "	200 × 137 =	27400
Feb. 24, 1871,	140 × 238 =	33320
April 1, "	190 × 274 =	52060
	$875	124720
		143 d.

Debits due 143 days from July 1, which is Nov. 21.

Credits.

Due.		
July 1, 1870,	$200 × 00 =	
Oct. 3, "	150 × 94 =	14100
Dec. 20, "	300 × 172 =	51600
	$650	)65700
		101 d.

Credits due 101 days from July 1 which is Oct. 10.

The account thus equated stands as before:

Dr.		*Cr.*	
Due, Nov. 21, ...	$875.	Due, Oct. 10, ...	$650.

3. The next step is to find when the balance of the account, as thus equated, becomes due.

Debits due Nov 21, ...	$875
Credits, ...	650
Balance, ...	$225

Difference in time, from Oct. 10 to Nov. 21 = 42 days.

Due, Oct. 10, 650
42
1300
2600
225)27300
121 days.

4. *Or thus, by Discount:*

Using 12% as the most convenient rate for computation, we have

Discount or interest of $650 for 42 da. = $650 × .014 = $9.10.

Time that the balance $225 should be kept = ($9.10 : $225) × 3 = 121 days; or $9.10 ÷ .075 (disc't of $225 for 1 day) = 121 days.

Thus the balance is due 121 days from Nov. 21, 1870, which is March 22, 1871.

5. *Explanation.*—Assume the account settled Nov. 21, the *latest* date. The credit side of the account has been due from Oct. 10 to Nov. 21, or 42 days. Nov. 21, the credit side is equal

to $650 and the interest of the same 42 days. That the debit side of the account may be increased by an equal amount of interest, it is evident that the balance of the account must remain unpaid 121 days after Nov. 21, or the 121 days must be counted *forward* from Nov. 21.

6. *Or thus:*

The above account may be stated as follows: Oct. 10, 1870, G. C. Cook paid D. B. Clark & Co. $650; Nov. 21, 1870, D. B. Clark & Co. paid G. C. Cook $875. Now, since D. B. C. & Co. had the use of $650 for 42 days, G. C. C. is entitled to the use of $225 (the balance) until its interest equals the interest of $650 for 42 days, which is 121 days. 121 days from Nov. 21, 1870, is March 21, 1871.

7. PROOF.

	Dr.		*Cr.*
Due Nov. 21, ...	$875.	Due Oct. 10, ...	$650.
Int. on $225 to March 21,	35.30	Int. on $650 to March 21,	35.30
	$910.30	Balance, ...	225.
			$910.30

Suppose the account be *reversed* so as to stand thus:

	Dr.		*Cr.*
Due Nov. 21, 1870 . .	$650.	Due Oct. 10, 1870, . .	$875.

8. What is the equated time for the payment of the balance now due?

Credits, ...	$875	875
Debits, ...	650	42
Balance, ...	$225	225)36750(163 days.
Difference in time, 42 days.		225
		1425
		1350
		750
		675

Balance due 163 days *previous* to Nov. 21, 1870, which is June 11, 1870.

9. *Explanation.*—Suppose the account settled Nov. 21. The credit side is equal to $875 and its interest from Oct. 10 to Nov. 21, or 42 days. That the debit side of the account may be increased by an equal amount of interest, the balance of the account must be regarded as due 163 days *previous* to Nov. 21, or June 11.

10. *Or thus:*

Oct. 10, 1870, G. C. Cook paid D. B. Clark & Co. \$875; Nov. 21, 1870, D. B. C. & Co. paid G. C. C. \$650. Since D. B. C. & Co. had the use of \$875 for 42 days, G. C. C. is entitled to the interest of \$225 (the balance) for 163 days. Hence, the balance must be regarded as due 163 days *previous* to Nov. 21. The simple question is: How long must \$225 be on interest to equal the interest of \$875 for 42 days?

NOTE.—If D. B. Clark & Co. should wish to give their note for the balance, it is evident the note may in equity be dated June 11, 1870, or may draw interest from that time.

689. RULE.

First find the equated time for each side of the account separately. Then multiply the amount due on that side of the account which falls due FIRST, *by the number of days between the dates of equated time, and divide the product by the balance of the account. The quotient will be the number of days to be counted* FORWARD *from the* LATEST DATE *when the* SMALLER *side of the account falls due* FIRST; *and* BACKWARD *when the* LARGER *side falls due* FIRST.

NOTE.—Some authors give the following *rule*:—Multiply the *smaller* side of the account by the number of days between the dates of equated time, and divide the product by the balance of the account. The quotient will be the time for consideration. From the equated date of the *larger* side, count FORWARD when that side becomes due *last*, but BACKWARD when it becomes due *first*.

690. *Method by Single Equation.* The equated time for the payment of the balance of an account may be found *directly* without averaging the debit and credit items separately, by the following method:

Dr.

Due.				
July 3, 1870,	\$220	× 2	=	440
Oct. 1, "	125	× 92	=	11500
Nov 15, "	200	× 137	=	27400
Feb 24, 1871,	140	× 238	=	33320
April 1, "	190	× 274	=	52060
	\$875			124720
	650			65700
	\$225			59020

Cr.

Due.				
July 1, 1870,	\$200	× 0	=	
Oct. 3, "	150	× 94	=	14100
Dec. 20, "	300	× 172	=	51600
	\$650			65700

$59020 \div 225 = 262$.

262 days from July 1, 1870, is March 21, 1870.

Explanation.—We assume July 1, 1870, (the earliest date upon which any item becomes due), as the focal date or time upon which *all* the items of the account become due. The interest of the debit items, from this assumed date of maturity to the time they respectively become due, equals the interest of \$1 for 124720 days; the interest of the credit items equals the interest of \$1 for 65700 days. Hence, the balance of interest in favor of the debit side equals the interest of \$1 for 59020 days, or of \$225 for $\frac{1}{225}$ of 59020 days—262 days. Since the balance of items is also in favor of the debit side, it is evident it may remain unpaid 262 days without interest, or will become due 262 days from July 1, 1870, which is March 21, 1871. If the balance of items had been on the credit side it would have been due 262 days *previous* to July 1, 1870.

691. RULE.

Assume the earliest date upon which any item of the account becomes due to be the time of maturity for all the items.

Multiply each item by the number of days intervening between this assumed date and the date upon which it becomes due, and find the sum of these products on each side of the account. Then divide the DIFFERENCE *between the sums of the debit and credit products by the balance of the account; the quotient will be the time for consideration or average term of credit.*

When the difference of products and the balance of the account fall on the SAME *side count* FORWARD; *when on* OPPOSITE *sides count* BACKWARD; that is, *when the balance of account and balance of interest or discount go to the same party count backward; when they go to opposite parties count forward.*

NOTE.—Either the *earliest* or the *latest* date may be used as a focal date.

692. 1. The following method is used by some accountants. It is a modification of the last method by single equation.

INTEREST RULE.

Compute the interest at 12% on both sides of the account, from the last day of the month preceding the date of the earliest transaction in the account, to the time when each item becomes due. Divide the balance of interest by the interest on the balance of the

account for one month. The quotient will be the average term of credit in months to be reckoned forward from the focal date, if the balances of interest and of the account be on the same side, and backward, if these balances be on different sides of the account.

NOTE.—If only *days* are concerned, divide by the interest on the balance for *one day* and the quotient will be the average term of credit in *days*.

2. EXAMPLE.

Dr.

	Time from focal date to date of items. Mos.	Days.					
1862.	0	July 27.	Mdse. 4 mos., . . .	$1350	$54.00	Int. for	4 mos.
					12.15	"	27 das.
	4	Nov. 12.	" 6 " . . .	2531	253.10	"	10 mos.
					10.12	"	12 das.
1863.	6	Jan. 18.	" 5 " . . .	1940	213.40	"	11 mos.
					11.64	"	18 das.
	9	Apr. 21.	Cash, ...	1170	105.30	"	9 mos.
					8.19	"	21 das.
				$6991	$667.90		

Cr.

1862.	1	Aug. 9.	Cash, ...	$750	$ 7.50	Int. for	1 mo.
					2.25	"	9 das.
	3	Oct. 5.	Dft. 90 d. (3 d. grace),	961	57.66	"	6 mos.
					2.56	"	8 das.
1863.	9	Apr. 6.	Cash, ...	850	76.50	"	9 mos.
					1.70	"	6 das.
	18	Aug. 15.	Note 60 d. (3 d. grace),	500	75.00	"	15 mos.
					3.00	"	18 das.
				$3061	$226.17		
			Balance of account,	$3930	$441.73	Balance of int.	

Interest on balance of account for 1 mo. is $39.30, and $441.73÷$39.30= 11.24; hence the balance is due 11 months 7 days from June 30, 1862; that is *June 7, 1863.*

693. EXAMPLES.

1. A has with B an account, which, when each side is equated, stands as follows:

Dr.		*Cr.*	
Due, June 5, . . .	$1285	Due, June 24, . .	$1080.

What is the equated time of payment for the balance?

2. C has with D an account, the debit and credit sides of which, when equated, are as follows:

	Dr.		*Cr.*
Due, Jan. 7	$325.	Due, Jan. 11, . . .	$1090.

What must be the date of a note for the balance?

3. What is the equated time for the payment of the balance of an account, which, when the two sides are equated, stands as follows:

	Dr.		*Cr.*
Due, July 12, . . .	$450.	Due, Sept. 1, . . .	$800.

4. At what time will the balance of the following account commence drawing interest?

	Dr.		*Cr.*
Due, Oct. 15, . . .	$1260.	Due, Nov. 20, . . .	$900.

5. What is the equated time for the payment of the balance of the following account, the merchandise items having a credit of 4 months?

Dr. R. BILL & CO. *in account with* ORVIL BLAKE. *Cr.*

1870.				1871.			
May 1	To Mdse.	$850	70	Jan. 1	By Cash.	$500	00
June 6	"	340	75	" 19	"	440	00
July 3	"	180	25	Feb. 1	"	100	00
Aug. 13	"	500	00	" 15	"	980	00
20	"	340	40				
30	"	80	00				

NOTE.—In finding the equated time, when the cents are less than 50 reject them; when more, add $1. The work will be sufficiently accurate.

6. When will the balance of the following account commence drawing interest, allowing that each item was due from date? What will balance the account Oct. 1?

Dr. A *in account with* B. *Cr.*

1870.				1870.			
July 10	To Mdse.	$120	00	Aug. 20	By Cash.	$350	00
" 30	"	450	00	Sept. 25	" Mdse.	250	00
Aug. 30	"	380	00	Oct. 3	" Cash.	950	00
Sept. 9	"	560	00				
" 30	"	400	00				

NOTE.—Since the balance of the above account commences to draw interest at the *equated time* of the account, it is evident that the *cash value*

this balance, at any date *subsequent* to the equated time of the account may be found *by adding to the balance its interest up to date*; and at any date *previous* to the equated time, *by deducting from the balance its interest for the intervening time.* By mercantile custom interest is deducted, instead of finding the *true present worth*, when money is paid before it is due.

7. When will the balance of the following account commence drawing interest? What will be the *cash value* of the balance, Jan. 1, 1869? Credit of 90 days on merchandise items.

Dr. B *in account with* C. *Cr.*

1868.					1868.				
Aug.	18	To Mdse.	$ 50	00	Oct. 7	By Cash.	$200	00
Sept.	15	"	140	00	" 30	"	100	00
"	30	"	80	00	Dec. 1	"	400	00
Oct.	8	"	200	00				
Nov.	1	"	350	00				

In the following eight examples are given the *footings* of accounts on A's Ledger. The footings of each side of the accounts are given, with the equated date when these footings are due, interest to be computed at 6%, as per agreement between A and each of his customers. When is the balance of each account due?

Dr. 8. B's Acct. *Cr.*
Due, Jan. 1st. . . Footings, $800. | Due, April 1st. . Footings, $200.

Dr. 9. C's Acct. *Cr.*
Due, Jan. 12. . . Footings, $1200. | Due, April 12. . Footings, $1800.

Dr 10. D's Acct. *Cr.*
Due, April 20. . Footings, $2800 | Due, Jan. 20. . . Footings, $2200.

Dr. 11. E's Acct. *Cr.*
Due, April 30 . Footings, $3200. | Due, Jan. 30. . . Footings, $3800.

Dr. 12. F's Acct. *Cr.*
Due, July 15/70. Footings, $896. | Due, Mar. 1/71. . Footings, $1240.

Dr. 13. G's Acct. *Cr.*
Due, Sept. 8/69. . Footings, $2946. | Due May 22/71. . Footings, $728.

Dr. 14. H's Acct. *Cr.*
Due, Aug. 17/71. Footings, $9876. | Due, Nov. 2/70. . Footings, $3684.

Dr. 15. I's Acct. *Cr.*
Due, June 1/70. Footings, $44.682. | Due, Jan. 1/72. Footings, $31.896.

16. The following questions are to be applied to each of the above accounts, standing on A's Ledger.

a. Which becomes indebted to the other first—A to B, or B to A? (A to C, or C to A, etc.)

b. For what amount?

c. How long was he owing this amount before the opposite side was due?

d. How much cash should be paid to settle the account in full, if the settlement be made at the later of the two dates?

e. For what amount should a note be given to settle the account, if it be dated at the later of the two dates?

f. Why is it necessary to date a note given in settlement of the *balance* of an account at any other than the later date at which the sides average due?

g. What should be the date of a note, if the note be given in settlement of an account, and the face of the note be for the difference in the footings of the account?

h. What should be the date of a note given to settle an account, the face of the note being for the sum of the side of the account having the earlier date, together with the interest on that side from its date to the date of the later side of the account?

CASH BALANCE.

694. The difference between the sum of the debit side of an account, plus the sum of the interest on each of its items up to the date of settlement, and the sum of the credit side, and interest on its items, is called the *cash balance* of the account.

The *cash* balance of an account at a particular date may be found directly as follows:

Ex.

Dr. MURRAY & CO. *in account with* JONES & SONS. *Cr.*

1871.			1871.		
April 10	To Mdse.	$150	April 12	By Cash.	$250
" 30	"	400	May 1	"	180
May 16	"	90	June 7	"	400
" 24	"	100	" 25	"	564
June 1	"	300			
" 10	"	340			
" 26	"	200			

What will be the true balance of the above account July 1, 1871, the time of settlement, allowing that each item draws interest from its date, at 6 per cent.?

695. 1. OPERATION.

Debits.

Due.		Days.	
April 10,	$150 × 82 =	12300	
" 30,	400 × 62 =	24800	
May 16,	90 × 46 =	4140	
" 24,	100 × 38 =	3800	
June 1,	300 × 30 =	9000	
" 10,	340 × 21 =	7140	
" 26,	200 × 5	1000	
	$1580	6)62180	
Add int.	10.36	$10.364	
	$1590.36		

Credits.

Due.			
April 12,	$250 × 80 =	20000	
May 1,	180 × 61 =	10980	
June 7,	400 × 24 =	9600	
" 25,	564 × 6 =	3384	
	$1394	6)43964	
Add int.	7.33	7.327	
	$1401.33		

Balance = $1590.36 − $1401.33 = $189.03.

Explanation.—Since each item of the debit side of the account was on interest from its date to the time of settlement, the total interest of the several debit items equals the interest of $1 for 62180 days, which is $10.364. (The interest of $1 for 6 days at 6% is 1 mill; hence, the interest of $1 for 62180 days is found by dividing 62180 by 6, and pointing off three decimal places.) The total interest of the several credit items equals the interest of $1 for 43964 days, which is $7.327. Now increase each side of the account by its interest, then find the balance.

2. METHOD BY INTEREST.

Due.		Days.	6% Int.
April 10,	$150 for 82	=	$2.05
" 30,	400 for 62		4.133
May 16,	90 for 46		.69
" 24,	100 for 38	=	.634
June 1,	300 for 30		1.50
" 10,	340 for 21	=	1.19
" 26,	200 for 5		.17
	$1580		$10.367
	10.37		
	$1590.37		

Due.		Days.	6% Int.
April 12,	$250 for 80	=	$3.333
May 1,	180 for 61	=	1.83
June 7,	400 for 24		1.60
" 25,	564 for 6		.564
	$1394		$7.327
	7.33		
	$1401.33		

$1590.37 − $1401.33 = $189.04, the true balance.

NOTE.—The "method by interest" will generally be found most convenient either for finding the equated time for the payment of the balance of accounts, or for finding the cash balance.

696. The above account, when balanced by interest, may be presented as follows:

Dr. MURRAY & Co. *in account with* JONES & SONS. *Cr.*

1871.		Am't.	Ds.	Int.	1871.			Ds.	Int.
April 10	To Mdse.	$150.00	82	$2.05	April 12	By Cash.	$250.00	80	$3.333
" 30	"	400.00	62	4.133	May 1	"	180.00	61	1.83
May 16	"	90.00	46	.69	June 7	"	400.00	24	1.60
" 24	"	100.00	38	.634	" 25	"	564.00	6	.564
June 1	"	300.00	30	1.50	July 1	bal. acc't.	189.04		
" 10	"	340.00	21	1.19					$7.327
" 26	"	200.00	5	.17					
July 1	bal. by int.	3.04							
		$1583.04		$10.367			$1583.04		

Errors excepted. PORTSMOUTH, July 1, 1871. JONES & SON.

697. RULE.

1. *Multiply each item of the account by the number of days intervening between the date on which it becomes due and the time of settlement. Divide the sums of the debit and credit products respectively by 6: the quotient will be the interest of the two sides of the account, at 6 per cent., expressed in* MILLS. *Reduce to the interest at any required rate.*

Add the interest on each side to that side of the account. The difference of the amounts of the two sides of the account thus found, will be the cash balance. Or,

2. *Find the interest of each item from the date on which it becomes due to the time of settlement. The difference between the sums of interests on the debit and credit sides of the account will be the* BALANCE OF INTEREST.

When the balance of interest falls on the same side as the balance of items, the cash balance will be their SUM; *when on opposite sides, their* DIFFERENCE.

EXAMPLES.

1. The following account was settled July 1, 1871. What was the cash balance, interest being computed on each item from date at 7%?

Dr. JAMES KEHOE *in account with* J. SMITH. *Cr.*

1871.			Ds.	Int. or prod.	1871.			Ds.	Int. or prod.
Jan. 7	To bal. of acc't.	$120.00			April 1	By cash.	$140.00		
" 15	" mdse.	96.75			" 30	" "	50.00		
" 24	" bills payable	130.50			May 20	" order on T. S.	140.00		
Feb. 27	" mdse.	200.80			" 31	" cash.	450.00		
March 7	" "	80.00			June 11	" mdse.	500.00		
May 10	" "	300.00							
June 9	" "	240.75							

NOTE.—It is usually more convenient to find the interest at 12%, and if it be desired at any other rate, as for instance 7%, add a sixth to one-half the sum of debit, and the same to the sum of the credit interest, or a sixth added to one-half the difference between the sums would produce the same final result.

2. What was due on the following account, Jan. 1, 1872, interest 6 per cent., and a credit of 90 days being allowed on each merchandise item?

Dr. JOHN SCOTT *in account with* GEO. FIELDS. *Cr.*

1871.				Days.	Int. or product.	1871.				Days.	Int. or product.
July 3	To Mdse.	$104	85			Aug. 12	By Mdse.	$300	00		
" 16	"	340	60			" 26	" "	116	80		
" 31	"	67	50			Sept. 15	" "	339	75		
Sept. 13	"	236	80			Oct. 13	" Cash.	50	00		
" 20	"	90	38			Nov. 1	" "	148	75		
" 27	"	60	64								
Oct. 1	"	260	40								

3. What would have been the true balance of the above account Jan. 1, 1872, at 7 per cent., no credit being allowed on merchandise items?

4. What was due on the following account Sept. 1, 1872, interest 10 per cent., and a credit of 60 days being allowed on each merchandise item?

Dr. WM. BROWN *in account with* ARTHUR ROY. *Cr.*

1872.				Days.	Int. or product.	1872.				Days.	Int. or product.
Jan. 2	To Mdse.	1250				Feb. 3	By Cash.	500			
" 23	"	987	50			Mar 4	" "	640	50		
Mar 4	"	846	25			Apr. 30	" "	750			
Apr 15	"	1000				Aug. 22	" Mdse.	965	75		
" 20	"	650	75			" 28	" Cash	675			
July 18	"	980	80								

5. What was due on the following account Dec. 31, 1872, interest 12 per cent., and a credit of 30 days being allowed on each merchandise item?

Dr. H. W. BRYANT *in account with* NELSON WILLIAMS. *Cr.*

1872.				Days.	Int. or product.	1872.				Days.	Int. or product.
Mar. 3	To Mdse.	1600				Jan. 2	By Cash.	1000			
Apr 9	"	970	45			" 20	" "	1200			
June 10	"	740	25			Mar. 10	Note 60 ds.	500			
Oct. 1	"	790	75			May 30	By Cash.	750	50		
" 21	"	650									

ACCOUNT OF SALES.

698. 1. An *account* of *sales* is a statement of the quantity and price of goods sold, the charges incurred in the sales, and the net proceeds, which a commission merchant or consignee makes to his employer or consignor.

2. In an *account sales* the *charges* for freight, commission, etc., are the *debits*, the proceeds of sales are the *credits*, and the difference between the sum of the credits and the sum of the debits is the net *proceeds*.

3. If the sales are made on time, the commission merchant or consignee is not compelled to account to the consignor for the net proceeds until the term of credit expires and the money has been paid.

4. In case the consignee has exercised that degree of care that those who are engaged in similar business take in giving credit and caring for the consignment, he is not responsible to the consignor for any loss. An exception to this is where he acts under a *delcredere commission*, or in other words, where he sells for an increased commission and guarantees the collection of time sales; in such case he becomes responsible if the purchaser fails to pay at the specified time.

5. All charges or expenses when paid to others on the consignment by the consignee are considered due from the consignor at the time of payment. The *after-charges*, or those which the consignee makes for commission, storage, etc., after a part or the whole of the consignment is closed out, should, in equity, take date at the *average date* of *sales*.

699. 1. The object of *equating* an *account* of *sales* is to make an equitable adjustment between the parties so that neither will gain an advantage over the other in *the use of money*. And it is well not to get the impression that the courts will enforce such an adjustment as that the consignor is entitled to the proceeds of any sale, minus the charges, the moment it comes into the hands of consignee, without taking into consideration the interest on money advanced on the consignment, unless there has been an express agreement to the contrary.

2. It would be inequitable to charge the consignee with the interest on the proceeds of sales unless he has mingled the money thus received with his own funds and derived some benefit from its use.

3. With the exception of finding the average date of the sales for a date for the after-charges, the process is exactly the same as in the equation of accounts.

700. EXAMPLES.

1. Required the equated date of the net proceeds of Edward Cook's consignment, the following being the account of sales:

Account of sales of wheat received of Edward Cook, Dubuque, Iowa, to be sold on his account and risk.

1871.						
July	12	Sold at 30 days, 3000 bush. @ $1.10,	$3300	00		
"	15	Sold 5000 bush. @ $1.15.				
		Received cash, . . . $3000.00				
		· Bal. @ 20 days, . . . 2750.00	5750	00		
"	25	Sold 10000 bush. @ $1.20.				
		Received cash, ...	12000	00		
Aug.	15	Sold 3000 bush. @ $1.15.				
		At 30 days, ...	3450	00	24500	00
		CHARGES.				
July	6	Pd. Freight in cash, . . $4200.00				
		" Insurance, ... 500.00				
		" Inspection, ... 50.00	4750	00		
		After-charges.				
		Commission @ 2%, . . . $490.00				
		Storage, ... 510.00				
		Adv. in city papers, . . 100.00	1100	00	5850	00
		Net proceeds at Eq. date, ...			$18650	00

The average date of the sales (Art. 681) is July 19.

When the items of this Account-Sales are arranged in the form of an Account, they stand as follows:

Dr. EDWARD COOK'S CONS'T. *Cr.*

1871.					1871.				
July	6	To Cash, . .	4200	00	Aug.	11	By Cash.	3300	00
"	"	" " . .	500	00	July	15	"	3000	00
"	"	" " . .	50	00	Aug.	4	"	2750	00
		" Com. . .	490	00	July	25	"	12000	00
"	19	" Storage, .	510	00	Sept.	14	"	3450	00
		" Adv., . .	100	00					
"	22	Net proceeds.	18650	00					
			$24500	00				$24500	00

By equating (690) we find the net proceeds are due July 22.

2. Account of sales of grain for Fisk, Cook & Co.

Date.		Purchaser.	Description.	Bush.	Price.	$
1871.						
Jan.	30	M. B. Gilbert.	Wheat, white.	250	$ .95	237.50
Feb.	3	Crest & Fisk.	Wheat, med.	1000	.88	880.00
"	16	Wheeler & Co.	Corn.	2000	.55	1100.00
"	28	C. A. Davis.	Oats.	1500	.37½	562.50
March	20	T. C. Skinner.	Wheat, Ky. white.	750	1.00	750.00
April	9	Talcott & Co.	Wheat, red.	1450	.85	1232.50
"	28	J. B. Howard.	Corn.	1300	.58	754.00
May	7	T. Benton.	"	450	.60	270.00
"	30	F. Hart.	Wheat, med.	955	.90	859.50
						$6646.00

CHARGES.

	Commission 2½ per cent. on $6646, . . .	$166.15
May 30—	Freight on 955 bushels wheat, ...	47.75
	Drayage and sacks, ...	51.00
	Advertising in "Tribune,"	7.50
		$272.40
	Net proceeds to credit of F. C. & Co., .	$6373.60

Errors excepted.

NEW YORK,
June 1, 1871.

SMITH & JONES.

3. Sales 544 barrels flour, for account of P. Rhodes & Son, Navarre, O., by Bryant & Stratton, Cleveland, O.

1871.			Monroe Extra.	Granite Mills.	Fine.						
July	3	P. Anderson.	400				830	3,320			
"	"	"			19		500	95			
"	5	Morgan & Co.		125			750	937	50		
			400	125	19					4,352	50
		CHARGES.									
June	15	Trans. Boat. "Kent."	400	125	19	=544 bl.	.16	87	04		
		Less am't de	duc	ted	for	damage					
		on boat.						10			
								77	04		
		Storage.					3	16	32		
		Insurance.						10	88		
		Commission	on	4352 50			2½%	108	81		
										213	05
		Proceeds to	cred	it as cash		July 10, 1871				4,139	45
										4,352	50

CLEVELAND, O.,
July 12, 1871.

(Signed) BRYANT & STRATTON.

4. What will be due P. Rhodes & Son on the above account January 1, 1872?

5. Sales, 100 barrels linseed oil, for account of Robert Miller, Warren, O., by Bryant & Stratton, Cleveland, O.

1871		Bbl.	Gal.					
May 14	Cash.	10	403	95¢	382	85		
" "	Gaylord & Co.	30	1,200	92	1,104			
" 15	Cash.	5	201½	95	191	43		
" 18	"	55	2,200	90	1,980			
		100	4,004½				3,658	28
	CHARGES.							
" 18	Tr. Boat Cuyahoga.	[redacted]		87½	87	50		
	Storage			8¢	8			
	Fire Insurance.			¼%	9	14		
	Cooperage				2	50		
	Com. on 3658.28.			2½%	91	46		
							148	60
Net proceeds due as cash May 18, 1871.							3,509	68

CLEVELAND, O., May 24, 1871. (Signed) BRYANT & STRATTON.

6. Sales of provisions for account of M. Fisher & Co., Cincinnati, O., by James & Co., St. Paul, Minn.

1871.		Boxes & kegs	Bbl.	Pieces.		Pounds			
Feb 7	Wheeler & Co.			29	Hams, plain.	1450	8¢		
" 22	"			[redacted]	" sugar cured	2300	10¢		
Mar. 6	"			18	Shoulders, plain.	846	7¢		
" 15	Altram & Co.		25		Mess Pork, No. 2.		16.50		
Apr 8	E. Miller.	[redacted]			Kegs butter, W. R	840	16¢		
" "	"	80			Cheese.	4200	6½¢		
" 10	Wheeler & Co.			150	Bacon sides	2432	7¢		
May 2	Altram & Co.		15		Mess Pork, No 2.		16.50		
" 5	Geo. Singer			87	Shoulders (city).	2512	7½¢		
		95	40	276		14,580			

CHARGES.

Feb. 1—Freight of 13,040, at 70¢ per 100.
April 2— " 5,040, at 68¢ "
Storage, 5.50. Cooperage, 3.30=6.60.
Fire Insurance, at ¼% on $.
Commissions, at 2½% on $.
Net proceeds as cash, due ——

ST. PAUL'S, MINNESOTA, May 15, 1871. JAMES & Co.

PARTNERSHIP, AND PARTNERSHIP SETTLEMENTS.

701. 1. A *partnership contract* is made by two or more persons to combine their capital, labor and skill, or any of them, for the purpose of carrying on a joint lawful undertaking, and acquiring a common profit in the *partnership* thus formed.

2. Each person thus associated is called a ***partner***, and a *partnership* combination is sometimes called a ***company, firm***, or ***house***.

3. The money or property invested by all the *partners* in the joint undertaking is called *capital stock*, or *joint-stock*, or *stock in trade*.

4. The profits to be shared by each of the partners are sometimes called *dividends*.

5. When a company is dissolved, either by the limitations of the contract or by mutual agreement, the adjustment of the accounts of the company and the division of effects is called a *partnership settlement*.

6. Each partner's stock increased by gain or diminished by loss at the time of settlement, is called his ***interest in the business*** at settlement.

702. 1. A *partnership contract* is the basis of the relationship that each partner bears to the others, in regard to the amount that each shall contribute toward carrying on the joint undertaking; that of which it shall consist, whether of capital, labor, or skill; the relation that each shall bear to the gains, and if there be losses, the proportion of loss each shall bear; also the time during which this relationship shall continue.

2. The true basis of all partnership settlements is the original agreement or contract between the parties. To avoid misapprehension and difficulty, such agreements should be explicit and comprehensive on all essential points; for, although the legal construction of such instruments aims at the "intent of the contracting parties," it is best to save the necessity of such construction, by putting the *intent* in the plainest English possible.

703. The following points should be embraced in a partnership contract:

1. The amount, time of investment, and continuation of each partner's capital.

2. The proportionate amount to be drawn by each partner for his private use.

3. The basis of gain or loss, and each partner's proportion thereof.

4. The limitation of copartnership.

5. In case of dissolution, what disposition shall be made of the partnership effects, whether they shall be sold at auction, or at private sale, or divided among the partners in certain proportions.

Other points may be added, according to the necessities of the case; but great care is necessary to avoid defeating the purposes of the contract by verbosity and ambiguity of terms.

704. 1. The object of a partnership settlement is to ascertain the relations in which the partners stand to the business and to each other. Such settlements should be effected at least as often as once every year.

2. The dissolution of a copartnership may be effected by the expiration of the terms of copartnership, the decease of one of the partners, the breaking out of a war between the two countries of which the partners are citizens, the mutual consent of the partners themselves, or the completion of the business contemplated in the contract.

3. No time having been given for terminating the contract, and the business having no natural termination, either of the partners may dissolve the partnership at any time, without incurring a liability to the others, by insolvency of the partnership and sale of its property, by insolvency of one of the partners and sale under execution of his interest in the partnership property, by a valid assignment by one partner of the partnership property or of his own separate interest in the partnership property, by the marriage of a female partner, by the voluntary act of either partner if he will pay damage to his fellow-partners, and by a judicial decree of a court of chancery if the business is found to be impracticable, or visionary, or if one of the partners be insane, or if one of the partners' conduct be grossly improper, so as to exclude his fellow-partners and patrons from the business.

4. In case a partnership expires by its own limitation or is dissolved by a decree of a court of chancery, or by the death of one of its partners, it is not necessary to give notice of its dissolution to those who have had previous dealings with the partnership, in

order to make the dissolution complete; but should a dissolution arise from any of the other causes, and two or more of the remaining partners continue the business under the old partnership name, those who have withdrawn would still be liable to third parties who have had previous dealings with the firm, unless they have been notified of the dissolution.

5. Third parties who have never had any previous dealings with the partnership are bound to make inquiry, and any statement made by one of the partners of the new firm would not affect the partner who had withdrawn.

6. After a partnership has been dissolved, and proper notice given, one member of the firm cannot bind the others by drawing or accepting drafts, or by making promissory notes, even for previously existing debts of the firm, although the partner drawing the same was authorized to settle the partnership affairs.

7. When notice is given of dissolution, and also of the appointment of one of the partners to settle up the business, a settlement made by a debtor of the firm with one of the other partners, without the knowledge and consent of the partner so appointed, would be fraudulent and void.

8. In case of a dissolution of a partnership where all evidence of the contract has been lost, and there are no circumstances to show how much each partner contributed to carry on the business, a court of chancery would assume that all the partners contributed equally, as in such cases "equality would seem to be equity," and the partnership effects, including the gains, if any, would be divided equally among the partners.

9. In case circumstances, or the partnership contract, shows how much each contributed to carry on the business, and the contract be silent in regard to the division of the gains, then a court of chancery will decide that each partner's share of the gains shall bear the same relation to the whole gain as his contribution bears to the total contribution; therefore if their contributions be equal, they will share the gains in equal proportions; if not, their proportions will be unequal.

NOTE.—The foregoing principles are questioned by many American decisions, and may be properly considered as open questions in this country, but they are well established in England. The courts that have decided to the contrary have held the presumption of law to be that in a case where

two or more combine their capital, labor and skill, if one's capital is not equal to the other, or others, his labor and experience may compensate for this lack of capital, and the fact of the contract being silent in regard to the division of gains is sufficient in itself to raise the presumption that they so considered it, or that, if there was any inequality, they waived it, therefore any circumstances going to show the contrary would not be admitted as evidence to change the presumption.

705. In the formation of a partnership the partners often adjust any inequality in their labor and skill by acting in a double capacity of partners and employees, receiving a fixed salary, which is carried to expense account, and which ultimately comes out of the profits of the business. Any inequality of capital by one who has furnished more than his share or proportion, is adjusted by allowing him to receive interest on his surplus capital from those who have failed to furnish their proportion, and after this adjustment of the contribution, to share the gains and losses in certain proportions. Any rate of interest, however high, is not regarded as usurious in such cases; but unless such an adjustment is agreed upon at the time of forming the partnership, or unless all are willing at the time of dissolution, a court of chancery will not sanction it.

706. *To find each partner's gain or loss when the capital of the several partners is invested for the same length of time,* and they are to share gains and losses in proportion to the capital invested.

Ex. A, B, and C enter into partnership in the lumber business for 3 years. A put in \$2400; B, \$3600; C, \$6000. At the time of the dissolution of the firm the net profits were \$4000. What is each partner's share of the profits?

1. FIRST METHOD.

\$12000 : \$2400	::	\$4000 : \$ 800,	A's profits.	
12000 : 3600	::	4000 : 1200,	B's	"
12000 : 6000	::	4000 : 2000,	C's	"
		\$4000,	Entire profits.	

2. SECOND METHOD.

\$4000 ÷ \$12000 = .33⅓. Hence the profits = 33⅓% of the stock.

A's share of profits	= \$2400 × .33⅓	=	\$ 800
B's " "	= 3600 × .33⅓	=	1200
C's " "	= 6000 × .33⅓	=	2000
Entire profits, ...			\$4000

707. RULE.

By proportion, as the whole stock is to each partner's stock, so is the whole profit or loss to each partner's profit or loss. Or,

Find what per cent. of the entire stock the total profits or loss may be; then multiply each partner's stock by the rate per cent. expressed decimally.

708. EXAMPLES.

1. A, B, and C traded in company. A put in \$8000; B, \$4500; and C, \$3500. Their profits were \$6400. What was each partner's share of the profits?

2. A and B, in trading for three years, make a profit of \$4800. A invested $\frac{3}{5}$ as much stock as B. What is each man's share of profit?

3. Two drovers, A and B, have been operating in company in buying and selling sheep. A made purchases to the amount of \$6780, and paid expenses amounting to \$274.12. B made purchases to the amount of \$3840, and paid expenses amounting to \$312. The sheep were sold by A for \$10482. How much was made or lost? How will A and B settle, the profits or losses to be shared in proportion to investment?

4. C and D agree to perform a certain piece of work for government, for which they are to receive \$4680, provided it passes inspection as No. 1. If it pass as No. 2, 15 per cent. is to be deducted; as No. 3, 20 per cent. is to be deducted.

The result of the inspection was as follows:

1st division,	which is	$\frac{3}{8}$	of contract,	is	No. 1.
2d	"	"	$\frac{1}{8}$	"	No. 3.
3d	"	"	$\frac{1}{2}$	"	No. 2.

C has advanced, for the prosecution of the work, \$1328; B has advanced \$987.45. Neither has received anything from government, and all the money advanced has been used. How much have they gained, and what is each man's share?

709. When the capital is invested for *different periods of time*, to find each partner's share of the profits or loss.

Ex. A, B, and C traded in company for three years. When they commenced business, A put in \$4000; B, \$3000; and C, \$5000. At the close of the first year A put in \$3000 more, and

C took out $1000. At the close of the second year B put in $2000. At the close of the third year they dissolve partnership, and the net profits of the firm are found to be $2100. What was each partner's share of the gain?

OPERATION.

$$
\begin{array}{llrl}
\text{A invested} & \$4000 \times 1 = \$ & 4000 & \multirow{2}{*}{$\Big\} = \textit{use} \text{ of } \$18000 \text{ for 1 yr.}$} \\
\quad " & 7000 \times 2 = & 14000 & \\
\text{B} \quad " & 3000 \times 2 = & 6000 & \Big\} = \quad " \quad 11000 \quad " \quad " \\
\quad " & 5000 \times 1 = & 5000 & \\
\text{C} \quad " & 5000 \times 1 = & 5000 & \Big\} = \quad " \quad 13000 \quad " \quad " \\
\quad " & 4000 \times 2 = & 8000 & \\
 & & & \qquad\quad \$42000
\end{array}
$$

The *use* of the entire investment was equivalent to the use of $42000 for 1 yr.

$$
\begin{array}{l}
\$42000 : \$18000 :: \$2100 : \text{A's gain} = \$900 \\
42000 : 11000 :: 2100 : \text{B's} \quad " \quad = 550 \\
42000 : 13000 :: 2100 : \text{C's} \quad " \quad = \underline{650} \\
\qquad\qquad \text{Entire profits,} \quad . \quad . \quad . \quad \$2100
\end{array}
$$

Or, $2100 ÷ $42000 = .05; that is, the gain is 5% of the capital; hence A's gain = $18000 × .05 = $900, etc.

Explanation.—Since A invested $4000 for 1 year and $7000 for 2 years = $14000 for 1 year, A used in trade the same as $4000 + $14000 = $18000 for 1 year; and, since B invested $3000 for 2 years = $6000 for 1 year, and $5000 for 1 year, B used in trade the same as $6000 + $5000 = $11000 for 1 year; and since C invested $5000 for 1 year, and $4000 for 2 years = $8000 for 1 year, C used in trade the same as $5000 + $8000 = $13000 for 1 year. Hence, A, B, and C, together, used in trade the same as $18000 + $11000 + $13000 = $42000 for 1 year.

NOTE.—The remaining portion of the solution is in accordance with either of the *two preceding rules;* for the *time* of each one's investment is now regarded the same, A's stock being equivalent to $18000, B's, $11000; and C's, $13000 for one year.

710. RULE.

Multiply each partner's stock by the time it was invested, and regard the product as his stock-in-trade, and the SUM *of the products as the entire stock-in-trade, and then proceed according to either of the two preceding rules.* (Art. 707.)

711. EXAMPLES.

1. A and B entered into partnership Jan. 1, 1858. A put in \$4500, and B, \$5500. July 1, 1858, B put in \$1500 more. Oct. 1, 1858, A took out \$500. Jan. 1, 1859, each put in \$1500. July 1, 1859, they dissolved partnership, and found they had lost \$846. What is each partner's share of the loss?

2. A, B, and C hired a pasture for 6 months, for \$245. A put in 40 sheep; B, 50 sheep; C, 80 sheep. At the close of 3 months A put in 20 more; at the close of 4 months B took out 20; and at the close of 5 months C took out 60. How much ought each to pay?

3. A and B enter into a partnership for 3 years. A put in \$10000, and B, \$2500. B is to do the business, and his services are to be regarded as worth the use of \$7500, the difference between his and A's stock. At the close of the first year A increased his stock to \$18000. At the close of the 3 years the partnership closed, and a net gain found of \$9500. What is each partner's share of the gain?

712. 1. When the partners' labor and skill are equal, and they have, in their partnership contract, agreed to invest equally and share the gains and losses equally, if one of the partners should fail to invest his proportion, he would be required to pay interest at some certain rate to the partner or partners that furnish more than their share.

2. The *use* or *interest* of each partner's stock will bear the same ratio to the *interest* of the whole stock as the actual stock to the entire stock; accordingly the interest may be used instead of stock, in computation.

Ex. A, B and C entered into a partnership Sept. 1st, 1870, to continue one year, for the purpose of carrying on the retail dry goods business. As each had about the same experience in business and each was to give his whole time, they agreed to divide the gains and losses equally; and should either partner put into the business more than his one-third of the capital, he was to be allowed 12 per cent. interest from those who failed to invest an equal amount.

Sept. 1, 1871, the partners' accounts stood as follows:

Dr.			A		*Cr.*
Oct. 1, 1870.	Cash,	$200.00	Sept. 1, 1870.	Cash,	$4000.00
Nov. 15,	" "	300.00	Nov. 20,	" "	2000.00
Dec. 16,	" "	500.00	April 15, 1871,	"	3000.00
Feb. 20, 1871,	"	800.00	June 20,	" "	2375.00
July 15,	" "	1500.00			
Amt. withdrawn,		$3300.00	Total investment,		$11375.00

Dr.			B		*Cr.*
Oct. 20, 1870.	Cash,	$800.00	Sept. 1, 1870.	Cash,	$3000.00
Jan. 15, 1871.	"	840.00	Mar. 15, 1871.	"	4000.00
July 20,	" "	560.00	May 20,	" "	1800.00
Aug. 1,	" "	2000.00			
Amt. withdrawn,		$4200.00	Total investment,		$8800.00

Dr.			C		*Cr.*
Dec. 20, 1870.	Cash,	$1000.00	Sept. 1, 1870.	Cash,	$3500.00
Mar. 25, 1871.	"	875.00	April 15, 1871.	"	2000.00
June 27,	" "	325.00	June 20,	" "	1500.00
Amt. withdrawn,		$2200.00	Total investment,		$7000.00

The net gain was found to be $11,223.90. How should the partners settle with each other, and how much was due to each Sept. 1, 1871?

SOLUTION.

Sept. 1, 1870.	$4000 for 365 da. affords	$480.00 Int.	
Nov. 20, "	2000 " 285 "	190.00 "	
Apr. 15, 1871.	3000 " 139 "	139.00 "	
June 20, "	2375 " 73 "	67.81 "	
Interest on A's total investment, .			$876.81
Oct. 1, 1870.	$200 for 335 da. affords	$22.00 Int.	
Mar. 15, "	300 " 290 "	27.00 "	
Dec. 16, "	500 " 259 "	43.17 "	
Feb. 20, 1871.	800 " 168 "	44.80 "	
July 15, "	1000 " 48 "	16.00 "	
Int. on A's withdrawals, . . .			152.97
Int. on A's net investment, . .			$723.84

Sept. 1, 1870.	$3000 for 365 da. affords	$360.00 Int.		
Mar. 15, 1871.	4000 " 170 "	228.67 "		
May 20, "	1800 " 104 "	62.40 "		
Interest on B's total investment, .			$651.07	
Oct. 20, 1870.	$800 for 316 da. affords	$84.27 Int.		
Jan. 15, 1871.	840 " 229 "	64.12 "		
July 20, "	560 " 43 "	8.43 "		
Aug. 1, "	2000 " 31 "	20.67 "		
Int. on B's withdrawals, . . .			177.49	
Int. on B's net investment, . .			$473.58	
Sept. 1, 1870.	$3500 for 365 da. affords	$420.00 Int.		
April 15, 1871.	2000 " 139 "	92.67 "		
June 20, "	1500 " 73 "	36.50 "		
Int. on C's total investment, . .			$549.17	
Dec. 20, 1870.	$1000 for 255 da. affords	$85.00 Int.		
Mar. 25, 1871.	875 " 160 "	46.67 "		
June 27, "	325 " 67 "	7.15 "		
Int. on C's withdrawals, . . .			138.82	
Int. on C's net investment, .			$410.35	

Interest on A's net investment, ...	$723.84
" B's " "	473.58
" C's " "	410.35
Interest at 12% on the net investment in the business,	$1607.77

According to contract, the interest on each one's investment should have been $\frac{1}{3}$ of $1607.77 or 535.92\frac{1}{3}$.

Interest on A's net investment, ...		$723.84
Int. on one-third of the capital invested in the business,		535.93
Int. on his surplus investment to be paid by the other partners, ...		$187.91
Interest on $\frac{1}{3}$ of the net investment, . .	$535.92	
" " B's net investment, . . .	473.58	
" due from B to A, ...		$62.34
" on $\frac{1}{3}$ of the net investment, . .	$535.92	
" " C's net investment, . . .	410.35	
" due from C to A, ...		125.57
" " B and C to A, . . .		$187.91

The following would be the final statement of the partners, on settlement:

A's total investment,	$11375.00	
" $\frac{1}{3}$ net gain,	3741.30	
" Interest due from B and C,	187.91	
" Total credits,		$15304.21
Amount withdrawn by A,		3300.00
A's present interest in the business,		$12004.21
B's total investment,	$8800.00	
" $\frac{1}{3}$ net gain,	3741.30	
" Total credits,		$12541.30
Amount withdrawn by B,	$4200.00	
Interest due A from B,	62.34	
B's total debits,		4262.34
" present interest in the business,		$8278.96
C's total investment,	$7000.00	
" $\frac{1}{3}$ net gain,	3741.30	
" total credits,		$10741.30
Amount withdrawn,	$2200.00	
Interest due A from C,	125.57	
C's total debits,		2325.57
" present interest in the business,		$8415.73

713. RULE.

1. *Compute the interest at the rate agreed upon, on each item of each partner's account from the date of the same to the time of settlement. The debit or credit balance of the entire interest computed for each one's account will be the interest on the net investment of each.*

2. *The sum of the interest of all the partners on their net investment will be the interest on the entire net investment. Such a part of this as each was to share of the gain or loss, will be the net interest that should have been credited to each, if each had furnished a corresponding part of the capital.*

3. *If any partner's interest on his net investment be more than this, the difference will show what is due him from the other partners on interest account; if any partner's interest on his net*

investment be less than this, the difference will show what is due from him on interest account.

4. *Each partner's net investment, increased by the interest due to him, or decreased by the interest due from him, will show his entire interest in the business at the time of settlement.*

5. *If one partner has withdrawn more than he has invested, the balance of his withdrawals, plus his balance of interest, plus the net interest required by contract from each partner, will show his entire indebtedness to the other partners.*

714. Ex. Wm. Johnson, Samuel Smith and Robt. Wilson entered into a copartnership Jan. 1, 1870, to continue at will, for the purpose of carrying on the wholesale hardware business, an interest account to be kept at 10%, and the gains to be divided equally. Oct. 1st, 1871, a dissolution took place, and the gains were found to be $9651.36. The partners' accounts stood as follows:

WM. JOHNSON.

Dr.				*Cr.*			
Jan.	25, 1870.	Cash.	$371.00	Jan.	1, 1870.	Cash.	$6000.00
Mar.	14, "	"	890.00	Feb.	15, "	"	7000.00
May	20, "	"	749.00	Apr.	10, "	"	5384.00
Oct.	25, "	"	371.43	Aug.	15, "	"	3843.71
Feb.	3, 1871.	"	1284.31	Dec.	21, "	"	1500.00
Aug.	23, "	"	349.45	May	14, 1871.	"	8473.00
Sept.	14, "	"	150.83				

SAML. SMITH.

Dr.				*Cr.*			
Mar.	24, 1870.	Cash.	$938.74	Jan.	1, 1870.	Cash.	$7000.00
Aug.	13, "	"	48.43	Mar.	15, "	"	8413.00
Sept.	20, "	"	500.00	July	13, "	"	9474.84
Apr.	23, 1871.	"	3700.00	Jan.	15, 1871.	"	3843.74

ROBT. WILSON.

Dr.				*Cr.*			
May	20, 1870.	Cash.	$100.00	Jan.	1, 1871.	Cash.	$5700.00
Jan.	13, 1871.	"	2000.00	Sept.	15, "	"	8730.00
Aug.	10, "	"	250.00	Nov.	20, "	"	3844.00
Sept.	8, "	"	370.00				
"	20, "	"	500.00				

Make out a final statement showing the present interest of each in the business, and how much interest was to be paid or received by each partner.

715. When the interest on the money withdrawn from the business by all the partners, exceeds the interest on their investments.

Ex. 1. Walter Brooks, H. H. Dean and James Maguire formed a partnership July 15, 1870, to carry on the milling business, to share the gains and losses equally. An interest account was kept in order to equalize their investments. Interest at 15%. A dissolution took place Sept. 20, 1871. Net loss $3600.

The partners' total investments, withdrawals, and interest, stood as follows:

Walter Brooks had withdrawn	$15000		Int. $1433.55
" " " invested	12000		" 1131.54
" " *insolvency in stock*	$3000	Dr.	" $302.01
H. H. Dean had withdrawn	$8400		Int. $784.33
" " " invested	5700		" 434.03
" " *insolvency in stock*	$2700	Dr.	" $350.30
James Maguire had withdrawn	$20000		Int. $1734.80
" " " invested	16000		" 1571.60
" " *insolvency in stock*	$4000	Dr.	" $163.20

How shall the interest be adjusted? And what is each one's insolvency?

SOLUTION.

Walter Brooks' debit interest is	$302.01
H. H. Dean's " " "	350.30
James Maguire's " " "	163.20
Total debit interest of the partners, . .	$815.51

The business has not cost the partners anything to carry it on, as the interest on the money withdrawn is more than the interest on the money invested. According to the partnership contract each should have received $\frac{1}{3}$ of the use of the money that was withdrawn from the business. And as the interest on the money withdrawn was equal to $815.51, each partner had a right to draw out such an amount that the interest upon it would equal $\frac{1}{3}$ of $815.51 or 271.83\frac{2}{3}$; and if any partner has not done so, he is entitled to the difference at settlement, which must be paid by the

others. Therefore, James Maguire is *entitled* to the difference between \$271.83⅔ and \$163.20=\$108.63⅔. Walter Brooks must *pay* the difference between \$302.01 and \$271.83⅔=\$30.17⅓. And H. H. Dean must *pay* the difference between \$350.30 and \$271.83⅔ =\$78.46⅓. The sum of \$30.17⅓ and \$78.46⅓=\$108.63⅔.

The following is a statement of the partners' account at the settlement:

Walter Brooks'	insolvency in stock, . .	\$3000.00	
" "	⅓ net loss, ...	1200.00	
" "	int. due Maguire, . . .	30.17⅓	
" "	total insolvency, . . .		\$4230.17⅓
H. H. Dean's	insolvency in stock, . .	\$2700.00	
" "	⅓ net loss, ...	1200.00	
" "	int. due Maguire, . . .	78.46⅓	
" "	total insolvency, . . .		3978.46⅓
James Maguire's	insolvency in stock, . .	\$4000.00	
" "	⅓ net loss, ...	1200.00	
" "	total debits, ...	\$5200.00	
" "	deduct int. due from B. & D.	108.63⅔	
" "	net insolvency, . . .		\$5091.36⅓
	Total insolvency of the firm, ...		\$13300.00

Ex. 2. G. Beach, J. W. Grubbs and W. A. Lewis have been doing business as partners, with the understanding that an interest account be kept for the purpose of equalizing their investments. At the close of the business the interest on the money withdrawn by G. Beach exceeded the interest on his investment by \$384.21. J. W. Grubb's balance of debit interest, \$437.63. And W. A. Lewis \$384.21. How shall they adjust their interest account?

716. When there is a debit balance of interest with some of the partners and a credit balance with the others. (Art. 713, 5.)

Ex. 1. S. A. Chase, Geo. K. Rix, A. B. Copp and James Carroll have been associated as partners in carrying on the wholesale dry-goods business. The partnership contract required that interest at 18 per cent. should be computed on each side of the partner's account. The partnership was dissolved Oct. 1, 1871, and the balance of interest on their accounts stood as follows:

S. A. Chase's credit balance of interest,	. .	$3785.24
Geo. K. Rix's " " "	. .	2848.48
A. B. Copp's debit " "	. .	1840.60
James Carroll's " " "	. .	4380.80

How should the partners have settled?

SOLUTION.

S. A Chase's credit interest, . . .	$3785.24	
Geo. K. Rix's " " . . .	2848.48	
Total credit interest, ...		$6633.72
A. B. Copp's debit interest, . . .	$1840.60	
James Carroll's " " . . .	4380.80	
Total debit interest, ...		6221.40
Interest on the net investment of the business, . .		$412.32
Each partner should have invested enough so that the interest at the required rate would have equalled ¼ of $412.32, ...		$103.08

S. A. Chase's interest	on his investment, . . .	$3785.24
"	on what he should have invested, .	103.08
"	due him, ...	$3682.16
Geo. K. Rix's "	on his investment, ...	$2848.48
"	on what he should have invested, .	103.08
"	due him, ...	$2745.40
A. B. Copp's "	on what he has withdrawn, . .	$1840.60
"	" " should have invested, .	103.08
"	he must pay, ...	$1943.68
James Carroll's "	on what he has withdrawn, . .	$4380.80
"	" " should have invested, .	103.08
"	he must pay, ...	$4483.88

Interest due S. A. Chase, ...	$3682.16
" " Geo. K. Rix, ...	2745.40
" to be received by both, ...	$6427.56
" " paid by A. B. Copp, ...	$1943.68
" " " James Carroll, ...	4483.88
" " " both, ...	$6427.56

A and B were partners and had an interest account. At the close of the partnership affairs, A's debit balance of interest was $376.20. B's credit balance was $273.20. How did they properly settle, gain or loss being shared equally?

717. Where the partners are not equal in labor and skill, and they wish to invest their capital in the same proportion and share the gains and losses in like manner.

Ex. 1. Y and Z were partners in the manufacturing business. Y's labor and experience were worth twice as much as Z's. Y was to invest $\frac{2}{3}$ of the capital and Z $\frac{1}{3}$. Should Y invest more than his share, Z was to pay him 20% interest on his excess. The gains and losses were to be divided as follows: Y $\frac{2}{3}$ and Z $\frac{1}{3}$. At the time of settlement it was found that there had been a gain of $30000. The partners' balances and their interest as shown by their accounts, were as follows:

Y's net investment, . . .	$80000.	Int. $15000.00.
Z's " " . .	30000.	" 6375.00.

How much interest was due Y? What was each partner's interest at closing?

SOLUTION.

Y's interest on his net investment, ...	$15000.00
Z's " " " "	6375.00
Interest on total investment, ...	$21375.00
The interest on Z's net investment should have been $\frac{1}{3}$ of $21375, or	$7125.00
The interest on Z's investment is	6375.00
Interest due from Z to Y, ...	750.00

Final Statement.

Y's net investment, ...	$80000	
" $\frac{2}{3}$ net gain, ...	20000	
" interest due from Z, ...	750	
" present interest in business, . .		$100,750.00
Z's net investment, ...	$30000	
" $\frac{1}{3}$ net gain, ...	10000	
" Cr. in the business, ...	$40000	
" interest due Y, ...	750	
" present interest in business, .		$39250.00

2. D, E, and F embarked, as partners, in the printing business, with the following understanding: that D, on account of his superior skill and experience, was worth as much to the business as both E and F; and that E and F were equal in skill, etc. D was to furnish $\frac{1}{2}$ the capital and have $\frac{1}{2}$ of the gains; E and F each was to furnish $\frac{1}{4}$ the capital and have $\frac{1}{4}$ of the gains. Each to have the privilege of adding to the capital or drawing from it as circumstances required. An interest account to be kept at 12% to sustain the foregoing relationship.

At the dissolution of the business, the partners' accounts showed the following results:

D's net investment,	$4375.	Cr. interest,	$732.25
E's " "	3748.	" "	284.30
F's " "	2876.	" "	348.60
Net gain of business, . .			3216.00

How shall the partners adjust the matter? Make out the final statement on settlement.

718. The investment and the resources and liabilities at closing being given, to find the net gain or loss.

RULE.

Subtract the sum of the liabilities (including the investment) from the sum of the resources, and the difference will be the net gain; or (if the liabilities are the larger) subtract the sum of the resources from the sum of the liabilities, and the difference will be the net loss.

Ex. 1. A and B are partners. At the close of one year's business, an inventory is taken showing the condition of affairs to be as follows, viz.: Cash on hand $3278. Merchandise in store valued at $1500. Five shares City Bank Stock $500. House and lot valued at $4000. The firm owes on their notes $2000, and to Wm. Brown on account, $1200. A invested $2426, B invested $2872. What is the net gain?

OPERATION.

RESOURCES.		LIABILITIES.	
Cash, ...	$3278	Firm's notes, ...	$2000
Merchandise, ...	1500	Due Wm. Brown, . .	1200
City Bank Stock, . .	500	A invested, ...	2426
House and Lot, . . .	4000	B "	2872
	9278		$8498
	8498		
Net gain, . . .	$780		

2. C, D, and E are partners. After conducting business one year they have the following resources and liabilities: Cash on hand \$4860. Mill and fixtures valued at \$6924. Bills receivable \$896. Brown and Co. owe \$2000. Ten shares R. R. stock \$1000. The firm owe on notes outstanding \$6400. C invested \$4500. D invested \$3800. E invested \$3600. What is the net loss?

719. The investment, the resources and liabilities at closing, and the proportion in which the partners share the gains or losses being given, to find each partner's interest in the concern at closing.

RULE.

Find the net gain or net loss by Art. 718. Then, if there is a gain, add each partner's share of gain to his investment and subtract the amount he owes the firm. Or, if there is a loss, find each partner's share of loss and subtract it from his investment; also subtract any amount that the partner owes the firm, as before.

Ex. 1. A and B are partners. A is to share $\frac{2}{5}$ of the gain or loss, and B $\frac{3}{5}$. At the close of business the following is shown to be the condition of their affairs, viz.: Cash on hand \$2680. Bills receivable on hand \$3620. Five shares United States stock valued at \$520. House and lot valued at \$6000. Sturgis & Co. owe on account \$1800. The firm owe on notes outstanding \$2840. They owe G. P. Carey on account \$890. A invested \$4610. B invested \$4860. What is A's interest in the concern? What is B's interest in the concern?

OPERATION.

RESOURCES.		LIABILITIES.	
Cash on hand, . . .	\$2680	Notes unpaid, . . .	\$2840
Bills Receivable, . . .	3620	Due G. P. Carey, . .	890
U. S. Stock, ...	520	A invested, ...	4610
House and Lot, . . .	6000	B "	4860
Sturgis & Co. owe, . .	1800		
	14620		\$13200
	13200		
Net gain, . . .	\$1420		

5)1420 Net gain.
284 $\frac{1}{5}$ " "
3
\$852 B's $\frac{3}{5}$ " "
\$568 A's $\frac{2}{5}$ " "

PROOF.

Cash, ...	$2680	Bills Payable, ...		$2840
Bills Receivable, .	3320	G. P. Carey, ...		890
U. S. Stock, . .	520	A invested, ...	$4610	
House and Lot, .	6000	" $\frac{2}{5}$ net gain, . . .	568	
Sturgis & Co., . .	1800	" present interest in concern,		5178
		B invested, ...	$4860	
		" $\frac{3}{5}$ net gain, . . .	852	
		" present interest in concern,		5712
Total resources, .	$14620	Total liabilities, ...		$14620

NOTE.—In the following examples the resources are supposed to be brought in at their actual cash value. No interest is allowed on the partners accounts unless so specified.

2. C, D, and E are partners. Each is to share one-third of the gains or losses. The resources and liabilities at the close of the year are found to be as follows, viz.: Money deposited in City Bank $8460. Copper Mine Stock valued at $10240. Bills Receivable on hand $6420. Fulton Bank Stock on hand valued at $3826. Block of buildings and lot valued at $35000. Hall & Co. owe on account $1344. L. M. Howard owes on account $960. The firm owe on their notes unredeemed $5680. To Mason & Austin on account $1700. C invested $18420. D invested $18460. E invested $184[illegible]2. What is each partner's present interest in the concern?

3. F, G, H, and I are partners. They share the gains or losses as follows, viz.: F and G $\frac{3}{12}$ each, H $\frac{4}{12}$ and I $\frac{2}{12}$. At the close of business the resources are Cash $4628, Merchandise $12620, Real Estate $5000, Bank Stock $3000, Wheat and Corn $2800, Horses and Harness $500, Lumber $520, Money deposited in Globe Bank $8620. F has drawn from the business $450. H has drawn $180. The liabilities of the concern are, Notes unredeemed $4600, due Simon Good on account $800, due S. S. Packard on account $1200. F invested $6682. G invested $6682. H invested $8908. I invested $4454. What is each partner's interest in the concern?

4. J, K, L, M, and N are partners. The gain or loss is to be divided as follows: J $\frac{4}{15}$, K $\frac{4}{15}$, L $\frac{3}{15}$, M $\frac{3}{15}$, N $\frac{1}{15}$. Upon examination the following is found to be the condition of affairs at the

close of business, viz.: Notes on hand against other persons \$12680, Ohio State Stocks \$8420, New York State Stock \$6000, City Bank Stock \$2800, Bonds and Mortgages \$9[illegible]0, Deposit in Ocean Bank \$6742, Attica Bank owes the firm \$4286, Brown & Bros. owe \$1520, Interest on Notes, Bonds and Mortgages in the hands of the firm \$688, Office Furniture on hand valued at \$824. The liabilities of the concern are as follows, viz.: Notes and Acceptances outstanding \$5486, Interest due on firm's Notes and Acceptances \$280, Bal. favor Trader's Bank \$2626, Bal. favor of Fulton Bank \$1500. N invested \$2287. M invested \$4575. K invested \$9150. L invested \$6861. J invested \$11455. What has been the net gain? What is J's interest in the concern? K's? L's? M's? N's?

5. There are four partners in a concern, O, P, Q, and R. Each partner to share $\frac{1}{4}$ of the gains or losses. At dissolution there is Cash on hand \$6820, Bills Receivable \$8922, Croton Water Stock \$4500, Deposit in Bank Commerce \$3860. O has drawn from the concern \$860, P has drawn \$575, Q has drawn \$630, R has drawn \$452. The liabilities are: Notes and Acceptances outstanding \$3680, Bal. in favor of Smith & Co. \$1264, in favor of Hall & Reed \$860, Geo. Carey \$575. O invested \$5590, P invested \$5322, Q invested \$5540, R invested \$5228. What has been the net gain or loss? What is each partner's interest in the business?

720. The resources, the liabilities (except the investment), and the net gain or loss being given, to find the net capital at commencing.

RULE.

When the resources are greater than the liabilities, deduct the given liabilities from the given resources; the difference will be the present worth of firm, and this, diminished by the net gain, or increased by the net loss, will give the capital. Or,

When the liabilities are greater than the resources, deduct the resources from the liabilities; the difference will be the net insolvency of firm, then deduct this remainder from the net loss.

NOTES.—1. The liabilities can never exceed the resources at closing when there is a capital at commencing and a net gain during business.

2. In the following examples it is supposed that the whole investment is made at the time of commencing business, and that it remains undisturbed until the date of partnership settlement.

Ex. 1. A and B are partners. A invested $\frac{2}{5}$ and B $\frac{3}{5}$ of the capital. They are to share equally in gains or losses. At the close of business the resources are: Cash $6800, Bills Receivable $4700, Merchandise $6400, Real Estate $5000, Bank Stock $900, Steamboat Stock $9000. A has drawn from the business $365, B has drawn $526. The liabilities are: Firm's Notes unredeemed $4680, Bal. favor of S. S. Packard $620, J. T. Calkins $476, R. H. Hoadley $326. The net gain during business has been $2644. What was the firm worth at commencing? What was each partner worth?

OPERATION.

Cash, ...	$6800	Bills Payable, ...	$4680
Bills Receivable, . .	4700	S. S. Packard, ...	620
Merchandise, . . .	6400	J. T. Calkins, ...	476
Real Estate, . . .	5000	R. H. Hoadley, ...	326
Bank Stock, . . .	900		$6102
Steamboat Stock, .	9000		
A is charged, . . .	365	$\frac{2}{5}$ of $24945 = $9978 = A's capital at first.	
B " . . .	526		
	$33691	$\frac{3}{5}$ of $24945 = $14967 = B's capital at first.	
Liabilities, ...	6102		
Present worth of firm,	27589		
Net gain, ...	2644		
Net Capital at first, .	24945		

PROOF.

Cash, ...	$6800	Bills Payable, ...		$4680
Bills receivable, ...	4700	S. S. Packard, ...		620
Merchandise, ...	6400	J. T. Calkins, ...		476
Real Estate, ...	5000	R. H. Hoadley, ...		326
Bank Stock, ...	900	A's Cap. at com., . . .	9978	
Steamboat Stock, ...	9000	" ½ Net gain, . . .	1322	
A is charged, ...	365	" Present worth, ...		11300
B "	526	B's Cap. at com., . .	14967	
		" ½ Net gain, . .	1322	
		" Present worth, ...		16289
	$33691			$33691

2. C, D, and E are partners. C invested $\frac{1}{8}$, D $\frac{3}{8}$, and E $\frac{4}{8}$, to share the gain or losses equally. At the close of business the resources are found to be: Wheat on hand valued at $2600, Corn on hand $3200, Flour $1600, Mill and Fixtures $8000. The firm owe Digby V. Bell $2600, to J. H. Goldsmith $1500, and on their Notes unredeemed $949. The net loss in the business has been $633. What was the net capital of the firm at commencing? What was each partner's net capital?

3. There are four partners engaged in business as a firm, F, G, H, and I. They have been unfortunate, the net loss being $15320. On examination the resources are found to be as follows, viz.: Live Cattle on hand valued at $9680, Packed Beef valued at $12600, Empty Barrels on hand valued at $500, Deposit in Drovers' Bank $2500. The firm owe on their Notes and Acceptances $22600, Warren P. Spencer on account $4000, J. C. Bryant on account $6000. The partners invested in equal amounts and are to share the gains or losses in the same proportion. What was the investment of the firm? What was each partner's investment?

721. *When the firm commence insolvent.*

The resources and liabilities at closing, and the net gain or loss being given, to find the net insolvency at commencing.

RULE.

When the liabilities are greater than the resources at closing, deduct the given resources from the given liabilities, and to this remainder add the net gain or from it subtract the net loss. Or,

When the resources are larger than the liabilities at closing, deduct the liabilities from the resources, and deduct this remainder from the net gain.

Ex. 1. A and B are partners. They commence business insolvent. The proportion of their insolvency is A $\frac{3}{4}$, B $\frac{1}{4}$. The gains or losses are to be equally divided. At the close of business the resources are, Cash on hand $3246, Lumber on hand valued at $6428, Timber and Logs valued at $3272, Bills Receivable $1800. The firm owe on their Notes and Acceptances $9400, to E. R. Felton on account $3684, to H. W. Ellsworth on account $2160. The net gain during business has been $1568. What was the net insolvency of the firm at commencing? What was each partner's net insolvency at commencing?

OPERATION.

Cash on hand, . . .	$3246	Bills Payable, ...	$9400
Lumber "	6428	E. R. Felton, ...	3684
Timber and Logs on h.	3272	H. W. Ellsworth, . .	2160
Bills Receivable, " .	1800	Liabilities, ...	15244
	$14746	Resources, ...	14746
4)2066 Net Insolv. at com.		Pres. Net Insolv. of firm,	498
516.50 B's ¼ " "		Net gain, ...	1568
3		Insolv. of firm at com., .	2066
$1549.50 A's ¾ " "			

PROOF.

Cash, ...		$3246.00	Bills Payable, ...		$9400.00
Lumber, ...		6428.00	E. R. Felton, ...		3684.00
Timber and Logs, . . .		3272.00	H. W. Ellsworth, . . .		2160.00
Bills Receivable, . . .		1800.00	B's ½ Net gain, .	784.00	
A's Insolv. at com.,	1549.50		" Ins. at com., .	516.50	
" ½ Net gain, .	784.		" Net Capital, ...		267 50
" Net Insolvency, . .		765.50			
Total Resources of firm, .		$15511.50	Total Liab. of firm, . . .		$15511.50

NOTE.—In the foregoing example the partners were both insolvent at commencing business. The business was profitable, and B's share of the gain was more than his insolvency at commencing, so that he ends with a *net capital*. A is still insolvent, but to a less amount than when he commenced.

2. C, D, E, and F are partners, commencing with equal insolvency, the gains or losses to be shared as follows, viz.: C $\frac{3}{12}$, D $\frac{4}{12}$, E $\frac{2}{12}$, F $\frac{3}{12}$. Two years having passed, an inventory is taken, showing the following condition of affairs: 20000 lb. Cheese on hand @ 8¢, $1600; 40000 lb. Butter @ 18¢, $7200; 2000 bu. Potatoes @ 40¢, $800; 3000 bu. Wheat @ 90¢, $2700. The firm owe on their Notes and Acceptances $8628. They owe E. B. Rockwell on account $3242. They owe W. H. Clark on account $4563. There has been a *net loss* during the business of $528. What was the net insolvency of the firm at commencing? What was the net insolvency of each partner? What is the net insolvency of the firm and of each partner at closing?

3. G, H, I, J, and K formed themselves into a copartnership for the purpose of carrying on the building and masonry business, the firm to assume the liabilities of the partners. The proportion in which the partners are insolvent at commencing is as follows, viz.: G $\frac{2}{20}$, H $\frac{3}{20}$, I $\frac{4}{20}$, J $\frac{5}{20}$, and K $\frac{6}{20}$. The gains or losses are to be divided in the proportion of their insolvency. At the close of business the following is the condition of affairs: Deposit in City Bank \$5428, Bonds and Mortgages Rec. \$3826, Notes and Drafts \$6294, Brick and Stone on hand valued at \$3688. J. C. Bryant owes on account \$4466. The firm owe on their Notes and Acceptances \$18000, and they owe Baldwin & Co. \$3620. The net gain has been \$5622. What was the net insolvency of firm at commencing? What was the insolvency of each partner? What is the net capital of firm at closing? Of each partner?

722. MISCELLANEOUS EXAMPLES.

1. D. V. Bell, J. H. Goldsmith, E. G. Folsom, and J. C. Bryant are partners. The two former furnish the capital, and the two latter are to bear the expenses of conducting the business, each one-half. The profits or losses are to be distributed as follows: Bell $\frac{7}{20}$, Goldsmith $\frac{6}{20}$, Folsom $\frac{4}{20}$, and Bryant $\frac{3}{20}$. Bell advanced at commencing business \$18423. Goldsmith advanced \$13142. At the close of the year it is ascertained that the profits have exceeded the losses (not including expenses) \$6823.80. The expense account has an excess of debits of \$2412.08. Bell has drawn out during business \$426. Goldsmith has drawn out \$2342.13. What is each partner's interest in the concern at the close of the year?

NOTE.—In the above example Mr. Goldsmith was allowed to draw a large amount from the business, and by consent of the other partners was not to pay interest upon it. Interest is not to be taken into account in solving this and the following examples unless it is so specified.

2. S. S. Packard, J. T. Calkins, and E. B. Rockwell are partners, to share the gains or losses equally. At the close of one year the following is the result of the business: Cash on hand \$8920, Bills Receiv. \$6273, Merchandise \$5682, Bank Stock \$896, Mr. Packard has drawn from the concern \$672.43, Mr. Calkins \$2471.04, Mr. Rockwell \$1896.06. Bills Payable outstanding \$5957.95.

Packard invested $7420, Calkins invested $6812, Rockwell invested $4635. What has been the gain or loss? What is each partner's present interest in the concern?

3. R. W. Hoadley, H. W. Ellsworth, and H. C. Spencer are partners. They invest in equal amounts, and share gains and losses equally. At the expiration of two years they have Cash on hand $7242, R. R. Stock $4860, Real Estate $4673, Produce $2921. They have Bills Payable outstanding $2326.41. During business Mr. Ellsworth has withdrawn from the concern $924, and Mr. Spencer has advanced to the concern $1138. The total losses have been $754.25, the total gains $3269.54. What is each partner's share of gain or loss? What was each worth at commencing? What is each partner's interest in the concern at closing?

4. R. C. Spencer, W. H. Clark, L. Fairbanks, and C. E. Wilber have been associated in business during the past three years. The books have remained unclosed to this date. At commencement of business R. C. S. invested $6824, W. H. C. $5982, L. F. $7126, C. E. W. $4998. They are to share equally in gains or losses. Since the books were opened the partners have made the following additional investments: R. C. S. $2128.40, W. H. C. $684.12, L. F. $1242.78, C. E. W. $946.64. The partners have each drawn from the concern the following amounts: R. C. S. $8126.42, W. H. C. $5274.18, L. F. $8232.64, C. E. W. $3178.26. There are no resources or liabilities at this date except such as are shown by the partners' accounts. Has the business been prosperous or adverse? If a dissolution now take place, how shall the partners settle with each other?

5. G. B. Collins, A. H. Redington, and Alonzo Gaston were partners in a manufacturing business, commencing July 1, 1869. At that date G. B. C. put into the concern $1600, A. H. R. put in $4000, A. G. made no investment, but was to superintend the business. They were to share equally in gains or losses. Six per cent. interest to be allowed on each side of the partners' accounts. The books are not closed until July 1, 1871, when the following statement is rendered by the bookkeeper: G. B. C. has drawn from the concern at different times to the amount of $14760, the average date at which it was drawn being September 12, 1860. A. H. R. has drawn $11380, average date January 22, 1871. A. G. has drawn $16240, average date May 16, 1870. G. B. C.'s total

investment has been $2982, average date August 17, 1870. A. H. R.'s total investment $6824, average date October 9, 1869. A. G.'s total investment $1528, average date April 24, 1871. Cash on hand $628, Cash in Bank, $2892, Bills Receivable on hand, $5462, Real Estate $7586, Manufactured Articles $4327, Personal Accounts $1523, R. R. Stocks $837, Bills Payable unredeemed $6248, Balance due on personal accounts $4895.

What has been the net gain or loss of the firm? What is each partner's present interest in the concern?

A. H. R. proposes to retire from the business, and the other partners agree to give him $900 more than the books show to be due him. How much will he receive?

6. A of New York, and B of Ohio, enter into an arrangement to buy and sell cattle, and share equally in gains and losses; B to make the purchases, and A to effect most of the sales. A forwarded to B a draft of $8000, B made purchases to the amount of $13682.24. B has forwarded cattle to A during the season, from which he has made sales to the amount of $9241.18. B has made sales to the amount of $2836.24. A has paid out for expenses $364.16. B has paid out for expenses $239.14. At the close of the season B has on hand a number of cattle the cost of which was $2327.34. A has a quantity on hand which are estimated to be worth, in the New York market, $3123.42. The parties now propose to dissolve the copartnership, each taking the stock he has in his possession at the figures given above, and the balance in their accounts, if any, to be paid in cash. What has been the gain or loss? What is each partner's share of gain or loss? What is the cash balance to be paid, and which partner is to receive it?

7. C and D make a contract with government to do a certain piece of work, which is divided into three sections, for which they are to receive as follows, provided the work all pass as No. 1 on being inspected: for Section 1, $1842, for Section 2, $1275, for Section 3, $1563. If any portion of the work pass as No. 2 on inspection, 15 per cent. will be deducted from the original estimate; if any portion as No. 3, 20 per cent. will be deducted. The following is the result of the inspection:

Section 1 passes as No 1; Section 2, as No. 3; and Section 3, as No. 2.

C has drawn from government $728.42. D has drawn $1226.14. D has made disbursements to the amount of $1342.25. C has made disbursements on the work to the amount of $987.45. What has been the gain or loss? How much is due C? How much is due D?

8. Two persons, E and F, enter into business under an agreement that E shall draw from the concern weekly $5 more than F. Subsequently F lends E $260 from his private funds, with the understanding that they were then to draw an equal sum weekly until the loan be liquidated. How long will it take?

9. Three mechanics are partners. They agree that each shall pay $2.25 per day for all working days that he is absent from the business. At the close of the year it is found that A has lost 44 days, B 28 days, C 12 days. How will the partners adjust the matter between them?

10. A, B, and C enter into a copartnership, each investing $5000. A is worth to the business $1500 a year; B $1200; C $1000. At the end of two months B draws out $500, and A adds to his capital $1000. At the end of five months, C withdraws $300. They close up their business at the end of a year, and find that a net profit has been realized of $3500. What proportion of this gain belongs to each partner, if money is worth 7 per cent. per annum?

11. Again: A, B, and C are partners, each investing at the commencement of business $5000, and each being of equal value to the business. They draw from and add to the capital, as before, and at the end of the year ascertain their gain to be, as before, $3500. How will the gain be equitably divided? And should the value of money, as in the former case, have any effect on the adjustment of gains?

12. Again: A, B, and C are partners, investing as in the former two instances, with the understanding that C shall conduct the business, for which he is to receive a commission of 25 per cent. on the net gain. The additions and withdrawals the same as above, and also the gain. How much of the gain should each have?

13. There are five partners in a concern, sharing the gains or losses equally. The liabilities of the firm have been canceled, after which the remaining effects are appropriated by the part-

ners without regard to the proper proportion that each should take. The following is the condition of the partners' accounts, as they now stand. A invested $5680, and has drawn from the concern $4700. B invested $4780, and has drawn $4400. C invested $4980, and has drawn $4600. D invested $3984, and has drawn $3300. E invested $5600, and has drawn $5346. How should the partners properly settle with each other?

14. A and B are partners. They have cash and Bills Receivable on hand to the amount of $5280.11. A has drawn from the concern $2446.80, B has drawn $905.98. A put into the concern $3127.25, B put in $448.75. The firm owe on Paper and Book Bebts $4005.48. What is each partner's present interest in the concern, if they share equally in gains and losses?

15. S. S. Guthrie and H. C. Walker purchased a vessel on joint account, for which they paid $8400, Mr. G. taking one-third interest and Mr. W. two thirds.

During the season G. paid for supplies, repairs and sundry expenses, ...	$956.00
And received cash from freight and passage receipts,	2686.40
W. paid for repairs, supplies, etc., ...	1548.26
And received cash from freight and passage receipts,	4862.48

At the close of the season they sell the vessel for $9000, receiving one half in cash, and the purchaser's note for one-half.

W. agrees to take this note, to apply on his account, at 2% discount, which G. assents to; and then the $4500 cash is properly divided between the two partners; how much is taken by each?

16. Alonzo Gaston and G. B. Collins take a contract of A. H. Redington to sink an aqueduct of a certain width 50 rods in length, and if it average 10 feet deep, they are to receive for constructing the same $26 per rod. If on measurement it average less than 10 feet, 3% will be deducted for the first 6 inches, 5% for the second 6 inches, 9% for the third 6 inches.

A. G. has paid out for wages and material . .	$158
G. B. C. " " " . .	536

A. H. R. has advanced $488, of which A. G. received $242.18, G. B. C. received $245.82. The average depth was to be ascertained by measurement at the end of every five rods, which resulted as follows:

			Ft.	In.				Ft.	In.
End of	1st	five rods	10	4	End of	6th	five rods	7	10
"	2d	"	10	9	"	7th	"	8	4
"	3d	"	9	8	"	8th	"	7	9
"	4th	"	9	4	"	9th	"	9	7
"	5th	"	8	3	"	10th	"	8	8

What has been the gain or loss? How much is due from A. H. R.? How will Mr. Gaston and Mr. Collins settle with each other?

17. A and B contracted with Russell & Co. to erect a Steam Flouring Mill for $11000. Not wishing to be burdened with the salary of a bookkeeper, it was arranged that each partner should keep a strict account of all his receipts and expenditures, and report at the completion of the contract, at which time they would have a general settlement. On the fulfillment of the contract they find their affairs standing as follows, viz.: A has paid out for building material and wages $2862.48. He has received from Russell & Co. at different times to the amount of $1324.08. B has paid out for building material and wages $4788.04. He has received from Russell & Co. $5024.44. There is due the hands for wages $410. What has been the profit? How much is due from Russell & Co.? And how much of it should be paid to A? How much to B?

18. E. C. Bradford, Joseph Dawson, and E. Young have been doing business together as partners, with the understanding that Mr. B. should receive a salary of $1200, for managing the concern, the other partners' time not to be required in the business. Interest to be allowed on both sides of each partner's account. The profits or losses to be divided equally between them. Mr. B. invested January 1, 1870, $6000, May 2, $350, October 12, $500. He drew out February 8, $250, April 4, $380, July 5, $620, November 20, $782. Mr. D invested January 1, $5400, June 12, $280, October 3, $365, December 18, $428. He drew out March 2, $468, May 21, $428, August 3, $542, September 15, $247, December 19, $388. Mr. Y. invested January 1, $4896, May 9, $356, July 2, $428. He drew out March 13, $355, June 3, $126, August 9, $281, October 6, $126, December 24, $439. On December 31, 1871, one year from the day of commencing business, the resources and liabilities (not including the partners' accounts) are as follows, viz.:

Cash on hand,	$5680
Bills Receivable on hand,	4366
Real Estate,	5200
Bank Stock,	5388
	$20634
Bills Payable unredeemed,	$1298.40

What is the net capital of the firm at closing? What is each partner's interest in the concern at closing?

19. The following "Statement," taken from a single entry ledger, in part, the balance being made up from inventories and estimates shows the present condition of the affairs of the firm of A & B.

RESOURCES TAKEN FROM THE LEDGER.

John Smith owes	$460.00
Wm. Brown "	680.00
Geo. Carey "	1260.00
Wm. Dudley "	870.00
Geo. Bryant "	260.00
Amos Dean "	890.00
A has drawn from the concern	2400.00
B " " "	1261.00

LIABILITIES TAKEN FROM THE LEDGER.

Due Baldwin & Co., on account	$546.00
A invested	11600.00
B "	13742.00

RESOURCES NOT SHOWN ON LEDGER, TAKEN FROM INVENTORIES AND ESTIMATES.

Merchandise on hand, per Inv.	$9685.00
Notes and Drafts on hand, per B. B. (Face)	5672.00
Store Fixtures on hand	384.00
Horses, Carriages, and Harnesses	865.00
Stable and Feed	1262.00
City Bank Stock	892.00
House and Lot valued at	6000.00
C. C. & C. R. R. Stock valued at	1820.00
Rent paid in advance	600.00

LIABILITIES NOT SHOWN ON LEDGER.

Firm's Notes and Acceptances outstanding (Face) .	$3826.00
Mortgage on House and Lot	500.00

ADDITIONAL ITEMS OF RESOURCE AND LIABILITY.

The interest upon the Notes and Drafts that are on hand, computed up to this date, is	$694.00
The interest upon the Notes and Drafts that the firm owe, computed to this date, is	148.00

A was to share $\frac{2}{5}$ of the gain or loss, and B $\frac{3}{5}$. What was the firm worth at commencing business? What is the firm worth at the close of business? What has been the net gain or net loss of firm? What is each partner's interest in the concern at closing?

20. Wm. H. Kinne and Edward Rice are partners in the Stone business. Their books are kept by single entry, and run four years before they are closed. An Inventory is taken and a Statement made up at the close of the first year. At the close of the second year, the party having charge of the books neglected to do this. At the close of the third year, the Inventory and Statement are made up, showing the result of two years' business. The Statement and Inventory are made up again at the close of the fourth year.

The profits or losses of the first year are to be divided as follows, viz.: Wm. H. Kinne $\frac{2}{3}$, Edward Rice $\frac{1}{3}$.

At the commencement of the second year J. G. Ranney is admitted as a partner, the three partners to be equally interested in gains or losses.

The following Statements were made out at the close of the first, third, and fourth years.

1867 TO 1868. 1ST YEAR'S BUSINESS.

	RESOURCES.	LIABILITIES.
Cash on hand, ...	$1260.11	
W. H. Kinne—paid him, ...	786.49	
Stone on hand, ...	430.66	
Balances on Ledger, ...	6945.00	
Edward Rice—advanced by him, . .		$2675.44
Gains, ...		6746.82
	$9422.26	$9422.26

1868–1869 TO 1870. 2D AND 3D YEAR'S BUSINESS.

	Resources.	Liabilities.
Edward Rice—paid him, ...	$2675.44	
" " advanced last year, . .		$2675.44
" " paid him, ...	839.58	
W. H. Kinne " "	2947.73	
J. G. Ranney " "	1535.39	
Balances on Ledger, ...	7039.67	
" " last year, . . .		6945.00
Gains, ...		5407.37
	$15027.81	$15027.81

1870 TO 1871. 4TH YEAR'S BUSINESS.

Edward Rice—paid him, ...	$1014.47	
W. H. Kinne " "	1543.16	
J. G. Ranney " "	557.95	
Balances on Ledger, ...	10137.06	
" " last year, . . .		$7039.67
Stone on hand, ...	981.49	
Gains, ...		7194.46
	$14234.13	$14234.13

The above Statements are given precisely as they were made up by one of the partners who handed them to us for adjustment. The student will please exercise his skill in producing the best form of Statement for showing clearly and conclusively each of the answers to the following questions:

How much is the firm worth at the close of each year, and what does the property consist of? What is each partner's interest in the concern at the close of each year?

21. A, B, and C are in the packing business. For several years preceding the last, A and B did a very profitable business. Last year, C, the confidential clerk of the house, was admitted to a copartnership interest, he preferring that to a stated salary. The terms upon which they united for the prosecution of this business were as follows: A and B to put in $30,000 each, no capital required of C; interest to be allowed A and B upon their credits monthly; the interest to be computed upon the last day of each month upon all the credits in the account, at the rate of

10 per cent. per annum; each of the three partners to draw a salary of $3000; the profits to be divided equally.

The interest was computed upon the credit side of A's and B's accounts at the close of each month, and the credits were given as per agreement. Expense account was charged at the close of each month to the amount of $750, and each partner was credited $250 on account of salary. The partners drew from the business during the year just the amount of their salaries, and the same was charged up to them.

At the close of the year the following is the condition of their affairs:

RESOURCES OF FIRM.

Merchandise on hand,	$52460.00
Cash on hand and in bank,	4620.00
Personal accounts,	8940.00
Other items of property, per inventory,	2880.00

LIABILITIES OF FIRM.

Firm notes unredeemed,	9126.00
Personal accounts,	4821.00

EXPENSES AND LOSSES OF FIRM.

As shown by Expense Account,	18126.00
" " Interest "	7120.00
" " other "	5172.00

PROFITS OF FIRM.

As shown by Merchandise Account,	17480.72
" " other "	1260.00

Is C really benefited by having a salary of $3000 allowed to each partner? How is C affected by the interest allowed on A's and B's capital and interest? How shall the partners settle up their matters if a dissolution take place? Could A and B collect from C if he set up the defence of usury on account of interest being compounded?

22. A and B became partners, A to receive $\frac{3}{5}$ of the profits and to take no active part in the business, B to receive $\frac{2}{5}$ of the profits and to travel and sell goods at the expense of the firm. A gave his checks for goods purchased to the amount of $830.05.

B furnished goods at his own expense to the amount of $526.44, paid travelling expenses $298.12, and sent remittances to A to the amount of $264.22, and B gave away samples worth $51.25 at the expense of the firm. B delivered to A the balance of goods unsold to the amount of $1860.46, and C's acc't for goods sold him to the amount of $85. How should the partners settle?

23. A and B enter into copartnership to conduct a manufacturing business. A invests $2846, and B invests $4732. Each partner is to give his whole time and attention to the business, and each one to forfeit $3.75 for each day that he is absent from the business. B, in consideration of his larger capital, is to be allowed 2 per cent. on total sales, in addition to an equal share of profit with A.

The following is a Trial Balance taken from the Ledger at this date.

Dr.		*Cr.*
$28,646.44	Cash,	$25,872.14
9,361.12	Bills Receivable,	4,186.73
30,143.92	Manufacturing Acct.,	24,372.41
8,439.46	Personal Accts.,	12,963.82
6,000.00	Real Estate,	10,928.18
7,924.73	Expense Acct.,	
5,405.86	A's Acct.,	2,846.00
7,126.48	B's Acct.,	4,732.00
13,420.00	Bills Payable,	26,842.48
828.44	Interest Acct.,	296.00
8,426.72	Bank Stock,	12,683.41
$125,723.17		$125,723.17

The manufacturing account is credited with the sales. No entry has been made for B's commission, or for the lost time. A has lost 72 days, B has lost 28 days. On taking an inventory this day, we find there is $18,000 worth of manufactured goods on hand. What has been the net gain? What is each partner's share of the gain? What is each partner's present interest in the concern? What per cent. profit has been made upon sales? What per cent. profit has been made upon the investment?

24. The following are the statements and queries made by an accountant. The student will make out a complete statement of the present condition of affairs if possible.

I am in a dilemma; an accident has happened to my books. Our store was destroyed by fire a few nights ago. All the books were burned except a portion of the ledger. The forepart of the ledger was very much damaged, but upon examination I find that I have all the accounts in a fair state of preservation, except the accounts of the three partners and the cash account. I have taken off the footings of these accounts as below, and now I wish to know if there is any way of ascertaining what was the actual condition of the partners' accounts. I can give you the proportion in which they invested, and it is known by the partners and by myself, that the accounts remained in about that proportion to date of fire.

Dr.		*Cr.*
	A,	
	B,	
	C,	
	Cash,	
$80,962.45	Merchandise,	$74,528.12
20,160.00	Bills Receivable,	16,120.00
42,980.00	Bills Payable,	50,240.00
624.00	Interest Acct.,	190.00
4,130.00	Expense Acct.,	
52,460.00	City Nat. Bank,	38,252.00
25,870.00	Personal Accts.,	22,168.15
2,680.46	Team Acct.,	146 12
	Commission,	2,423.48
896.12	Freight Acct.,	
300.00	Insurance,	
8,900.00	U. S. Bonds,	1,200.00
26,480.00	Real Estate,	3,186.40
5,000.00	Bank Stock,	460.00
1,240.00	Advertising,	
8,960.00	Suspense Account,	5,894.25
1,140.00	Fixtures Acct.,	
2,980.00	Profit and Loss,	4,672.00
940 00	Traveling Expense,	
828 40	Exchange Acct.,	271.18
25,480.00	3d Nat. Bank, N. Y.,	20,140.16
462 00	Office Furniture,	
$313,773.43		$239,896.86

We have adjusted the losses with insurance companies; they pay us $48,200, as follows: To cover loss on merchandise, $25,000; on fixtures, $800; on office furniture, $400; on building, $22,000. None of this property was insured at its full value. None of the goods were saved; the fragments of the building have been sold by the firm to a contractor for $1,500 cash. The lot on which the building stood is valued at $10,000. The money-box came out of the safe all right; it contained $12,241.57. The burnt safe has been sold for $45. The $5,000 of bank stock is worth 12 per cent. premium. The U. S. bonds on hand are worth (face, interest and premium) $8,394. The teams, harnesses and wagons have been sold, since the fire, to Smith & Co., builders, on credit, for $2,294.

The partners invested in this proportion: A $\frac{6}{15}$, B $\frac{5}{15}$, and C $\frac{4}{15}$. They would like to know how each of their accounts stood previous to the fire, if possible, and to have a complete statement or balance-sheet showing the present assets and liabilities of the firm; what each partner's interest or ownership in the effects is at the present time, and what the profits and losses have been. Can all of this be done?

25. A, B, C, D and E are partners—to share the gains and losses equally. As they invested in unequal amounts, it is agreed that their capital shall be equalized by allowing the partners interest on their accounts, the interest to be at the rate of 10 per cent. per annum. As an inducement to the partners to attend strictly to the partnership business, it was agreed that each partner should be charged 4.00 for each day that he was absent from the copartnership business. In consideration of the great personal popularity and influence of A and B, and their ability to command trade, it was agreed that A should be allowed for this superiority over his copartners a sum equal to one-half ($\frac{1}{2}$) per cent. on the sales, and B to be allowed for same a sum equal to one (1) per cent. on the net profits of the business.

The credits of the partners have not been changed during the year. By average the date of each partner's debits is found to be as follows:

A's debits, average date, May 4, 1870; B's September 20, 1870; C's June 16, 1870; D's November 10, 1870; E's March 25, 1870. The copartnership was formed January 1, 1870; the business is brought up to December 31, 1870. The partners that lost time

during the year are as follows: B lost 22 days, D 13 days, and E 41 days.

TRIAL BALANCE TAKEN DECEMBER 31, 1870.

Dr.		*Cr.*
$2,240	A,	$8,000
2,460	B.	12,000
1,200	C,	15,000
1,896	D,	17,000
1,340	E,	25,000
568,900	Merchandise,	562,324
824.728	Cash,	822,465
148,128	Bills Receivable,	122,240
328,160	Bills Payable,	356,250
453.170	City Nat. Bank,	421,640
5,860	Expense,	
9,240	Wages Acct.,	
4,000	Rent Acct.	213
242,080	Personal Accts.,	238,420
8,000	Ill. Cent. R. R. Bonds,	400
1,940	Profit and Loss,	12,672
48,270	Manufacturing Acct.,	45,620
4,260	Engine Acct.,	
2,680	Barn and Team Acct.,	428
520	Fuel Acct.,	
$2,659.672		$2,659,672

INVENTORY.

Merchandise, $90,000; Illinois Central R. R. Bonds, $8,640; numerous articles for schedule of property belonging to "Manufacturing Account," $12,460; items per schedule belonging to "Barn and Team Account," $1,980; items per schedule belonging to "Expense Account," $8[illegible]; items per schedule belonging to "Engine Account," $2,926. The firm also own a dock privilege that was granted them for the term of two years by one of our customers, without any consideration, and for which they could command $800 for the unexpired time, having been offered that a few days ago in cash. No account has been opened with this on the books.

B retires from the copartnership at the close of the year, the remaining partners to continue their relations in every respect as

before. On retiring B is to receive $\frac{1}{3}$ of the amount due him by check on City National Bank, $\frac{1}{3}$ in Bills Receivable belonging to firm, and the remaining $\frac{1}{3}$ by the note of new firm at 60 days.

What shall be the settlement at the close of the year?

26. A, B, and C were partners, manufacturing a patented article that was very salable and profitable. Other parties wished to become interested with them, and it was thought best to change into a stock company. An inventory was taken, which resulted as follows:

Merchandise manufactured, ...	$4,000
Engine and Machinery, ...	6,400
Tools and Implements, ...	600
Material unmanufactured, ...	1,500
Teams, Wagons, Harnesses, etc., . . .	2,500
Total, ...	$15,000

The original partners, A, B, and C, owned this property as follows: A, $7,740; B, $3,160; and C, $4,100. They decided to stock the new company at $50,000, which was "watering" the old stock or capital $35,000. D and E propose to take $10,000 each of the new stock, and pay for it as follows: D pays cash $2000, four notes of 2,000 each, at two, four, six, and eight months from date. E pays cash $3,000, and seven notes of $1,000 each, payable at three, five, seven, eight, nine, ten, and twelve months from date. 100 shares of $100 each, of the new stock, is issued to each of the five stockholders. It was the understanding that $3,000 of the cash paid by D and E for their stock should be left in the business three months before A, B, and C would be entitled to appropriate it to their own use.

A new set of books is to be opened for the new company. How will the new books be opened, and what entries are necessary to be made in the old books in order to close up the business of the old firm?

27. A and B entered into copartnership June 1, 1867, for a term of one year, A to invest $10,000, and B $5,000, each to devote his whole time to the business. Each to be charged when absent from business—A at the rate of $4 per day, and B at the rate of $2. Interest to be computed on partners' accounts at the rate of 10 per cent. per annum, the gains and losses to be divided—A $\frac{2}{3}$ and B $\frac{1}{3}$.

At the close of the year the following is the condition of their affairs:

Jan. 1,	1867,	A invested,	$10,000
"	"	B invested,	5,000
Feb. 15,	"	A put in,	500
Mar. 16,	"	A drew out,	1,000
Mar. 30,	"	A put in,	2,500
Apr. 20,	"	B put in,	900
Dec. 31,	"	Gains on Merchandise,	7,200
"	"	Merchandise inventory,	9,900
"	"	Expense (loss),	2,200
"	"	Interest (loss),	400
"	"	Bills Payable, notes out,	500
"	"	Bills Receivable, notes on hand,	6,000
"	"	Fixtures, inventory,	500
"	"	A lost 20 days,	
"	"	B lost 30 days,	

What is each one's interest in the business at the close of the year?

28. Mr. A is a merchant doing business on his own account. He also formed a copartnership with B to run a tannery—the style of the tannery firm to be "B & Co." A owned the tannery buildings and machinery, and was to rent the same to B & Co. for $900 per annum; the firm to keep them in as good repair as when they were taken. Of the necessary capital to carry on the business, A furnished $600 in cash and B furnished $900. This was placed to each partner's credit on the tannery books. A few days afterwards $1,200 of this money was taken to the store by A to purchase hides with. No entry was made on the books at the tannery when this money was taken, but it was placed to the credit of B & Co. on the store books. A and B were to share equally in gains or losses. Both sets of books were kept by single entry—the books at the tannery being very single, or singular, but little bookkeeping being done there.

B's family do their trading at A's store and have an open account there.

A buys hides at the store and sends them to the tannery. Most of the leather manufactured is sent to the store to be sold.

A gives the firm credit for it at the wholesale price when he receives it. B sells some leather at the tannery at retail. B's books consist only of memoranda. At the close of the year the partners wish a settlement. B makes up a statement of his transactions as follows: He has received for leather sold at tannery \$246.12. He has paid for wages to the men employed \$748; has paid for bark and lime \$264; has paid for repairs \$116.50. He has taken an account of stock, and finds there is on hand, finished leather to the value of \$348.25, and unfinished leather to the value of \$861.25; lime, bark, etc., on hand to the value of \$68.

B & Co. were credited on the books at the store for all leather that was taken by A, and they were charged for all hides purchased by A and sent to the tannery. They were also charged for amounts traded by the workmen at the tannery on account of wages. B & Co.'s account upon the store books is charged \$2,684, and is credited \$4,732.50. B is charged on the store books for family expenses, \$621. It is estimated that it would cost \$75 to put the buildings and machinery in as good condition as they were at the commencement of the year. A owes B for a horse bought of him for private use, \$150. If a dissolution were to take place at the close of the year, what would be a proper settlement between the partners?

20. A and B are partners in the milling business. A invested $\frac{1}{4}$ and B $\frac{3}{4}$ of the capital, which is to be kept in this proportion by a settlement at the end of each year's business. Current expenses to be borne and profits and losses to be shared in the same proportion as the investment. The profits of the business are not to go to increase the working capital, but this is to remain unchanged; if a loss occur it is to be made good by an additional investment—A one-fourth, B three-fourths. Expenses for permanent improvement to be proportioned—A one-fourth, B three-fourths, and at the annual settlement no account is to be made of the gain or loss by the rise or fall of real estate (lot, mill and machinery), or of the amount paid for permanent improvement, save that it shall be so equalized that each shall bear his proportion of the whole amount thus expended—A one-fourth, B three-fourths.

A takes full charge of the business in consideration of a salary of \$1000 for his services. He keeps no separate set of books for this business, but makes the record with that of another business

in which B is not interested. He keeps a "Mill Account," which he treats as a personal account, debiting it with all purchases of grain and crediting it with sales of grain, flour, etc. He keeps, however, a "Mill Expense" and "Permanent Expense" account, expressly to represent current and permanent expenses in the mill business. He keeps his books by a single entry system, or rather by no particular system, and yet keeps such memoranda as enables him to exhibit in a statement at the required time the running affairs of the mill business, and consequently how he stands in relation to the same at the time of settlement. At the end of the present business year he rendered to B the following statement:

Paid cash for Wheat, ...	$8,116.75
" " Corn, ...	5,586.74
" " Oats, ...	3,842.60
" " Barley, ...	2,160.90
" Current Expenses, ...	1,438.01
" Permanent Expenses, ...	371.94
Let B have flour, etc., ...	129.72
Paid B's orders in cash, ...	120.00
Sold Wheat for cash, ...	2,674.50
" " on account, ...	462.25
" Flour for cash, ...	4,120.50
" Oats "	1,202.20
" Corn "	1,680.40
" Corn Meal, cash, ...	4,280.40
" Barley for cash, ...	2,821.04

INVENTORY.

Flour on hand, ...	$1,569.00
Oats on hand, ...	2,852.50

B is transacting a distinct and separate business, and on his books is found the following record of affairs pertaining to the mill business:

Paid out during the year for current expenses,	$140.25
Paid for permanent expenses, ...	187.50
Sent Wheat to mill from farm, . . .	28.00
Advanced cash to A, ...	146.45

In the current expenses paid by A is included $816 on his own salary. What is each partner's interest in the year's profits?

ANNUITIES.

NOTE.—For Annuity Tables, see Part Third, Arts. 787, 788, 798.

723. An Annuity originally signified an "annual income," but in a more general sense, it is now applied to *repeated payments* of various kinds. In this sense, an Annuity is the annual, semi-annual, quarterly, monthly, weekly, or daily payment of a certain or regularly varying sum, whether the payment be regular or intermittent, for a given term of years, for life, or forever.

724. 1. Regular annuities involving the periodical payment of fixed sums are *certain* or *contingent*, *perpetual* or *limited*, *immediate* or *deferred*.

2. A *certain Annuity* is one that is unconditionally payable for a definite time.

3. A *contingent Annuity* is one whose commencement or continuance, or both, depend on some specified contingency, generally the life or death of one or more persons.

4. A *perpetual Annuity* or a *perpetuity* is one that continues forever. A *limited annuity* ceases at a certain time.

5. An *immediate Annuity* or an *Annuity in possession*, is one that begins immediately.

6. A *deferred Annuity* or an *Annuity in reversion*, is one that does not begin immediately; the term of reversion may be definite or contingent.

7. An annuity is said to be *foreborne* or *in arrears* if it has not been paid when due.

725. The subject of annuities is one of great practical importance. Its principal applications are leases, life-estates, rents, dowers, reversions, life-insurance, etc. The problems are readily solved by means of tables in Part Third which give the present and final values of $1 for a given number of years at the ordinary rates of interest, or each problem may be solved by analysis, by applying the principles of compound interest and geometrical progression.

726. 1. The *amount*, or *final value*, of an annuity is the sum of the amounts of all its payments, at compound interest, from the time each is due, to the end of the annuity.

This is readily found for $1 by finding the sum of a geomet-

rical series (Art. 395) whose *first term* is \$1, *ratio* \$1 plus the rate of interest, and *last term* the ratio raised to a power one less than the number of years. Thus the amount of an annuity of \$1 for 8 years at 5% would be $\$1 + 1.05 + 1.05^2 + \ldots\ldots + 1.05^7$; for the amount of the \$1 due the 8th year would be \$1, the amount of the \$1 due the 7th year would be \$1.05, etc., and the sum of this series is equal to $\frac{\$1.05^8-1}{1.05-1} = \$.477455 \div .05 = \$9.549$.

2. On this principle tables are constructed showing the amount of \$1 annuity for any number of years. (Art. 787.)

3. Ex. Suppose a rental of \$500 a year remain unpaid for 8 years; what is the amount due at 5% compound interest?

Solution.—$500 + 500 \times 1.05 + \ldots\ldots + 500 \times 1.05^7 =$ Amount. Factoring this, it becomes $500 \times (1 + 1.05 + 1.05^2 + \ldots\ldots + 1.05^7)$ $= 500 \times \frac{1.05^8-1}{1.05-1} = 500 \times .477455 \div .05 = \$4774.55$ or the value of $\frac{1.05^8-1}{1.05-1}$ may be taken from the table, giving $\$500 \times 9.5491 =$ \$4774.55.

727. To find the amount of an annuity for any length of time at compound interest.

RULE.

Multiply the given annuity by the amount of one dollar for the required time at the required rate and intervals as given in the table. Or,

Multiply the given annuity by the sum of the geometrical series whose first term is 1, ratio 1 plus the rate for one interval of time, and last term that power of the ratio whose index is one less than the number of years or other periods of time concerned.

NOTE.—If the amount be required at simple interest, multiply the annuity by the sum of the *arithmetical series* whose first term is 1, common difference the rate for one interval, and the number of terms the number of intervals of time.

Ex. 1. What is the final value or amount of an annuity of \$150 for 12 yr. at 4% compound interest?

2. What is the value of \$200 annuity forborne for 15 yr. at 6% compound interest?

3. What is the value of a semi-annual income of $380 in arrears for 6 yr. at 8% compound interest?

4. What is the value of a monthly due of $12 foreborne 6 months at 12% simple interest?

5. How much greater is the compound amount of $150 annuity for 10 yr. at 6% than the amount of the same at simple interest?

728. 1. The *present value* of an annuity, at compound interest, is the sum of the present values of all its payments; or the present worth of its final value. The present value, put out at compound interest, will amount, at the time of the expiration of the annuity, to its final value.

2. To find the present value of an *annuity certain,* at compound interest.

The present value of $1 annuity for 8 years at 5% is equal to the amount of the *annuity* divided by the *amount per cent.* or the amount of $1 for the same time and rate (Arts. 525 and 530); that is, $9.549109 ÷ 1.477455 = $6.463213.

On this principle tables are constructed showing the present value of $1 annuity for any number of years. (Art. 788.)

729. RULE.

Multiply the annuity by the present value of $1 annuity at the required time and rate as given in the table. Or,

Divide the amount of the annuity for the given time by the amount of $1 for the same time and rate.

Ex. 1. What is the present value of $120 annuity for 25 yr. at 6%?

Solution.—$120 × 12.783356 (Art. 788) = $1534.00272; or, as the *amount* of the annuity for the time and rate is $658.374144 and the amount of $1 principal is $4.291871, therefore $658.374144 ÷ 4.291871 = $1534. (Art. 530.)

2. What is the present value of an annuity of $650, to continue 15 years at 5 per cent.?

3. What is the present worth of a leasehold of $1200, payable annually for 50 years, at 6 per cent.?

4. A widow is entitled to $140 a year, payable semi-annually, for 18 years; what is the present value of her interest, at 10 per cent. compound interest?

5. I wish to purchase an annuity which shall secure to my ward, at 4 per cent. compound interest, \$250 a year for 14 years. What must I deposit in the annuity office?

730. 1. To find the present value of a *perpetuity* we have only to consider that the perpetual annual interest on any principal constitutes a perpetuity of which the present value is the principal.

RULE.

Divide the sum periodically due, by the interest on one dollar for one interval of time at the given rate.

Ex. 1. What is the present value of a perpetual leasehold of \$1200 a year, at 5 per cent.?

NOTE.—The present value must evidently be a principal which yields an annual interest of \$1200 at 5 per cent

$$\$1200.00 \div .05 = \$24000, \text{ present value.}$$

2. What is the present value of the perpetual lease of \$4800 a year, at 8 per cent. interest?

3. The ground rent of an estate yields an annual income of \$2400, payable quarterly, at 4 per cent. per annum. What is the value of the estate?

4. What is the present value of a perpetual leasehold of \$1600 a year, at 6 per cent.?

5. What is the present value of a perpetual leasehold of \$1600 a year, payable semi-annually, at 6 per cent. interest per annum, compounded semi-annually?

NOTE.—In this case there is a perpetuity of \$800 per half-year at a rate of interest equal to 3 per cent. for the same time.

6. What is the present value of a perpetual leasehold of \$1600 a year, payable annually, at 6 per cent. interest per annum, compounded semi-annually?

NOTE.—6 per cent. compounded semi-annually is equivalent to 6.09 per cent per annum.

7. What is the present value of a perpetual leasehold of \$1600 per annum, payable semi-annually, at 6 per cent. interest per annum?

NOTE.—6 per cent. per annum is equivalent to 5.912602 per cent. per annum, compounded semi-annually, or 2.956301 per cent per half year, or the present value of the amount of the *annual income* (\$1624) may be found.

731. To find the present value of a *deferred annuity certain.*

Ex. 1. What is the present value of an annuity of $250, deferred 12 years and to continue 10 years, allowing 6 per cent. compound interest?

OPERATION.

$12.041582	=present worth of $1 for 22 yr.				
8.383844	"	"	12 yr.		
$3.657738	"	"	10 yr. deferred 12 yr.		
250					
$914.434500	"	$250	"	"	

Explanation.—The present worth of an annuity of $1 for 22 years must be equal to its present worth for 12 years, *plus* its present worth for the 10 succeeding years. Hence the present worth of an annuity of $1 for 10 years deferred 12 years, must equal its present worth for 22 years, *minus* its present worth for 12 years. The present worth of $250 is evidently 250 times the present worth of $1.

RULE.

Find from the table the present value of an annuity of $1, commencing at once and continuing till the TERMINATION *of the annuity, and also till the reversion* COMMENCES. *Multiply the difference of these present values by the given annuity.*

2. What is the present value of a leasehold of $1800, deferred 10 years and to run 20 years, at 5 per cent. compound interest?

732. To find the present value of a *deferred perpetuity.*

Ex. 1. What is the present value of a perpetuity of $500, to commence in thirty years (first payment 31 years hence), compounding interest at 5 per cent.?

OPERATION.

Value of perpetuity 30 years hence, . . $10000.00
Present value of $1 due in 30 years, . . .231377

$10000 \times .231377 = 2313.77.$

Explanation.—$2313.77 improved for 30 years at 5 per cent. compound interest (see Discount Table, Art. 783), will produce $10000, which will yield $500 per annum perpetually thereafter. By proportion,

$1 : .231377 :: 1000 : 2313.77.

RULE.

Multiply the value of the perpetuity at the time of beginning by the present value of $1 due at that time.

NOTE.—The present value of an *immediate annuity limited* (Art. 724) will be the present value of a perpetuity, minus the present value of a perpetuity deferred till the limited annuity ends.

Ex. 2. A lease, whose rental is $1000 a year, is left to two sons. The elder is to receive the rent for 9 years and the youngest for the 12 years succeeding. What is the present value of each son's interest, allowing 6 per cent. compound interest?

3. What is the present value of a perpetuity of $900, to commence in 30 years, allowing 4 per cent. compound interest?

733. To find the annuity which can be purchased for a given sum, allowing a certain rate of interest.

Ex. 1. What annuity for 20 years will $15000 purchase, allowing interest at 7 per cent.?

OPERATION.

$$\$15000 \div \$10.593997 = 1415.89.$$

Explanation.—Since $10.593997, at 7 per cent. compound interest for 20 years, yields an annuity of $1, $15000 will yield an annuity equal to $15000 ÷ 10.593997.

RULE.

Divide the given sum by the present value of an annuity of $1, for the given rate and time.

NOTE.—To find an annuity that will *amount* to a certain sum, divide by the *amount* of $1 annuity for the rate and time.

2. The present value of a lease, running 25 years, at 6 per cent. compound interest, is $15340.037; what is the annual income?

3. An annuity in arrears for 8 years, at 5 per cent. compound interest, amounts to $47745.545; what is the annuity?

4. A yearly pension, unpaid for 12 years, at 6 per cent. compound interest, amounted to $1591.7127; what was the pension?

5. What perpetual annuity deferred for 10 yr. could be purchased for $5000, allowing 5% compound interest?

6. What semi-annual income could be secured for 15 yr. by the permanent investment of $25000 in securities bearing 3½ per cent. semi-annually?

LIFE ANNUITIES.

734. 1. A contingent annuity terminating with the life of a given person is called a *life annuity.*

2. Limited to a given number of years of such life, it is a *temporary life annuity.*

3. Beginning a certain number of years hence, it is a *deferred life annuity.*

4. Depending on the joint continuance of two or more lives, it is a *joint life annuity.*

5. There are many possible varieties of contingent annuities, but those mentioned are the most usual and the most important.

735. 1. Various tables have been constructed to illustrate the probabilities of life at various ages. Of these the one most generally accepted is known as the "Actuaries' Life Table," based on the experience of 17 English life insurance offices.

2. The tables given in Part Third, Arts. 791 and 2, enable us to answer readily almost all questions relating to life contingencies. The table of "numbers living" is the basis of the other tables.

3. From it we can perceive at a glance what amount of probability there is of a person of average health surviving any given number of years. Thus, out of 86292 persons living at age 30, we perceive that there will be 69517 living at age 50—or in other words, that a person now aged 30 has 69517 chances out of 86292 of living 20 years. Expressed technically, it is said that the "probability" $= \frac{69517}{86292}$.

4. The "expectation," or average number of years which will be enjoyed, is found by adding together the "numbers living" at all the ages succeeding the given age, including half the number now living, and dividing the sum by the number now living. Thus the "expectation" at age 94 is $\frac{92+89+37+13+4+1}{184} = \frac{236}{184} = 1\frac{13}{46}$ years.

5. All these calculations of "expectation" are, however, curious rather than useful, being utterly valueless for any practical purpose. Some persons suppose that by finding the amount of an "annuity certain" for a term equal to the "expectation," they can obtain the value of a contingent or life annuity; but they are mistaken.

736. To find the present value of a *simple endowment.*

A *simple endowment* is a contract by which a certain sum is to be paid in case a certain person survives to a certain age.

Ex. 1. What is the present value of \$1 receivable at 50, the present age being 30?

OPERATION.

NOTE.—All the following operations are based on the rate of 4 per cent. interest.

$$0.456387 \times 69517 \div 86292 = 0.36767.$$

Explanation.—\$0.456387 (see Discount Table, Art. 783) improved at 4 per cent. compound interest for 20 years will produce \$1; and (number living at 50) 69517 times as much, or \$31726.66, will produce \$69517. If, therefore, 86292 persons living at age 30 deposit equal sums amounting in all to \$31726.66, it will accumulate in 20 years to \$69517, an amount sufficient to furnish \$1 to each survivor at 50 years of age. Whence the sum deposited by each will be \$31726.66 ÷ 86292 = \$0.36767.

Proof.—\$0.36767 × 86292, compounded for 20 years = \$69517.

RULE.

Multiply the present value of the desired sum (using the Discount Table) by the number living at the age of endowment named, and divide the product by the number living at the present age.

2. How much should be deposited to secure \$1 in case a child now 10 years old lives till 30?

OPERATION.

$$0.456387 \times 86292 : 100000 = 0.393825.$$

NOTE.—If examples similar to Ex. 2 be worked out for each age, and the results tabulated, we shall obtain the column of Payments or Deposits to secure \$1 at each age in Table, Art. 792. This column may therefore be described as containing *deposits required at age* 10 *to secure* \$1 *at the ages specified, at* 4%.

3. Find the present value, at age 43, of a *simple endowment* of \$1000 at 60.

OPERATION.

Deposit at 10 to secure \$1 at 60, . . .	0.078761
Divided by deposit at 10 to secure \$1 at 43, .	0.208786
Gives value at 43 of \$1 due at 60, . . .	0.377234
	× 1000
Value at age 43 of \$1000 due at 60, . .	\$377.234

Explanation.—The deposit securing $1 at 60 would previously secure *its value* at 43.

Deposit for $1 at 43 : Deposit for Value required :: $1 : Value required.

RULE.

Divide the deposit in the table for the age specified, by that for the present age, and multiply the quotient by the amount of endowment desired.

4. What is the value at age 34 of a *simple endowment* of $5000 at age 50?

737. To find the value of a *contingent annuity, immediate or deferred.*

Ex. 1. What is the present value, at age 43, of a life annuity of $1, first payment at 44?

OPERATION.

Deposit at 10 to secure annuity of $1 at 44, .	3.000992
Divided by deposit at 10 to secure $1 at 43, .	0.208786
	14.3735

Explanation.—The amount ($3.000992) of deposit at 10 to secure an annuity of $1 beginning at 44 is taken from the *annuity-deposit table*, Art. 792, which is formed by the successive addition of the figures in the previous column. (An "annuity at age 43" means an endowment of $1 at age 44, *plus* an endowment of $1 at 45, *plus*, etc., 0.198497 + 0.188630 + 0.179160 + 0.170057 + 0.161306 + etc., to age 99 = 3.000992.) Then we have the proportion:

Deposit for $1 : Deposit for annuity :: $1 : Value of annuity

RULE.

Divide the "annuity deposit" in the table for the age when the first payment is to be made, by the "endowment deposit" at the present age, and multiply the quotient by the amount of the given annuity.

2. What is the value, at age 25, of an annuity of $500, first payment at age 50?

3. I pay $453.76 per annum on my life insurance policy, and am 47 years old. One payment is now due. What is the present value of this and all future payments?

738. To find the value of a *temporary life annuity.*

Ex. 1. What is the present worth, at age 43, of an annuity of $1, to begin at age 46 and run ten years?

OPERATION.

Annuity-deposit for age 46, ...	2.613865
Less " " 56, ...	1.192834
	1.421031
Divided by deposit for $1 at 43, . . .	0.208786
Gives	6.8062

Explanation.—The temporary annuity in question is obviously the same as a deferred life annuity beginning at 46, *minus* a similar annuity beginning at 56.

RULE.

Subtract the "annuity-deposit" for the age succeeding the last payment from that for the age at first payment; divide the remainder by the "deposit for $1" for the present age, and multiply the quotient by the amount of the annuity.

Ex. 2. A man aged 46 has six more annual premiums of $211.16 each to pay on his life policy, including one now due; what is their present value?

739. To find the *annual premium* for a term of years, required to secure a *simple endowment* or *deferred annuity*.

Ex. 1. It is required to find the annual premium payable for ten years to secure a *simple endowment* of $1000 at 70—the present age being 46.

OPERATION.

To find deposit corresponding to the benefit desired:

Deposit for $1 at age 70, ...	.034067
	1000
Deposit corresponding to benefit, . .	34.067

To find deposit corresponding to premiums of $1 each:

Annuity-deposit for age 46, . . .	2.613865
Less " " 56, . . .	1.192834
Deposit for 10 premiums of $1, . .	1.421031

$$34.067 \div 1.421031 = 23.973.$$

RULE.

Find the deposit at age 10 to secure the benefit desired, and use it as a DIVIDEND. *Find the annuity-deposit for the age at*

which the first premium is payable, and also for the age succeeding the last payment, and use their DIFFERENCE *as a* DIVISOR *and multiply the quotient by the annuity required.*

Ex. 2. What is the annual premium payable for 5 years to secure an *annuity* of $1000 to begin at age 50 and run 17 years—present age 23 ?

OPERATION.

Deposit to secure $1 per annum, beginning at					50= 1.950452
"	"	"	"	"	67= 373994
"	"	"	"	"	23=10.449837
"	"	"	"	"	28= 7.950490

$$\frac{1.950452-0.373994}{10.449837-7.950490}\times 1000=630.748.$$

LIFE INSURANCE.

740. 1. A *life insurance policy* is a contract by which the insurer agrees, in return for certain *premiums* received, to pay a certain sum to the heirs or assigns of the insured, on the death of a person designated in the policy. Insurance of this nature is transacted mostly on the mutual plan, the profits being divided periodically among the assured themselves.

2. Insurance is granted either for the whole of life, or for a limited term; the policies being known as *whole-life* or *term* policies respectively.

3. Term policies are either *simple term policies*, or are combined with a *simple endowment* (Art. 736), thus forming a compound contract called an *endowment assurance.* Endowment assurance policies (often called merely "endowments") are contracts under which the sum assured is payable at a certain age, if the party live, or at death, if that occur sooner.

4. Other and more complicated forms of policies are also issued.

741. 1. The *sum assured* is usually made payable either two or three months after receipt of the proofs of death. In theory, however, the calculations are made on the supposition that all claims are paid at the end of the policy-years in which the deaths occur.

2. The *premiums* upon all these various forms of policies may be payable in one sum at starting (*single premiums*); in a limited number of payments (*five, ten,* or *twenty premiums*), or during the whole continuance of the policy. In practice, the premiums may be paid annually, semi-annually, or quarterly; but the calculations are always made on the supposition that payments are made not oftener than once a year.

3. The *net premiums* are calculated without making any provision for expenses or profit—that is, having regard only to the net cost of insurance as depending on compound interest. The *net premiums* thus computed are increased by a margin or *loading* to cover expenses and contingencies; these are called *office premiums* and are published, and used in making contracts.

742. 1. As the premiums chargeable on all kinds of policies increase with every year of age—a man at 41, for example, being charged more for the same benefit than one aged 40, it follows that the company must reserve something out of each premium received, in order to place the contract in as favorable a position as a new policy issued later at an advanced price. Thus, a policy is issued at 30 at a premium, say, of $100. For a similar policy at 50 the company cannot afford to charge less than $200 per annum. As the age of the party increases, therefore, such sums must be reserved as, when he reaches 50, will place the policy issued at 30 for a premium of $100 in as safe a position as a new policy requiring $200. These sums thus periodically set aside, and duly compounded, constitute what is called the *reserve* or *net value* of the policy.

2. The amount of reserve may be ascertained at any time by *finding the present value of the benefits (assurance, endowment, etc.) secured by the policy, and subtracting from it the present value of an annuity corresponding to the future net premiums receivable.*

3. The *surrender value* of a policy is equal to the reserve, diminished by a charge sufficient to pay the expense of replacing the risk withdrawn.

743. 1. DISTRIBUTION OF SURPLUS.—The surplus of most of the American mutual life insurance companies is divided annually among the policy-holders on what is known as the "Contribution Plan."

2. The annual "contribution" of any policy to the company's surplus arises from three sources:

1st. *Interest.*—The "reserve" is supposed to be compounded at 4 per cent. It actually earns 6 or 8, and the difference goes towards the general surplus.

2d. *Vitality.*—The amount of losses by death in the company is not usually so great as that provided for in the calculation of premiums. The difference, whatever it is, is presumed to be "contributed" by the several policies in the proportion of the actual amount of insurance enjoyed by each during the year.

3d. *Loading.*—The loading on the premium paid, diminished by a charge for expenses, and improved by interest, also goes to help make up the surplus.

3. The "Contribution Plan" requires each of these three elements of surplus on each policy to be separately computed, and their sum, together with a share of the miscellaneous profits of the company, if any, is returned to the policy-holder as a dividend.

4. There are several popular methods of computing dividends at a certain per cent. on the premiums paid, but the actual amount of dividends is determined by the actuaries according to the general principles explained.

5. The plan known as the "Partnership Plan of Dividends" is based on the principle of distributing the surplus in proportion to the *present worth* or *reserve value* of each policy, this representing the interest of the policy-holder as a partner in the company.

744. 1. Insurance is afforded on the *stock plan* at rates somewhat above the *net premiums* but less than the mutual premiums, the policy-holders receiving none of the profits, but only the insurance paid for.

2. By making premiums sufficiently high to insure both policy and premiums, some companies return all the premiums on certain conditions.

3. The various and complicated forms of insurance granted by the various companies in this and in foreign countries, can only be learned by actual practice, or from an experienced actuary, and a patient study of Life Insurance Reports and textbooks.

745. To find the premium to insure $1 during any given future year.

1. Ex. What is the net premium to insure $1 during the year succeeding age 50—the present age being 30?

OPERATION.

Number living at age 50, ...	69517
" " " 51, ...	68409
" dying during the year, ...	1108
Multiplied by present value of $1 due in 21 years,	0.438834
Present value of $1108, ...	$486.228
Divided by number living at 30, ...	86292
Gives a net premium = ...	.0056347

Explanation.—Out of 86292 persons now living aged 30, 1108 will die during the year succeeding age 50. The present value of $1108 payable 21 years hence, or $486.228, divided among 86292 persons now living, gives the premium which would secure to each of them an assurance of $1 in case of death during the year in question.

2. RULE.

Divide the "number dying" for the year specified, by the "number living" at the present age, and multiply the quotient by the present value of $1 due at the end of the year in question.

NOTE.—The present value of $1 is shown in the Discount Table, Art. 783.

3. Ex. What is the value, at age 10, of an insurance of $1 during the year succeeding age 30?

4. If we wished to know the value at age 10 of *two* years' insurance, beginning at age 30, we should go through this operation for ages 30 and 31, and add the results; and so for any other number of years. The column, Art. 792, headed "Assurance Deposit," contains the deposit required at age 10 to secure assurance for life, beginning at the ages specified. Thus the deposit required for life assurance beginning at age 30 is (see Table) $0.12058; beginning at age 31, $0.11739; difference (or deposit for the year's assurance from 30 to 31), $0.00319, as above.

746. To find the net premium upon any policy.

Ex. 1. What is the premium payable for ten years to secure an *endowment assurance* of \$1 at age 50 (or death, if previous)—present age 24?

OPERATION.

	Deposit at 10 for life assurance at 24, . .	0.141936
Less	" " " " " 50, . .	0.069779
	" " " assurance from 24 to 50, .	0.072157
Add	" " " endowment at 50, . .	0.144796
	" " " total benefits, . . .	0.216953

	Deposit at 10 for annuity of \$1 at 24, . .	9.901999
Less	" " " " " 34, . .	5.636021
	" " " 10 premiums of \$1 each, .	4.265978

$$0.216953 \div 4.265978 = 0.05086.$$

Explanation.—\$0.216953 deposited at age 10 would secure the benefits desired, and therefore would pay the 10 premiums whose amount is to be determined. Then we have the proportion:

Deposit for premiums of \$1 each : Deposit for actual premiums : : \$1 : actual premium.

RULE.

Find the deposit at age 10 to secure the benefits desired, and use it as a DIVIDEND. *Find the deposit at age 10 to secure the premiums payable (supposing them to be \$1 each) and use it as a* DIVISOR. *The quotient will be the net premium required.*

Ex. 2. What is the *annual premium*, payable for life, to secure assurance of \$5000 for life, both beginning at age 32?

OPERATION.

Assurance-deposit for age 32 to secure \$1, . .	0.114290
	× 5000
" " " " " \$5000, .	571.45
Divided by annuity-deposit for age 32, . . .	6.335141
Net annual premium, ...	\$90.20

Ex. 3. What is the *annual premium* payable at 27 to assure \$1000 for 20 years (without endowment)?

OPERATION.

Assurance-deposit for age 27, ...	0.130747
Less " " 47, ...	0.076414
" " 27 to 47, . . .	0.054333
	× 1000
" for $1000, ...	$54.333
Annuity-deposit for age 27, ...	8.404486
Less " " 47, ...	2.434705
" " 27 to 47, ...	5.969781

$54.333 \div 5.969781 = \$9.101.$

Ex. 4. What is the *single premium*, at age 23, for an endowment-assurance of $1000, payable at death or 40?

OPERATION.

Assurance-deposit for age 23, ...	0.145921
Less " " 40, ...	0.092403
" 23 to 40, . . .	0.053518
Add deposit at 10 for $1 at 40, ...	0.242502
	0.296020
	× 1000
Deposit corresponding to total benefits, . . .	$296.020
Divided by deposit at 10 for $1 at 23, . . .	0.547838
	$540.34

Ex. 5. Required the annual premium for *ten* years to secure assurance of $1000 for life, present age 37.

6. Required the annual premium for *life* to secure assurance of $5000 for the same term, present age 25.

7. Required the value at age 40 of a paid-up life policy of $1000.

8. Required the total value at age 40 of all the future *net* premiums of $18.04 each, receivable on a certain life-policy, including one just due. (See Art. 727, "Annuities.")

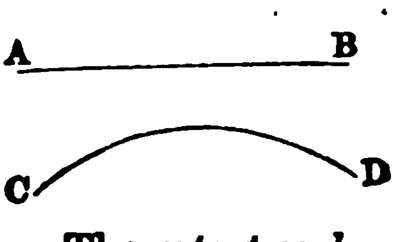
9. Required the *reserve* which should be held on a life policy of $1000 on which $18.04 is the *net* annual premium, one premium being just due, present age being 40.

NOTE.—The answer to the last example is obtained by subtracting that to Ex. 8 from that to Ex. 7. (See Art. 742, 1.) All the above premiums are *net*.

MENSURATION.

747. 1. *Mensuration* is the process of computing the length of lines, the area of surfaces, and the volume of solids.

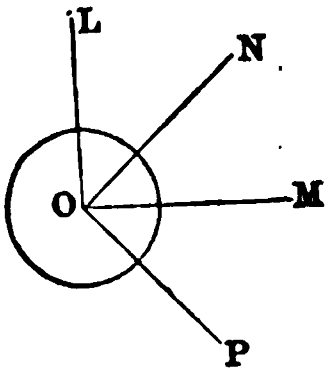
2. A *point* has position only, but no extent.



3. A *line* has extension in *length* only; a *straight* line has the same direction throughout as the line A B; a *curved* line changes its direction at every point, as the line C D.

The *extent* or *length* of a line is estimated in linear units, as inches, etc.

4. An *angle* is the divergence or difference in direction of two lines from a common point, as the angle of the lines F E and F G. F, the point of divergence, is called the vertex of the angle. The extent of an angle is estimated in units of circular measure, applied to the arc of a circle intercepted by the lines forming the angle, and having the vertex of the angle as a center.



5. A *right* angle is formed by two lines perpendicular to one another; it is measured by a quarter of the circumference of a circle and is called an angle of 90°, as L O M. An *acute* angle is one of less extent than a right angle, as L O N; an *obtuse* angle is one of greater extent than a right angle, as L O P.

6. *Parallel* lines are lines having the same direction, thus being equally distant from one another throughout their whole extent.

748. 1. A *surface* has *length* and *breadth* or extension in *area* only. A *plane* surface or figure extends in the same direction throughout, and a line joining any two points in it is a straight line lying wholly within the surface. A *curved* surface changes its direction.

2. The *area* of a surface is estimated in units of area, or *square* units, as sq. ft., etc. Art. 284.

749. 1. A *solid* or *body* has length, breadth and thickness, or extension in *volume*.

2. The *volume* or contents of a solid is estimated in units of volume or cubical units, as cu. inches, etc. Art. 285.

NOTES —1. A solid is bounded or limited by a curved surface or by several plane surfaces; a surface by a continuous curved line or several straight or curved lines, and a line is limited by points or if curved may be continuous.

2. The *perimeter* of a plane figure is the length of the line or of all the lines that bound it.

750. 1. A *rectilineal* figure or *polygon* is a plane figure bounded by straight lines.

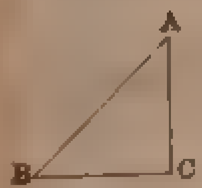
2. A *regular polygon* is one whose sides are all equal and whose angles are all equal, that is, it is an equilateral and equiangular figure. An *irregular* polygon has unequal sides and angles.

3. A polygon of *three* sides is called a *triangle*, of *four* sides a *quadrilateral*, of *five* sides a *pentagon*, of *six* sides a *hexagon*, and so on.

4. *Similar* figures have their corresponding angles equal and the sides including those angles proportional.

5. The areas of similar figures are to each other as the *squares* of their corresponding linear dimensions, and the volumes of similar solids are to each other as the *cubes* of their corresponding linear dimensions.

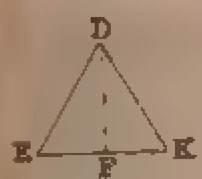
751. Triangles.—1. A *right-angled* triangle has two of its sides perpendicular to one another, as A C B with a right angle at C.



NOTES.—1 In such a triangle B C is called the *base*, A C the *perpendicular*, and A B the *hypothenuse*.

2. *The square of the hypothenuse is equal to the sum of the squares of the other two sides*; thus $BC^2 + AC^2 = AB^2$.

2. An *oblique-angled* triangle does not contain a right angle, as D E K, which contains only acute angles, or L M N, which contains an obtuse angle at N.



NOTES 1 Either side of a triangle may be taken as the *base*.

2 The *altitude* of any triangle is the length of a straight line drawn from the vertex of the angle opposite the base, perpendicular to the base or to base extended, as A C, D F, or L P.

752. To find the area of a ***Triangle.***

Multiply the base by half the altitude. Or,

From half the sum of the three sides subtract each side separately; then multiply together the half sum and the three remainders, and take the square root of the product.

EXAMPLES.

1. How many square yards in a piece of ground of triangular shape, one side measuring 50 yards, and the shortest distance from this side to the opposite angle being 24 yards?

2. The three sides of a triangle measure respectively 10, 12, and 14 feet; what is the area?

3. How much greater would be the area if we double the linear dimensions in the last example?

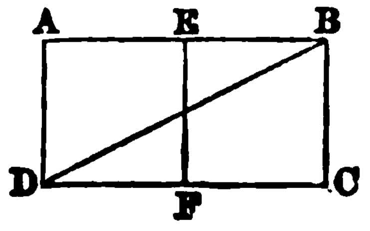
4. What should be the dimensions of a triangle similar to the one proposed in Example 1, to make the area 5400 sq. yards?

5. If one side of a field containing 50 acres is 50 rods, what must be the length of the corresponding side of a field of similar shape to contain 112½ acres?

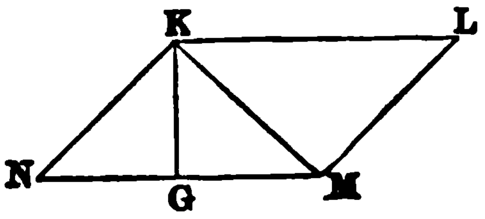
6. The area of a certain triangular field is $8\frac{3}{4}$ acres, and one of its sides is $37\frac{1}{2}$ rods long; what is the length of a perpendicular from the opposite corner?

7. What is the side of a square containing the same area as a triangle whose base is 36.1 feet, and altitude 5 feet?

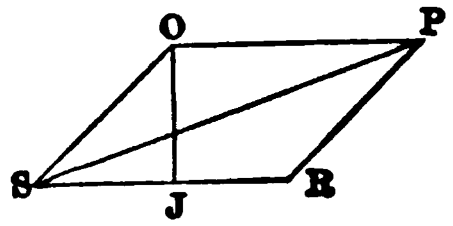
753. *Quadrilaterals, Pentagons, etc.*—1. A *parallelogram* is a quadrilateral whose opposite sides are parallel. The opposite sides are also equal. A *rectangle* is a right-angled parallelogram, as A B C D. A *square* is an equilateral rectangle, as A E F D.



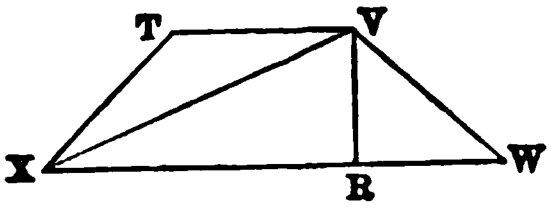
2. A *rhomboid* is an oblique-angled parallalogram, as K L M N.



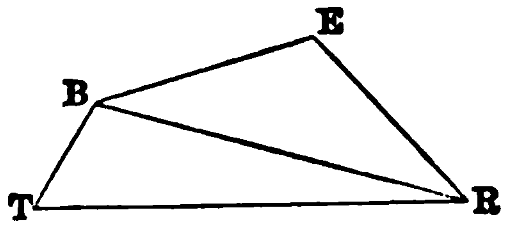
3. A *rhombus* is an equilateral rhomboid, as O P R S.



4. A *trapezoid* is a quadrilateral having only two sides parallel, as T V W X.



5. A *trapezium* is a quadrilateral having no two sides parallel, as B E R T. It is a species of irregular polygons.



NOTES.—1 Any side of a quadrilateral may be taken as the base.

2. The altitude of a parallelogram or trapezoid is the perpendicular distance between the parallel sides, as A D, K G, O J, and V R.

3. The diagonal of a quadrilateral is a straight line joining the vertices of any two opposite angles, as B D, K M, S P, V X, and B R. The two diagonals of a rectangle are equal.

754. 1. To find the area of any ***Quadrilateral*** *having two sides parallel.*

Multiply half the sum of the two parallel sides by the altitude, or perpendicular distance between those sides.

NOTES.—1. This rule is equally applicable to the square, rectangle, rhombus, rhomboid. and trapezoid.

2. The area of any rectangle is also thus equal to the product of its length and breadth.

2. To find the area of a ***Regular Polygon.***

Multiply the perimeter by half the perpendicular drawn from the center to one of its sides. Or

Multiply the square of one of the sides by the appropriate number, as given in the following

TABLE.

Triangle, . . .	.433013	Octagon, ...	4.828427
Square, ...	1.000000	Nonagon, . . .	6.181824
Pentagon, . . .	1.720477	Decagon, . . .	7.694209
Hexagon, . . .	2.598076	Undecagon, . . .	9.365640
Heptagon, . . .	3.633912	Dodecagon, . . .	11.196152

8. To find the area of an ***Irregular Polygon*** of four or more sides.

Divide the figure into triangles by diagonals connecting some one angular point with each of the others; then compute the area of each triangle, and find their sum.

NOTE.—This method may be used for finding the area of any irregular plat of land by measuring each side, and the diagonals from any angle.

EXAMPLES.

1. How many square feet in a board 14 feet long and 10 inches wide?

2. How many square feet in a board 14 feet long, it being 15 inches wide at one end, and 9 inches at the other?

3. If the same board be cut in two in the middle, making each piece 7 feet long, how much more would one piece contain than the other?

4. If the parallel sides of a trapezoid are 48 and 52 feet, and the perpendicular breadth 17 feet, what is the area?

5. What is the area of a regular decagon, one of its sides being 10 feet, and the perpendicular let fall from the center upon one of the sides being 15.3884 feet?

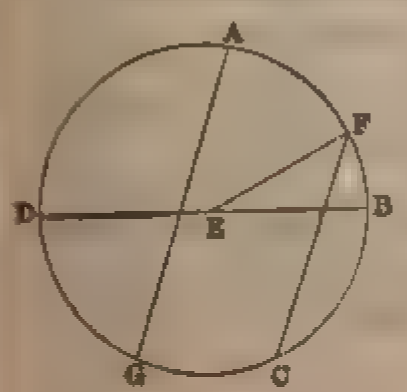
6. What is the area of a regular pentagon, one of its sides being 20 rods?

7. What must be the side of a regular octagonal field to contain 3 acres, 2 roods, 14 rods, 10 yards?

8. The sides of a certain trapezium measure 10, 12, 14, and 16 rods respectively, and the diagonal which forms a triangle with the first two sides named is 18 rods; what is the area?

9. How much more fencing will it require to enclose an acre in the form of a square than in the form of a hexagon?

755. Circles.—1. A *circle* is a plane figure bounded by a curved line, every part of which is equally distant from a certain point in the figure called the center, as the figure bounded by the line A B C D A, with the point at E as a center.



2. The *circumference* is the bounding line.

3. The *diameter* is a straight line drawn through the center and terminated by the circumference, as D B.

4. The *radius* is a straight line drawn from the center to any point in the circumference, as E F, E B, or E D.

5. An *arc* is any portion of the circumference, as F B or F B C.

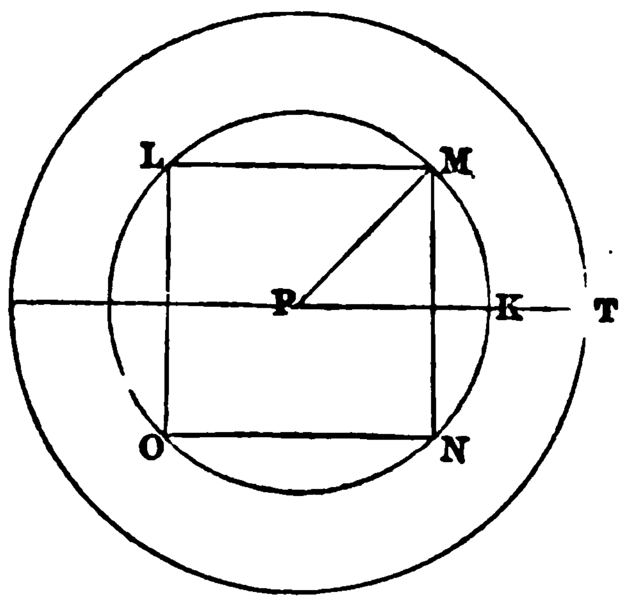
6. A *sector* is bounded by two radii and the intercepted arc, as the figure bounded by the radii E F, E B, and the arc F B.

7. A *chord* is the straight line joining the extremities of an arc, as F C.

8. A *segment* is bound by an arc and its chord, as the figure bounded by F B C and F C.

9. A *zone* is a portion of a circle bounded by two parallel chords, as the space between F C and A G.

10. Any plane figure is said to be *inscribed* in a circle when the vertices of its angles are in the circumference of the circle, as L M N O is inscribed in the circle whose radius is P M.



NOTE.—The side of an inscribed *hexagon* is equal to the radius.

11. *Concentric circles* have the same center but different radii, as the circles whose radii are P K and P T.

12. A *circular ring* is the space enclosed between the circumferences of two concentric circles.

756. The ratio between the diameter and circumference is an important number in problems relating to the circle, and its approximate value should be retained in the memory. That ratio is very nearly equivalent to the fraction $\frac{355}{113}$, which may easily be remembered from its containing the first three odd numbers each repeated, and found in their natural order, if we read the denominator first. If expressed decimally, and the approximation be carried to thirty places, we have the following, 3.14159265358979323846264338328.

757. 1. To find the ***Circumference*** of a circle.

Multiply the diameter by $\frac{355}{113}$ *or* 3.1416.

2. To find the ***Diameter,*** the *circumference* being known.

Divide the circumference by $\frac{355}{113}$ *or* 3.1416, *or multiply it by* .318309.

3. To find the ***area of a Circle,*** the *diameter* being known.

Multiply the square of half the diameter by $\frac{355}{113}$ *or* 3.1416.

4. To find the area of a circle, the *circumference* being known.

Divide the square of half the circumference by $\frac{355}{113}$ *or* 3.1416, *or multiply the square of the circumference by* .079577.

5. To find the area of a circle, both the *circumference* and *diameter* being known.

Multiply the circumference by one-fourth of the diameter.

6. To find the *diameter* or *circumference* of a circle, the *area* being known

Divide the area by $\frac{355}{452}$ *or* 3.1416; *the square root of the quotient will be equal to half the diameter, and the diameter multiplied by* $\frac{355}{113}$ *or* 3.1416, *will equal the circumference.*

EXAMPLES.

1. Suppose the earth to be distant from the sun 95,000,000 of miles, and to revolve in a circular orbit, how far does it move in an hour?

2. What is the diameter of a peach which measures 12 inches in circumference?

3. What must be the inside measure of a square box to exactly contain a globe 56 inches in circumference?

4. If a horse be tied to a stake in a meadow, with a halter 20 feet long, upon how many square yards can he feed?

5. If a circular fish pond is to be laid out containing just half an acre, what must be the radius or length of the cord needed to describe the circle?

758. 1. To find the area of a **sector** of a circle.

Multiply half the length of the arc by the radius, or take the same part of the area of the circle as the number of degrees in the arc are of 360°.

2. To find the area of a **segment.**

From the area of a corresponding sector, subtract the area of the triangle formed by the chord and radii, for a segment less than a semicircle, but add these areas for a segment greater than a semicircle.

3. To find the area of a **Zone.**

From the area of the circle subtract the areas of the segments not included in the zone.

4. To find the area of a **Circular Ring.**

Find the difference in the areas of the two circles, or find the product of the sum of the diameters, the difference of the diameters and .7854.

5. To find the side of an **Inscribed Equilateral Triangle.**

Multiply the diameter by .866025, *or the circumference by* .275664, *or the radius by the square root of three* ($\sqrt{3}$).

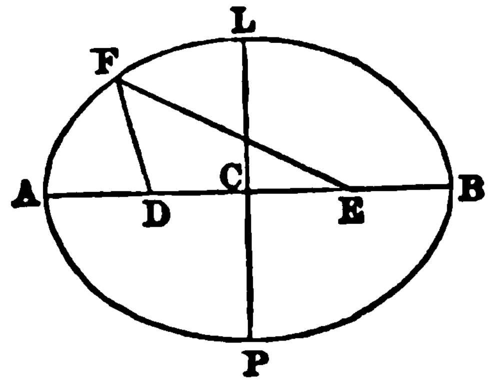
6. To find the side of an **Inscribed Square.**

Multiply the diameter by .707106, *or the circumference by* .225079, *or the radius by the square root of two* ($\sqrt{2}$).

7. To find the side of a *square that shall contain the same area* as a given circle

Find the square root of the area, or multiply the diameter by .886227, *or multiply the circumference by* .282094.

759. 1. An *ellipse* is a plane figure bounded by a line so curved that the sum of the distances from any point in it to two fixed points is equal to the longest diameter, which diameter passes through those points. Thus F D + F E = A B.



2. The points D and E are called the foci; the point C, at the center of A B, is called the center of the ellipse, and the shortest diameter, L P, passes through the center, perpendicular to the longest diameter.

3. To find the area of an *Ellipse*.

Find the product of half the longest diameter, half the shortest diameter, and 3.1416.

760. EXAMPLES.

1. How many square inches in a pane of glass in a circular window of 16 inches radius, if the curved edge of the pane of glass measures 8 inches. and comes to a point at the center?

2. What is the area of the sector of a compass plate of 6 inches radius, the arc of which is bounded by the East and North-east points?

3. On a circular map of 4 ft. radius, what is the area of a segment whose arc is 47 degrees and chord 3.19 ft.?

4. On the same map, what is the area of a segment whose arc is 227 degrees and chord 7.336 ft.?

5. On the same map, what is the area of a zone bounded by two segments, the arc of each segment being 133°?

6. What is the area of a walk around a circular pond 30 ft. in diameter if the width of the walk be 4 ft.?

7. What is the length of one side of the largest equilateral triangular piece of glass that can be cut from a circular piece whose radius is $4\frac{1}{2}$ inches?

8. How large a square timber can be cut from a log whose circumference is 100 inches?

9. What would the side of a square coin measure, the surface of which should equal that of a five-cent piece $\frac{8}{10}$ of an inch in diameter?

10. What is the area of an elliptical pond whose longest diameter is 84 ft. and shortest diameter 54 ft.?

SOLIDS.

761. *Prisms* and *Cylinders.* 1. A *prism* is a solid whose sides or faces are parallelograms, and whose ends or bases are equal and parallel polygons. A prism is triangular, quadrangular, pentagonal, etc., according as its bases are triangles, squares, or pentagons, etc.

2. A *parallelopiped* is a prism whose bases are parallelograms.

3. A *cylinder* is a solid resembling a prism, but having, instead of polygons, for its bases, equal parallel circles; its surface otherwise being uniformly curved instead of being made up of several plane faces.

4. The *lateral* or *convex surface* of a prism or cylinder does not include the two ends or bases.

5. A solid is said to be *right* when its axis or general direction is at right angles with the base; otherwise it is *oblique*.

6. A *cube* is a right prism the base and each side of which is a square.

762. 1. To find the entire *surface* of a *right prism* or *right cylinder*.

Multiply the perimeter, or circumference of the base, by the height, and to the product add the area of the two bases.

2. To find the *solidity* of a *prism* or *cylinder*.

Multiply the area of the base by the perpendicular height.

NOTES.—1. In this case it matters not whether the solid be *right* or *oblique*.

2. For a right prism it is the product of the three dimensions of length, breadth, and thickness.

EXAMPLES.

1. What is the entire surface of a right cylinder 10 feet long, the diameter of the base being 2 feet?

2. What is the solidity of a triangular prism whose perpendicular height is 150 feet, the sides of the base being 60, 80, and 100 feet?

763. Pyramids and Cones.—1. A *pyramid* is a solid whose base is a polygon, and whose sides are triangles meeting in a common point called the *vertex*.

2. A *right* pyramid is one whose vertex is in the line perpendicular to the base at its center.

3. A *right cone* is a solid resembling a right pyramid, but having a curved surface and a circular base.

4. The *altitude* or *height* of a pyramid, or of a cone, is the perpendicular distance from the vertex to the plane of the base.

5. The *slant height* of a regular pyramid or cone is the shortest distance from the vertex to the boundary of the base.

6. The *frustum* of a pyramid or cone is that part that remains after cutting off the top by a plane parallel to the base.

764. 1. To find the entire surface of a regular pyramid, or of a cone.

Multiply the perimeter or the circumference of the base by half of the slant height, and to the product add the area of the base.

2. To find the solidity of any pyramid or cone.

Multiply the area of the base by one-third of the altitude.

3. To find the entire surface of a frustum of a right pyramid, or of a cone.

Multiply the sum of the perimeters, or of the circumferences of the two ends, by half of the slant height, and to the product add the areas of the two ends.

4. To find the solidity of the frustum of any pyramid, or of a cone.

Multiply the areas of the two bases together, and extract the square root of the product. This root will be the area of a base which is a mean between the other two. Take the sum of the areas of the three bases, and multiply it by one-third of the altitude; the product will be the solidity.

EXAMPLES.

1. What is the entire surface of a right cone, the diameter of the base and the slant height being each 40 feet? What its solidity?

2. What are the contents of a stick of round timber whose length is 20 feet, the diameter of the larger end being 12 inches, and of the smaller end 6 inches?

3. What is the entire surface of a triangular pyramid whose slant height is 25 ft. and each side of whose base is 10 ft.?

765. ***Spheres.***—1. A *sphere* is a solid bounded by a curved surface, all the points of which are equally distant from a point within called the center.

2. The *diameter* or *axis* of a sphere is a line passing through the center, and terminated each way by the surface.

3. The *radius* is a line extending from the center to the surface, and is equal to half the diameter.

766. 1. To find the *surface* of a *sphere.*

Multiply the diameter by the circumference. Or,

Multiply the square of the diameter by 3.1416.

2. To find the *solidity* of a *sphere.*

Multiply the cube of the diameter by 3.1416, *and take one-sixth of the product.* Or,

Multiply the area of the surface by one-sixth of the diameter.

EXAMPLES.

1. How many square miles on the surface of the earth, it being 7912 miles in diameter?

2. What are the solid contents of a globe whose diameter is 10 inches?

3. The surface of a certain sphere is 1648 square feet; what is the surface of another whose diameter is three times as great?

4. What is the diameter of a sphere containing $\frac{8}{27}$ of the solidity of another sphere $7\frac{1}{2}$ feet in diameter?

767. *Lumber* and *Timber*. (Art. 285.)

NOTE.—One foot of lumber measure is 12 inches long, 12 inches wide, and 1 inch thick, and all kinds of lumber and timber are estimated by this unit, except that timber is sometimes estimated by the cubic foot.

1. To find the contents of an *inch board.*

Find the product of the length and breadth in feet.

NOTES.—For a two-inch board, multiply the area by 2, for a half-inch board divide the area by 2, etc

2. If the board tapers, take half the sum of the two ends for the average width.

2. To find the contents of a plank, joist, etc., estimated in board measure.

Find the product of the width in inches, the thickness in inches, and the length in feet; and take one-twelfth of this.

NOTE.—If the timber tapers in width, take the average width; if it tapers also in thickness, the contents in *cubic feet* may be found by *multiplying half the sum of the areas of the two ends in inches by the length in feet and dividing the product by 144.*

3. To find the cubic feet in round timber.

Multiply the length in feet by the square of one-fifth the mean circumference in inches and divide the product by 72.

NOTE. For perfect accuracy deduct $\frac{1}{10}$ of the estimated contents.

768. *Gauging.*—Gauging is the art of measuring the capacity of casks and vessels of any form. In commerce, most of the gauging is done by the use of technical rules and instruments, which give only an approximate result; perfect accuracy by a long process being less desirable than a tolerable approximation requiring but little skill and labor.

To gauge accurately use the following general

RULE.

Having taken the necessary linear measurements, compute by the rules under MENSURATION *heretofore given, the volume of the inside of the cask or vessel in cubic inches. Divide this by* 2150.42 *for the measurement in bushels, by* 282 *for beer gallons, by* 231 *for wine gallons.*

769. SPECIAL GOVERNMENT RULES.

NOTE.—*Casks* are of four varieties, according to the curvature of the staves. Their contents or capacity is estimated by using gauging instruments and extensive tables adapted to the different measurements and varieties.

To estimate the capacity of *cisterns* or *tubs.*

1ST RULE.—*Multiply together the mean diameter in inches, the decimal* .0034, *and the depth in inches; the product will indicate the number of wine gallons.*

2D RULE.—*Find the sum of the square of the top diameter, the square of the bottom diameter, and four times the square of the middle diameter; one-sixth of this sum will equal the square of the true mean diameter, and this quotient multiplied by the height will give the capacity in cylindrical inches. To get wine gallons divide by* 294.

NOTES.—1. The measurements are to be made in inches and tenths of an inch.

2. A cylindrical inch is the volume of a cylinder one inch in diameter and one inch high, and is equal to .785398 + cu. in.

3D RULE.—*Divide the outside mean circumference by* 3.1416; *from the quotient subtract twice the thickness of the staves; the remainder will be the mean diameter; then proceed as in the 1st Rule.*

MECHANICS.

770. 1. ***Gravitation*** is that force by which portions of matter at sensible distances are attracted toward each other.

2. The measure or amount of force with which a body is attracted toward the earth constitutes its *weight*. (Art. 288.)

3. The weight of a body upon or above the earth *varies inversely as the squares of its distances from the earth's center.*

Ex. 1. What would a body, weighing 500 lb. on the surface of the earth, weigh, if elevated 16000 miles?

2. How far must it be elevated to weigh 5 lbs.?

771. 1. The ***Specific Gravity*** (s. g.) of any substance is the ratio of its weight to the weight of an equal volume of some other substance taken as a standard. (See Table, Art. 941.)

2. The standard for solids and liquids is pure water, one cu. ft. of which weighs 1000 ounces Avoirdupois.

3. The specific gravity of liquids is commonly tested by the *Hydrometer*, which bears a scale so graduated as to indicate 100 when placed in *Proof Spirit*. Proof Spirit is alcoholic liquor, containing for one-half its volume, alcohol having a specific gravity of .7939 at 60° F., and water for the other half.

4. To find the specific gravity of a solid or liquid heavier than water.

Divide the weight of a given bulk of the substance by its loss of weight in water.

Ex. 1. A body weighed 48 lb. in air and 36 lb. in water; what is its specific gravity?

2. What is the specific gravity of a piece of iron which weighs 34 lb. in water and 5 lb. more in air?

772. 1. To find the solid feet in a body from its weight.

Divide the weight in ounces by 1000 *times the specific gravity, as given in the table.*

2. To find the weight in ounces from the solid contents.

Multiply the solid contents in cubic feet by 1000 times the specific gravity, as given in the table.

3. To find the quantity of each substance in a compound of two substances. *Compute by Alligation* (Art. 327), or

Multiply the difference between the s. g.'s of the two elements by the s. g. of the mixture; then multiply the s. g. of the element required, by the difference between the s. g. of the mixture and the other element; finally, find the weight of the entire mixture in ounces; then by proportion.

1st Prod. : 2d Prod. : : Whole weight : weight of element required.

Ex. 1. What is the volume of an irregular piece of iron weighing 16 lb., if its specific gravity be 7?

2. What is the weight of a cubical block of marble, one edge of which measures 3½ ft., if its specific gravity be 2.75?

3. How much cast gold s. g. 19.258, and cast silver s. g. 10.474, would be required to make 100 lb. coin metal s. g. 17.647?

773. ***Falling Bodies.***—1. A body falling from a height will fall 16 feet in the first second, three times that distance in the second, and so on, the space passed over in each successive second increasing as the odd numbers, 1, 3, 5, 7, 9, 11, etc. *The entire space passed over is as the square of the time.*

2. To find the height from which a body falls.

Multiply the square of the time occupied in falling by 16.

Ex. 1. A stone let fall from the top of a steeple reached the ground in three seconds; how high was the steeple?

2. What space will a body fall through in one minute, and what in the last second?

3. How long would it take a body to fall one mile?

774. 1. ***Velocity*** is the speed or rate at which a body moves.

2. To find the velocity.

Divide the space passed over by the time consumed in passing over it.

Ex. The moon is 240000 miles from the earth; suppose it take light 1¼ seconds to reach us, what is its velocity?

775. 1. The ***Momentum*** of a body is its quantity of motion.

2. To find momentum.

Multiply the weight by the velocity.

Ex. What is the momentum of a body weighing 25 lb. moving at the rate of 80 ft. per second?

776. ***Mechanical Powers.***—1. The mechanical powers are the Lever, the Wheel and Axle, the Pulley, the Inclined Plane, the Wedge, and the Screw.

2. The *Lever* is a solid bar turning upon a pivot.

3. The pivot is called its fulcrum.

4. To find the weight which can be moved by a lever, no account being taken of the weight of the lever.

Multiply the power by its distance from the fulcrum, and divide by the distance of the weight from the fulcrum.

777. 1. The *Wheel* and *Axle* consist of a cylinder and a wheel of larger diameter immovably attached to each other, and revolving upon the same axis.

This may be considered a perpetual lever, the radius of the wheel being the long arm, and the radius of the cylinder the short arm of the lever.

2. To find the weight which can be raised by the wheel and axle.

Multiply the power by the radius of the wheel and divide the product by the radius of the cylinder or axle.

778. 1. A *Pulley* is a small wheel fixed in a block and turning on an axis by means of a cord which runs in a groove formed on the edge of the wheel.

2. A *system of pulleys* is a number of pulleys so arranged that great weight may be raised by a small expenditure of power.

3. To find the weight which may be raised by a system of pulleys.

Multiply the power by twice the number of movable pulleys, or, when the end of the rope is attached to the movable block, by twice the number of movable blocks, plus one.

NOTE.—In practice, allowance must be made for friction in estimating the advantage gained in the use of the pulley, inclined plane, wedge, and screw.

779. 1. The *Inclined Plane* is a rigid surface, sloping at any angle between the horizontal and vertical.

2. To find the weight which can be raised on the inclined plane.

Multiply the power by the length of the inclined plane and divide the product by its perpendicular height.

3. The *Wedge* is composed of two inclined planes, and the advantage gained by its use may be approximately reached by the above rule.

780. 1. The *Screw* is a cylinder worked by a lever, and having upon it a thread, which is a winding inclined plane.

2. To estimate the advantage of the screw.

Multiply the power by the circumference of the circle which it describes, and divide the product by the distance the weight is raised by one revolution.

781. EXAMPLES.

Ex. 1. The long and short arms of a lever are respectively 23 ft. and 5 ft.; what weight can be raised by a power of 18 lb. applied to the long arm, and what if applied to the short arm?

2. A lever is 24 ft. long. At what distance from the fulcrum must a weight of 40 lb. be placed in order that it may be raised by a power of 8 lb.?

3. A power of 6 lb. is required to raise a weight of 42 lb. by means of the wheel and axle. What must be the ratio of the diameter of the wheel to that of the axle?

4. A power of 63 acts on a wheel of 7 ft. radius; what weight suspended from a rope winding round an axle 18 inches in diameter will balance this power?

5. Suppose a power of 80 lb. applied to a set of 4 movable pulleys; what weight will it move, allowing a deduction of two thirds for friction?

6. If a man has just strength enough to lift a cask weighing 196 pounds perpendicularly into a wagon 4 feet high, what weight could he raise by means of a plank 10 feet long, with one end resting upon the wagon and the other on the ground?

7. The length of a plane is 12 feet, the height is 3 feet; what is the ratio of the power to the weight to be raised?

8. The distance between the threads of a screw being half an inch, and the circumference described by the power 20 feet, what ratio will exist between the power and the weight?

9. A power of 40 pounds acting at the end of a lever attached to a screw describes a circle of 100 inches; what resistance will the power overcome, the distance between the threads of the screw being 2 inches?

782. PRACTICAL PROBLEMS FOR FARMERS.

1. *Measuring Grain.*—By the United States standard, $2150\frac{42}{100}$ cubic inches make a bushel. As a cubic foot contains 1728 cubic inches, a bushel is to a cubic foot nearly as 2150 to 1728; or, for practical purposes, as 4 to 5. Therefore, to convert cubic feet into bushels, it is only necessary to multiply by $\frac{4}{5}$.

Ex. How much grain will a bin hold which is 10 feet long, 4 feet wide, and 4 feet deep?

Solution.—$10 \times 4 \times 4 = 160$ cubic feet. $160 \times \frac{4}{5} = 128$, the number of bushels.

2. *To measure grain on the floor.*—Make the pile in form of a pyramid or cone, and multiply the area of the base by one-third the height. To find the area of a circular base, multiply the square of its diameter by the decimal .7854.

Ex. A conical pile of grain is 8 feet in diameter, and 4 feet high; how many bushels does it contain?

Solution.—The square of 8 is 64, and $64 \times .7854 \times \frac{4}{3} = 67.02$, and $67.02 \times \frac{4}{5} = 53.616$ bushels. *Ans.*

3. *To ascertain the Quantity of Lumber in a Log.*—Multiply the diameter in *inches* at the small end, by one-half the number of inches, and

this product by the length of the log in *feet*, which last product divide by 12.

Ex. How many feet of lumber can be made from a log which is 36 inches in diameter and 10 feet long?

Solution.—36 × 18 = 648; 648 × 10 = 6480; 6480 ÷ 12 = 540. *Ans.*

4. *To ascertain the Capacity of a Cistern or Well.*—Multiply the square of the diameter in inches by the decimal .7854, and this product by the depth in inches; divide this product by 231, and the quotient will be the contents in gallons.

Ex. What is the capacity of a cistern which is 12 feet deep and 6 feet in diameter?

Solution.—The square of 72 (the diameter in inches), is 5184; 5184 × .7854 = 4071.51; 4071.51 × 144=586297.44, the number of cubic inches in the cistern. There are 231 cubic inches in a gallon, therefore, 586297.44 ÷ 231 = 2538 gallons. To reduce the number of gallons to barrels, divide by $31\frac{1}{2}$.

5. *To Ascertain the Weight of Cattle by Measurement.*—Multiply the girth in feet, by the distance from the bone of the tail immediately over the hinder part of the buttock, to the fore part of the shoulder-blade; and this product by 31, when the animal measures *more than 7 and less than 9 feet in girth;* by 23, when *less than 7 and more than* 5; by 16, when *less than 5 and more than* 3; and by 11, when *less than* 3.

Ex. What is the weight of an ox whose measurements are as follows: girth, 7 feet 5 inches; length, 5 feet 6 inches?

Solution.—$5\frac{1}{2} \times 7\frac{5}{12} = 40\frac{19}{24}$; $40\frac{19}{24} \times 31 = 1264 +$. *Ans.*

NOTE.—A deduction of one pound in 20 must be made for half-fatted cattle, and also for cows that have had calves. It is understood, of course, that such standard will at best give only the *approximate* weight.

6. *Measuring Land.*—To find the number of acres of land in a rectangular field, multiply the length by the breadth, and divide the product by 160, if the measurement is made in rods, or by 43560 if made in feet.

Ex. How many acres in a field which is 100 rods in length, by 75 rods in width?

Solution.—100 × 75 = 7500; 7500 ÷ 160 = $46\frac{14}{16}$. *Ans.*

7. To find the contents of a triangular piece of land, having a rectangular corner, multiply the two shorter sides together, and take one-half the product.

8. *Measurement of Hay.*—10 cubic yards of meadow hay, weigh a ton. When the hay is taken out of old, or the lower part of large stacks, 8 or 9 cubic yards will make a ton. 10 or 12 cubic yards of clover, when dry, make a ton.

Hay stored in barns, requires from 300 to 400 cubic feet to make a ton, if it be of medium coarseness, and greater or less quantity, varying from 300 to 500 solid feet, according to its quality.

PART THIRD.

INTEREST, EXCHANGE, AND MISCELLANEOUS TABLES.

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783. *Rates of Interest, Penalties for Usury, and Statute Limitations in the United States.*

NOTES.—1. The legal rate is that rate of interest recoverable by law in the absence of a definite contract specifying the rate.

2. The different terms of limitation apply to different cases.
3. The time of limitation of accounts is generally reckoned from the date of the last item.
4. The mark refers to sealed and to witnessed instruments.
5. The mark (?) indicates that there are exceptions.
6. The marks (a), (b), (c), etc., refer to notes following the table.

STATES AND TERRITORIES.	Legal rate of interest.	Rate allowed by contract.	PENALTIES FOR USURY.	STATUTE LIMITATIONS. Open Accts.	Notes.	Judgments.
	%	%		Yr	Yr.	Yr
Alabama	8	8	Forfeiture of entire interest	3	6	10*, 20(?)
Alaska						
Arizona	10	Any				
Arkansas	6	Any		2, 3	7, 21	10
California	10	Any	(a)	2	4	5
Colorado	10	Any		3	4	5
Connecticut	6	6	(b) Forfeiture of entire interest.	6	17	3, 6
Dakota	7-10	Any		6	15	6
Delaware	6	6	Forfeiture of the principal.	3 (?)	6, 20*	20 (?)
District of Columbia	6	10	Forfeiture of entire interest	3	3	12
Florida	8	Any		5	5	
Georgia	7	10	(c). Forfeiture of excess	3	3	3, 12
Idaho	10	Any				
Illinois	6	10	Forfeiture of entire interest.	5	6	16 (?)
Indiana	6	10	Forfeiture of excess	6	20	5, 10
Indian Territory						
Iowa	6	10	Forfeiture of entire interest; 10% of it to school fund.	5	10	20
Kansas	7	12	Forfeiture of entire interest	3	5	5
Kentucky	6	10	Forfeiture of excess	1, 2	7	14
Louisiana	5	8	(d). Forfeiture of entire interest.	3	5	10
Maine	6	Any		6	6 (?)	20
Maryland	6	6	Forfeiture of excess	3	3	3
Massachusetts	6	Any		6, 20*	6, 20*	20
Michigan	7	10	Forfeiture of excess	6	6	10
Minnesota	7	12	Forfeiture of excess	6	6	10
Mississippi	6	10	Forfeiture of excess	3	6	7
Missouri	6	10	Forfeiture of entire interest to public schools.	5	10	20
Montana	10	Any				
Nebraska	10	12	Forfeiture of entire interest (?)	4	5	5
Nevada	10	Any				
New Hampshire	6	6	Forfeiture of thrice the excess, and costs.	6	6	20

Rates of Interest, Penalties for Usury, etc.—**Continued.**

States and Territories.	Legal rate of interest.	Rate allowed by contract.	Penalties for Usury.	Statute Limitations. Open Accts.	Notes.	Judgments.
	%	%		Yr.	Yr.	Yr.
New Jersey	7	7	(e). Forfeiture of entire interest.	6	16	20
New Mexico	6					
New York	7	7	(f). Forfeiture of excess.	6	6	20
North Carolina	6	8	Forfeiture of entire interest.	3	3	7, 10
Ohio	6	8	Forfeiture of excess.	6	15	5
Oregon	10	12		6	6	10
Pennsylvania	6	Any	(g).	6	6, 20*	20
Rhode Island	6	Any		6 (?)	6	20
South Carolina	7	Any		6	6	20
Tennessee	6	10	Forfeiture of excess; fine and imprisonment.	6	6	10 (?)
Texas	8	Any		2	4	10
Utah	10	Any				
Vermont	6	6	Forfeiture of excess.	6	6, 14*	8
Virginia	6	12	Forfeiture of excess, in action of *equity*. Forfeiture of principal and interest, on *proof of usury*.	5	5, 20*	10 (?)
Washington	10	Any				
West Virginia	6	6	(h). Forfeiture of excess.	5	5, 20*	10
Wisconsin	7	10	(i.) Forfeiture of entire interest.	[illegible]	6	10, 20
Wyoming						

Notes—(*a*.) On judgment for *money lent* only 7% recoverable.

(*b*.) May contract also to pay taxes and insurance on the obligation.

(*c*.) Only 6% recoverable in an action at law, even on a contract.

(*d*.) Only 8% allowed after *maturity* of an obligation, but more may be taken as *discount*.

(*e*.) May contract also to pay taxes on the obligation.

(*f*.) Usurious contracts void; action for usury must be brought within one year; no corporation may interpose the defense of usury in any action.

(*g*.) Interest above 6% *may be* recovered *at law*, except in the cases of discount, sale of canal or railroad bonds, and special rates allowed to savings banks.

(*h*.) Corporations excepted. (*i*.) Usurious contract void. May recover thrice the excess.

784. *Number of Days from any day of any Month to the same day of any Month not more than One Year later.*

Notes.—1. Add one day if Feb. of a leap year be included in the interval.

2. For intervals of more than one year add 365 days or more as the case may require.

3. If the interval ends at a later or an earlier day of the month than it begins, add or subtract for the difference required.

Ex. Thus, Oct. 9 to the next March 28 = 151 + 19 = 170 *days*; and April 17 to the next Oct. 9 = 183 − 8 = 175 *days*.

From	To Jan.	Feb.	Mch.	April	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
January	365	31	59	90	120	151	181	212	243	273	304	334
February	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	242	273	303	334	365	30	61	91
October	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	90	121	151	182	212	[illegible]	274	304	335	365

783. *Present Value of $1, due at the end of any number of years not exceeding 50, allowing compound interest.* (Art. 532.)

End of Year.	4 per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.	9 per ct.	10 per ct	End of Year.
1	.961538	.952381	.943396	.934579	.925926	.917431	.909091	1
2	.924556	.907029	.889996	.873439	.857339	.841680	.826446	2
3	.888996	.863838	.839619	.816298	.793832	.772183	.751315	3
4	.854804	.822702	.792094	.762895	.735030	.708425	.683013	4
5	.821927	.783526	.747258	.712986	.680583	.649931	.620921	5
6	.790315	.746215	.704961	.666342	.630170	.596267	.564474	6
7	.759918	.710681	.665057	.622750	.583490	.547034	.513158	7
8	.730690	.676839	.627412	.582009	.540269	.501866	.466507	8
9	.702587	.644609	.591898	.543934	.500249	.460428	.424098	9
10	.675564	.613913	.558395	.508349	.463193	.422411	.385543	10
11	.649581	.584679	.526788	.475093	.428883	.387533	.350494	11
12	.624597	.556837	.496969	.444012	.397114	.355535	.318631	12
13	.600574	.530321	.468839	.414964	.367698	.326179	.289664	13
14	.577475	.505068	.442301	.387817	.340461	.299246	.263331	14
15	.555265	.481017	.417265	.362446	.315242	.274538	.239392	15
16	.533908	.458112	.393646	.338735	.291890	.251870	.217629	16
17	.513373	.436297	.371364	.316574	.270269	.231073	.197845	17
18	.493628	.415521	.350344	.295864	.250249	.211994	.179859	18
19	.474642	.395734	.330513	.276508	.231712	.194490	.163508	19
20	.456387	.376889	.311805	.258419	.214548	.178431	.148644	20
21	.438834	.358942	.294155	.241513	.198656	.163698	.135131	21
22	.421955	.341850	.277505	.225713	.183941	.150182	.122846	22
23	.405726	.325571	.261797	.210947	.170315	.137781	.111678	23
24	.390121	.310068	.246979	.197147	.157699	.126405	.101526	24
25	.375117	.295303	.232999	.184249	.146018	.115968	.092296	25
26	.360689	.281241	.219810	.172195	.135202	.106393	.083905	26
27	.346817	.267848	.207368	.160930	.125187	.097608	.076278	27
28	.333477	.255094	.195630	.150402	.115914	.089548	.069343	28
29	.320651	.242946	.184557	.140563	.107328	.082155	.063039	29
30	.308319	.231377	.174110	.131367	.099377	.075371	.057309	30
31	.296460	.220359	.164255	.122773	.092016	.069148	.052099	31
32	.285058	.209866	.154957	.114741	.085200	.063438	.047362	32
33	.274094	.199873	.146186	.107235	.078889	.058200	.043057	33
34	.263552	.190355	.137912	.100219	.073045	.053395	.039143	34
35	.253415	.181290	.130105	.093663	.067635	.048986	.035584	35
36	.243669	.172657	.122741	.087535	.062625	.044941	.032349	36
37	.234297	.164436	.115793	.081809	.057986	.041231	.029408	37
38	.225285	.156605	.109239	.076457	.053690	.037826	.026735	38
39	.216621	.149148	.103056	.071455	.049713	.034703	.024304	39
40	.208289	.142046	.097222	.066780	.046031	.031838	.022095	40
41	.200278	.135282	.091719	.062412	.042621	.029209	.020086	41
42	.192575	.128840	.086527	.058329	.039464	.026797	.018260	42
43	.185168	.122704	.081630	.054513	.036541	.024584	.016600	43
44	.178046	.116861	.077009	.050946	.033834	.022555	.015091	44
45	.171198	.111297	.072650	.047613	.031328	.020692	.013719	45
46	.164614	.105997	.068538	.044499	.029007	.018984	.012472	46
47	.158283	.100949	.064658	.041587	.026859	.017416	.011338	47
48	.152195	.096142	.060998	.038867	.024869	.015978	.010307	48
49	.146341	.091564	.057546	.036324	.023027	.014659	.009370	49
50	.140713	.087204	.054288	.033948	.021321	.013449	.008519	50

NOTES. 1. The table shows what per cent. of any principal the present value is; hence to get P. W. multiply by the number given. Thus the present worth of $10000 due in 15 years at 4% = $10000 × .555265 = $5552.65.

2. The per cent. of present worth for any number of years may be found by dividing $1.00 by the amount of $1.00 at compound interest as given in the next table. Thus P. W. of $1.00 for 55 years at 3% = $1.00 ÷ 5.08214850.

782. 1. *Amount of $1 at Compound Interest for any number of years not exceeding 55.* (Arts. 523–530.)

Yrs.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.
1	1.0200 0000	1.0250 0000	1.0300 0000	1.0350 0000	1.0400 0000	1.0450 0000
2	1.0404 0000	1.0506 2500	1.0609 0000	1.0712 2500	1.0816 0000	1.0920 2500
3	1.0612 0800	1.0768 9062	1.0927 2700	1.1087 1787	1.1248 6400	1.1411 6613
4	1.0824 3216	1.1038 1289	1.1255 0881	1.1475 2300	1.1698 5856	1.1925 1860
5	1.1040 8080	1.1314 0821	1.1592 7407	1.1876 8631	1.2166 5290	1.2461 8194
6	1.1261 6242	1.1596 9342	1.1940 5230	1.2292 5533	1.2653 1902	1.3022 6012
7	1.1486 8567	1.1886 8575	1.2298 7387	1.2722 7926	1.3159 3178	1.3608 6183
8	1.1716 5938	1.2184 0290	1.2667 7008	1.3168 0904	1.3685 6905	1.4221 0061
9	1.1950 9257	1.2488 6297	1.3047 7318	1.3628 9735	1.4233 1181	1.4860 9514
10	1.2189 9442	1.2800 8454	1.3439 1638	1.4105 9876	1.4802 4428	1.5529 6942
11	1.2433 7431	1.3120 8666	1.3842 3387	1.4599 6972	1.5394 5406	1.6228 5305
12	1.2682 4179	1.3448 8882	1.4257 6089	1.5110 6866	1.6010 3222	1.6958 8143
13	1.2936 0663	1.3785 1104	1.4685 3371	1.5639 5606	1.6650 7351	1.7721 9610
14	1.3194 7876	1.4129 7382	1.5125 8972	1.6186 9452	1.7316 7645	1.8519 4492
15	1.3458 6834	1.4482 9817	1.5579 6742	1.6753 4883	1.8009 4351	1.9352 8244
16	1.3727 8571	1.4845 0562	1.6047 0644	1.7339 8604	1.8729 8125	2.0223 7015
17	1.4002 4142	1.5216 1826	1.6528 4763	1.7946 7555	1.9479 0050	2.1133 7681
18	1.4282 4625	1.5596 5872	1.7024 3306	1.8574 8920	2.0258 1652	2.2084 7877
19	1.4568 1117	1.5986 5019	1.7535 0605	1.9225 0132	2.1068 4918	2.3078 6031
20	1.4859 4740	1.6386 1644	1.8061 1123	1.9897 8886	2.1911 2314	2.4117 1402
21	1.5156 6634	1.6795 8185	1.8602 9457	2.0594 3147	2.2787 6807	2.5202 4116
22	1.5459 7967	1.7215 7140	1.9161 0341	2.1315 1158	2.3699 1879	2.6336 5201
23	1.5768 9926	1.7646 1068	1.9735 8651	2.2061 1448	2.4647 1555	2.7521 6635
24	1.6084 3725	1.8087 2595	2.0327 9411	2.2833 2849	2.5633 0416	2.8760 1383
25	1.6406 0599	1.8539 4410	2.0937 7793	2.3632 4498	2.6658 3633	3.0054 3446
26	1.6734 1811	1.9002 9270	2.1565 9127	2.4459 5856	2.7724 6978	3.1406 7901
27	1.7068 8648	1.9478 0002	2.2212 8901	2.5315 6711	2.8833 6858	3.2820 0956
28	1.7410 2421	1.9964 9502	2.2879 2768	2.6201 7196	2.9987 0332	3.4296 9999
29	1.7758 4469	2.0464 0739	2.3565 6551	2.7118 7798	3.1186 5145	3.5840 3649
30	1.8113 6158	2.0975 6758	2.4272 6247	2.8067 9370	3.2433 9751	3.7453 1813
31	1.8475 8882	2.1500 0677	2.5000 8035	2.9050 3148	3.3731 3341	3.9138 5745
32	1.8845 4059	2.2037 5694	2.5750 8276	3.0067 0759	3.5080 5875	4.0899 8104
33	1.9222 3140	2.2588 5086	2.6523 3524	3.1119 4235	3.6483 8110	4.2740 3018
34	1.9606 7603	2.3153 2213	2.7319 0530	3.2208 6033	3.7943 1634	4.4663 6154
35	1.9998 8955	2.3732 0519	2.8138 6245	3.3335 9045	3.9460 8899	4.6673 4781
36	2.0398 8734	2.4325 3532	2.8982 7833	3.4502 6611	4.1039 3255	4.8773 7846
37	2.0806 8509	2.4933 4870	2.9852 2668	3.5710 2543	4.2680 9986	5.0968 6049
38	2.1222 9879	2.5556 8242	3.0747 8348	3.6960 1132	4.4388 2385	5.3262 1921
39	2.1647 4477	2.6195 7448	3.1670 2698	3.8253 7171	4.6163 7680	5.5658 9908
40	2.2080 3966	2.6850 6384	3.2620 3779	3.9592 5972	4.8010 2063	5.8163 6454
41	2.2522 0046	2.7521 9043	3.3598 9893	4.0978 3381	4.9930 6145	6.0781 0094
42	2.2972 4447	2.8209 9520	3.4606 9589	4.2412 5799	5.1927 8391	6.3516 1548
43	2.3431 8936	2.8915 2008	3.5645 1677	4.3897 0202	5.4004 9527	6.6374 3818
44	2.3900 5314	2.9638 0808	3.6714 5227	4.5433 4160	5.6165 1508	6.9361 2290
45	2.4378 5421	3.0379 0328	3.7815 9584	4.7023 5855	5.8411 7568	7.2482 4843
46	2.4866 1129	3.1138 5086	3.8950 4372	4.8669 4110	6.0748 2271	7.5744 1961
47	2.5363 4352	3.1916 9713	4.0118 9503	5.0372 8404	6.3178 1562	7.9152 6849
48	2.5870 7039	3.2714 8956	4.1322 5188	5.2135 8898	6.5705 2824	8.2714 5557
49	2.6388 1179	3.3532 7680	4.2562 1944	5.3960 6459	6.8333 4937	8.6436 7107
50	2.6915 8803	3.4371 0872	4.3839 0602	5.5849 2686	7.1066 8335	9.0326 3627
51	2.7454 1979	3.5230 3644	4.5154 2320	5.7803 9930	7.3909 5068	9.4391 0490
52	2.8003 2819	3.6111 1235	4.6508 8590	5.9827 1327	7.6865 8871	9.8638 6463
53	2.8563 3475	3.7013 9016	4.7904 1247	6.1921 0824	7.9940 5226	10.3077 3853
54	2.9134 6144	3.7939 2491	4.9341 2485	6.4088 3202	8.3138 1435	10.7715 8677
55	2.9717 3067	3.8887 7303	5.0821 4859	6.6331 4111	8.6463 6692	11.2563 0817

NOTES.—1. The amount for any number of years not given in the table may be computed by finding the product of the amounts for any two numbers of years whose sum

2. *Amount of $1 at Compound Interest for any number of years not exceeding 55*—Continued.

Yrs.	5 per cent.	6 per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	1.0500 000	1.0600 000	1.0700 000	1.0800 000	1.0900 000	1.1000 000
2	1.1025 000	1.1236 000	1.1449 000	1.1664 000	1.1881 000	1.2100 000
3	1.1576 250	1.1910 160	1.2250 430	1.2597 120	1.2950 290	1.3310 000
4	1.2155 063	1.2624 770	1.3107 960	1.3604 890	1.4115 816	1.4641 000
5	1.2762 816	1.3382 256	1.4025 517	1.4693 281	1.5386 240	1.6105 100
6	1.3400 956	1.4185 191	1.5007 304	1.5868 743	1.6771 001	1.7715 610
7	1.4071 004	1.5036 303	1.6057 815	1.7138 243	1.8280 391	1.9487 171
8	1.4774 554	1.5938 481	1.7181 862	1.8509 302	1.9925 626	2.1435 888
9	1.5513 282	1.6894 790	1.8384 592	1.9990 046	2.1718 933	2.3579 477
10	1.6288 946	1.7908 477	1.9671 514	2.1589 250	2.3673 637	2.5937 425
11	1.7103 394	1.8982 986	2.1048 520	2.3316 390	2.5804 264	2.8531 167
12	1.7958 563	2.0121 965	2.2521 916	2.5181 701	2.8126 648	3.1384 284
13	1.8856 491	2.1329 283	2.4098 450	2.7196 237	3.0658 046	3.4522 712
14	1.9799 316	2.2609 040	2.5785 342	2.9371 936	3.3417 270	3.7974 983
15	2.0789 282	2.3965 582	2.7590 315	3.1721 691	3.6424 825	4.1772 482
16	2.1828 746	2.5403 517	2.9521 638	3.4259 426	3.9703 059	4.5949 730
17	2.2920 183	2.6927 728	3.1588 152	3.7000 181	4.3276 334	5.0544 703
18	2.4066 192	2.8543 392	3.3799 323	3.9960 195	4.7171 204	5.5599 173
19	2.5269 502	3.0255 995	3.6165 275	4.3157 011	5.1416 613	6.1159 090
20	2.6532 977	3.2071 355	3.8696 845	4.6609 571	5.6044 108	6.7275 000
21	2.7859 626	3.3995 636	4.1405 624	5.0338 337	6.1088 077	7.4002 499
22	2.9252 607	3.6035 374	4.4304 017	5.4365 404	6.6586 004	8.1402 749
23	3.0715 238	3.8197 497	4.7405 299	5.8714 637	7.2578 745	8.9543 024
24	3.2250 999	4.0489 346	5.0723 670	6.3411 807	7.9110 832	9.8497 327
25	3.3863 549	4.2918 707	5.4274 326	6.8484 752	8.6230 807	10.8347 059
26	3.5556 727	4.5493 830	5.8073 529	7.3963 532	9.3991 579	11.9181 765
27	3.7334 563	4.8223 459	6.2138 676	7.9880 615	10.2450 821	13.1099 942
28	3.9201 291	5.1116 867	6.6488 384	8.6271 064	11.1671 395	14.4209 936
29	4.1161 356	5.4183 879	7.1142 571	9.3172 749	12.1721 821	15.8630 930
30	4.3219 424	5.7434 912	7.6122 550	10.0626 569	13.2676 785	17.4494 023
31	4.5380 395	6.0881 006	8.1451 129	10.8676 694	14.4617 695	19.1943 425
32	4.7649 415	6.4533 867	8.7152 708	11.7370 830	15.7633 288	21.1137 768
33	5.0031 885	6.8405 899	9.3253 398	12.6760 496	17.1820 284	23.2251 544
34	5.2533 480	7.2510 253	9.9781 135	13.6901 336	18.7284 109	25.5476 699
35	5.5160 154	7.6860 868	10.6765 815	14.7853 443	20.4139 679	28.1024 369
36	5.7918 161	8.1472 520	11.4239 422	15.9681 718	22.2512 250	30.9126 805
37	6.0814 069	8.6360 871	12.2335 181	17.2456 256	24.2538 353	34.0039 486
38	6.3854 773	9.1542 524	13.0792 714	18.6252 756	26.4366 805	37.4043 434
39	6.7047 512	9.7035 075	13.9948 204	20.1152 977	28.8159 817	41.1447 778
40	7.0399 887	10.2857 179	14.9744 578	21.7245 215	31.4094 200	45.2592 556
41	7.3919 882	10.9028 610	16.0226 699	23.4624 832	34.2362 679	49.7851 811
42	7.7615 876	11.5570 327	17.1442 568	25.3394 819	37.3175 320	54.7636 992
43	8.1496 669	12.2504 546	18.3443 548	27.3666 404	40.6761 098	60.2400 692
44	8.5571 503	12.9854 819	19.6284 596	29.5559 717	44.3369 597	66.2640 761
45	8.9850 078	13.7646 108	21.0024 518	31.9204 494	48.3272 861	72.8904 837
46	9.4342 582	14.5904 875	22.4726 234	34.4740 853	52.6767 419	80.1795 321
47	9.9059 711	15.4659 167	24.0457 070	37.2320 122	57.4176 486	88.1974 853
48	10.4012 697	16.3938 717	25.7289 065	40.2105 731	62.5852 370	97.0172 338
49	10.9213 331	17.3775 040	27.5299 300	43.4274 190	68.2179 083	106.7189 572
50	11.4673 998	18.4201 543	29.4570 251	46.9016 125	74.3575 201	117.3908 529
51	12.0407 698	19.5253 635	31.5190 169	50.6537 415	81.0496 969	129.1299 382
52	12.6428 083	20.6968 853	33.7253 480	54.7060 409	88.3441 696	142.0429 320
53	13.2749 487	21.9386 985	36.0861 224	59.0825 241	96.2951 449	156.2472 252
54	13.9386 961	23.2550 204	38.6121 509	63.8091 260	104.9617 079	171.8719 477
55	14.6356 309	24.6503 216	41.3150 015	68.9138 561	114.4082 616	189.0591 425

equals the number of years required. Thus, the amount for 60 years at 2% = 2.97173067 (the amount for 55 years) × 1.1040808 (the amount for 5 years).

3. For amounts for fractional parts of a year, see Art. 527.

787. The amount or final value of $1 annuity per annum, at compound interest, for any number of years not exceeding 50.

Yrs.	3 per ct.	3½ per ct.	4 per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.	10 per ct.	Yrs.
1	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.0000	1.0000	1
2	2.030 000	2.035 000	2.040 000	2.050 000	2.060 000	2.070 000	2.0800	2.1000	2
3	3.090 900	3.106 225	3.121 600	3.152 500	3.183 600	3.214 900	3.2464	3.3100	3
4	4.183 627	4.214 943	4.246 464	4.310 125	4.374 616	4.439 943	4.5061	4.6410	4
5	5.309 136	5.362 466	5.416 323	5.525 631	5.637 093	5.750 739	5.8666	6.1051	5
6	6.468 410	6.550 152	6.632 975	6.801 913	6.975 319	7.153 291	7.3359	7.7156	6
7	7.662 462	7.779 408	7.898 294	8.142 008	8.393 838	8.654 021	8.9228	9.4872	7
8	8.892 336	9.051 687	9.214 226	9.549 109	9.897 468	10.259 803	10.6366	11.4359	8
9	10.159 106	10.368 496	10.582 795	11.026 564	11.491 316	11.977 989	12.4876	13.5795	9
10	11.463 879	11.731 393	12.006 107	12.577 893	13.180 795	13.816 448	14.4866	15.9374	10
11	12.807 796	13.141 992	13.486 351	14.206 787	14.971 643	15.783 599	16.6455	18.5312	11
12	14.192 030	14.601 962	15.025 805	15.917 127	16.869 941	17.888 451	18.9771	21.3843	12
13	15.617 790	16.113 030	16.626 838	17.712 983	18.882 138	20.140 643	21.4953	24.5227	13
14	17.086 324	17.676 986	18.291 911	19.598 632	21.015 066	22.550 488	24.2149	27.9750	14
15	18.598 914	19.295 681	20.023 588	21.578 564	23.275 970	25.129 022	27.1521	31.7725	15
16	20.156 881	20.971 030	21.824 531	23.657 492	25.672 528	27.888 054	30.3243	35.9497	16
17	21.761 588	22.705 016	23.697 512	25.840 366	28.212 880	30.840 217	33.7502	40.5447	17
18	23.414 435	24.499 691	25.645 413	28.132 385	30.905 653	33.999 033	37.4502	45.5992	18
19	25.116 868	26.357 180	27.671 229	30.539 004	33.759 992	37.378 965	41.4463	51.1591	19
20	26.870 374	28.279 682	29.778 079	33.065 954	36.785 591	40.995 492	45.7620	57.2750	20
21	28.676 486	30.269 471	31.969 202	35.719 252	39.992 727	44.865 177	50.4229	64.0025	21
22	30.536 780	32.328 902	34.247 970	38.505 214	43.392 290	49.005 739	55.4568	71.4027	22
23	32.452 884	34.460 414	36.617 889	41.430 475	46.995 828	53.436 141	60.8933	79.5430	23
24	34.426 470	36.666 528	39.082 604	44.501 999	50.815 577	58.176 671	66.7648	88.4973	24
25	36.459 264	38.949 857	41.645 908	47.727 099	54.864 512	63.249 038	73.1059	98.3471	25
26	38.553 042	41.313 102	44.311 745	51.113 454	59.156 383	68.676 470	79.9544	109.1818	26
27	40.709 634	43.759 060	47.084 214	54.669 126	63.705 766	74.483 823	87.3508	121.0999	27
28	42.930 923	46.290 627	49.967 583	58.402 583	68.528 112	80.697 691	95.3388	134.2099	28
29	45.218 850	48.910 799	52.966 286	62.322 712	73.639 798	87.346 529	103.9659	148.6309	29
30	47.575 416	51.622 677	56.084 938	66.438 848	79.058 186	94.460 786	113.2832	164.4940	30
31	50.002 678	54.429 471	59.328 335	70.760 790	84.801 677	102.073 041	123.3459	181.9434	31
32	52.502 759	57.334 502	62.701 469	75.298 829	90.889 778	110.218 154	134.2135	201.1378	32
33	55.077 841	60.341 210	66.209 527	80.063 771	97.343 165	118.933 425	145.9506	222.2515	33
34	57.730 177	63.453 152	69.857 909	85.066 959	104.183 755	128.258 765	158.6267	245.4767	34
35	60.462 082	66.674 013	73.652 225	90.320 307	111.434 780	138.236 878	172.3168	271.0244	35
36	63.275 944	70.007 603	77.598 314	95.836 323	119.120 867	148.913 460	187.1021	299.1268	36
37	66.174 223	73.457 869	81.702 246	101.628 139	127.268 119	160.337 402	203.0703	330.0395	37
38	69.159 449	77.028 895	85.970 336	107.709 546	135.904 206	172.561 020	220.3159	364.0434	38
39	72.234 233	80.724 906	90.409 150	114.095 023	145.058 458	185.640 292	238.9412	401.4478	39
40	75.401 260	84.550 278	95.025 516	120.799 774	154.761 966	199.635 112	259.0565	442.5926	40
41	78.663 298	88.509 537	99.826 536	127.839 763	165.047 684	214.609 570	280.7810	487.8518	41
42	82.023 196	92.607 371	104.819 598	135.231 751	175.950 545	230.632 240	304.2435	537.6370	42
43	85.483 892	96.848 629	110.012 382	142.993 339	187.507 577	247.776 496	329.5830	592.4007	43
44	89.048 409	101.238 331	115.412 877	151.143 006	199.758 032	266.120 851	356.9496	652.6408	44
45	92.719 861	105.781 673	121.029 392	159.700 156	212.743 514	285.749 311	386.5056	718.9048	45
46	96.501 457	110.484 031	126.870 568	168.685 164	226.508 125	306.751 763	418.4261	791.7953	46
47	100.396 501	115.350 973	132.945 390	178.119 422	241.098 612	329.224 386	452.9002	871.9749	47
48	104.408 396	120.388 257	139.263 206	188.025 393	256.564 529	353.270 093	490.1322	960.1723	48
49	108.540 648	125.601 846	145.833 734	198.426 663	272.958 401	378.999 000	530.3427	1057.1896	49
50	112.796 867	130.997 911	152.667 084	209.347 976	290.335 905	406.528 929	573.7702	1163.9085	50

NOTE.—If the amount be payable semi-annually and compound interest is to be allowed semi-annually, take the amount for double the number of years at one-half the rate per cent. Thus, for a semi-annual annuity of $1 for 10 years at 10 per cent., take the amount of $1 for 20 years at 5 per cent. $33.065954. For a quarterly annuity, take the amount for four times the number of years at one-fourth the rate per cent.

788. The Present Value of $1 Annuity per annum, at Compound Interest, for any number of years not exceeding 50. (Art. 728.)

Yr.	3 per ct.	3½ per ct.	4 per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.	10 p. ct.	Yr.
1	0.970 874	0.966 184	0.961 538	0.952 381	0.943 396	0.934 579	0.9259	0.9091	1
2	1.913 470	1.899 694	1.886 095	1.859 410	1.833 393	1.808 017	1.7833	1.7355	2
3	2.828 611	2.801 637	2.775 091	2.723 248	2.673 012	2.624 314	2.5771	2.4869	3
4	3.717 098	3.673 079	3.629 895	3.545 951	3.465 106	3.387 209	3.3121	3.1699	4
5	4.579 707	4.515 052	4.451 822	4.329 477	4.212 364	4.100 195	3.9927	3.7908	5
6	5.417 191	5.328 553	5.242 137	5.075 692	4.917 324	4.766 537	4.6229	4.3553	6
7	6.230 283	6.114 544	6.002 055	5.786 373	5.582 381	5.389 285	5.2064	4.8684	7
8	7.019 692	6.873 956	6.732 745	6.463 213	6.209 744	5.971 295	5.7466	5.3349	8
9	7.786 109	7.607 687	7.435 332	7.107 822	6.801 692	6.515 228	6.2469	5.7590	9
10	8.530 203	8.316 605	8.110 896	7.721 735	7.360 087	7.023 577	6.7101	6.1446	10
11	9.252 624	9.001 551	8.760 477	8.306 414	7.886 875	7.498 669	7.1390	6.4951	11
12	9.954 004	9.663 334	9.385 074	8.863 252	8.383 844	7.942 671	7.5361	6.8137	12
13	10.634 955	10.302 738	9.985 648	9.393 573	8.852 683	8.357 651	7.9038	7.1034	13
14	11.296 073	10.920 520	10.563 123	9.898 641	9.294 984	8.745 472	8.2442	7.3667	14
15	11.937 935	11.517 411	11.118 387	10.379 658	9.712 249	9.107 898	8.5595	7.6061	15
16	12.561 102	12.094 117	11.652 296	10.837 770	10.105 895	9.446 632	8.8514	7.8237	16
17	13.166 118	12.651 321	12.165 669	11.274 066	10.477 260	9.763 206	9.1216	8.0216	17
18	13.753 513	13.189 682	12.659 297	11.689 587	10.827 603	10.059 070	9.3719	8.2014	18
19	14.323 799	13.709 837	13.133 939	12.085 321	11.158 116	10.335 578	9.6036	8.3649	19
20	14.877 475	14.212 403	13.590 326	12.462 210	11.469 921	10.593 997	9.8181	8.5136	20
21	15.415 024	14.697 974	14.029 160	12.821 153	11.764 077	10.835 527	10.0168	8.6487	21
22	15.936 917	15.167 125	14.451 115	13.163 006	12.041 582	11.061 241	10.2007	8.7715	22
23	16.443 608	15.620 410	14.856 842	13.488 574	12.303 379	11.272 187	10.3711	8.8832	23
24	16.935 542	16.058 368	15.246 963	13.798 642	12.550 358	11.469 334	10.5288	8.9847	24
25	17.413 148	16.481 515	15.622 080	14.093 945	12.783 356	11.653 583	10.6748	9.0770	25
26	17.876 842	16.890 352	15.982 769	14.375 185	13.003 166	11.825 779	10.8100	9.1609	26
27	18.327 031	17.285 365	16.329 586	14.643 034	13.210 534	11.986 709	10.9352	9.2372	27
28	18.764 108	17.667 019	16.663 063	14.898 127	13.406 164	12.137 111	11.0511	9.3066	28
29	19.188 455	18.035 767	16.983 715	15.141 074	13.590 721	12.277 674	11.1584	9.3696	29
30	19.600 441	18.392 045	17.292 033	15.372 451	13.764 831	12.409 041	11.2578	9.4269	30
31	20.000 428	18.736 276	17.588 494	15.592 811	13.929 086	12.531 814	11.3498	9.4790	31
32	20.388 766	19.068 865	17.873 552	15.802 677	14.084 043	12.646 555	11.4350	9.5264	32
33	20.765 792	19.390 208	18.147 646	16.002 549	14.230 230	12.753 790	11.5139	9.5694	33
34	21.131 837	19.700 684	18.411 198	16.192 904	14.368 141	12.854 009	11.5869	9.6086	34
35	21.487 220	20.000 661	18.664 613	16.374 194	14.498 246	12.947 672	11.6546	9.6442	35
36	21.832 252	20.290 494	18.908 282	16.546 852	14.620 987	13.035 208	11.7172	9.6765	36
37	22.167 235	20.570 525	19.142 579	16.711 287	14.736 780	13.117 017	11.7752	9.7059	37
38	22.492 462	20.841 087	19.367 864	16.867 893	14.846 019	13.193 473	11.8289	9.7327	38
39	22.808 215	21.102 500	19.584 485	17.017 041	14.949 075	13.264 928	11.8786	9.7570	39
40	23.114 772	21.355 072	19.792 774	17.159 086	15.046 297	13.331 709	11.9246	9.7791	40
41	23.412 400	21.599 104	19.993 052	17.294 368	15.138 016	13.394 120	11.9672	9.7991	41
42	23.701 359	21.834 883	20.185 627	17.423 208	15.224 543	13.452 449	12.0067	9.8174	42
43	23.981 902	22.062 689	20.370 795	17.545 912	15.306 173	13.506 962	12.0432	9.8340	43
44	24.254 274	22.282 791	20.548 841	17.662 773	15.383 182	13.557 908	12.0771	9.8491	44
45	24.518 713	22.495 450	20.720 040	17.774 070	15.455 832	13.605 522	12.1084	9.8628	45
46	24.775 449	22.700 918	20.884 654	17.880 067	15.524 370	13.650 020	12.1374	9.8753	46
47	25.024 708	22.899 438	21.042 936	17.981 016	15.589 028	13.691 608	12.1643	9.8866	47
48	25.266 707	23.091 244	21.195 131	18.077 158	15.650 027	13.730 474	12.1891	9.8969	48
49	25.501 657	23.276 564	21.341 472	18.168 722	15.707 572	13.766 799	12.2122	9.9063	49
50	25.729 764	23.455 618	21.482 185	18.255 925	15.761 861	13.800 746	12.2335	9.9148	50

Note.—If the annuity be payable semi-annually and semi-annual interest is to be allowed, take the present value for double the number of years, at one-half the given rate. Thus, for an annuity of $1 every six months, for 10 years, at 8%, the present value would equal that of $1 per annum for 20 years, at 4% = $13.590326.

789. *Present Value of a $1000 Bond due in from 1 to 20 years, and bearing semi-annual interest, when money is worth 6 per cent., payable annually.* (Arts. 671, 672.)

Due in Years	5% Bond.	6% Bond.	7% Bond.	8% Bond.	10% Bond.	Due in Years
1	$991.27	$1000.84	$1010.44	$1020.00	$1039.15	1
2	983.04	1001.64	1020.27	1038.86	1076.09	2
3	975.27	1002.40	1029.53	1056.66	1110.98	3
4	967.94	1003.11	1038.80	1073.45	1143.80	4
5	961.02	1003.78	1046.55	1089.30	1174.81	5
6	954.51	1004.42	1054.34	1104.24	1204.07	6
7	948.35	1005.02	1061.69	1118.34	1231.67	7
8	942.55	1005.58	1068.62	1131.64	1257.71	8
9	937.08	1006.12	1075.16	1144.19	1282.27	9
10	931.91	1006.62	1081.38	1156.03	1305.44	10
11	927.04	1007.09	1087.15	1167.20	1327.31	11
12	922.44	1007.54	1092.64	1177.73	1347.93	12
13	918.11	1007.96	1097.82	1187.07	1367.39	13
14	914.02	1008.36	1102.71	1197.05	1385.74	14
15	910.16	1008.74	1107.32	1205.90	1403.06	15
16	906.52	1009.09	1111.67	1214.24	1419.39	16
17	903.08	1009.43	1115.77	1222.11	1424.80	17
18	899.84	1009.74	1119.64	1229.54	1440.34	18
19	896.78	1010.04	1123.29	1236.55	1463.06	19
20	893.90	1010.32	1126.74	1243.16	1476.00	20

790. *Present Value of a $1000 Bond due in from 1 to 20 years, and bearing semi-annual interest, when money is worth 7 per cent., payable annually.*

Due in Years	5% Bond.	6% Bond.	7% Bond.	8% Bond.	10% Bond.	Due in Years
1	$982.13	$991.63	$1001.15	$1010.65	$1029.67	1
2	965.42	983.81	1002.22	1020.61	1057.40	2
3	949.81	976.50	1003.22	1029.91	1083.32	3
4	935.22	969.67	1004.15	1038.61	1107.54	4
5	921.53	963.29	1005.02	1046.74	1130.18	5
6	908.84	957.33	1005.84	1054.33	1151.34	6
7	896.93	951.76	1006.60	1061.46	1171.11	7
8	885.80	946.55	1007.32	1068.09	1189.59	8
9	875.40	941.68	1007.98	1074.29	1206.86	9
10	865.67	937.13	1008.61	1080.08	1223.00	10
11	856.59	932.88	1009.19	1085.00	1238.08	11
12	848.10	928.91	1009.73	1090.54	1252.18	12
13	840.16	925.19	1010.24	1095.27	1265.35	13
14	832.74	921.72	1010.72	10[illegible]9.[illegible]0	1277.67	14
15	825.81	918.48	1011.16	1103.83	1289.17	15
16	819.33	915.45	1011.58	1107.09	1299.93	16
17	813.28	912.61	1011.96	1111.30	1309.98	17
18	807.62	909.97	1012.33	1114.67	1319.37	18
19	802.33	907.49	1012.66	1117.82	1328.16	19
20	797.39	905.18	1012.98	1120.77	1336.36	20

MORTALITY AND NET PREMIUM TABLES.

791. ACTUARIES' LIFE TABLE.

Age.	No. Living.	Age.	No. Living.
10	100,000	61	54,275
11	99,324	62	52,505
12	98,650	63	50,661
13	97,978	64	48,744
14	97,307	65	46,754
15	96,636	66	44,693
16	95,965	67	42,565
17	95,293	68	40,374
18	94,620	69	38,128
19	93,945	70	35,837
20	93,268	71	33,510
21	92,588	72	31,159
22	91,905	73	28,797
23	91,219	74	26,439
24	90,529	75	24,100
25	89,835	76	21,797
26	89,137	77	19,548
27	88,434	78	17,369
28	87,726	79	15,277
29	87,012	80	13,290
30	86,292	81	11,424
31	85,565	82	9,694
32	84,831	83	8,112
33	84,089	84	6,685
34	83,339	85	5,417
35	82,581	86	4,306
36	81,814	87	3,348
37	81,038	88	2,537
38	80,253	89	1,864
39	79,458	90	1,319
40	78,653	91	892
41	77,838	92	570
42	77,012	93	339
43	76,173	94	184
44	75,316	95	89
45	74,435	96	37
46	73,526	97	13
47	72,582	98	4
48	71,601	99	1
49	70,580		
50	69,517		
51	68,409		
52	67,253		
53	66,046		
54	64,785		
55	63,469		
56	62,094		
57	60,658		
58	59,161		
59	57,600		
60	55,973		

792. 4 PER CENT DEPOSITS AT AGE 10, TO SECURE $1.

Age.	Endowment Deposit.	Annuity Deposit.	Assurance Deposit.
20	0.630 085	12.235 379	0.158 724
21	0.601 434	11.625 307	0.154 807
22	0.574 036	11.028 873	0.150 041
[redacted]	0.547 898	10.449 847	0.145 921
24	0.522 782	9.901 900	0.141 996
25	0.498 821	9.379 217	0.138 083
26	0.475 910	8.880 390	0.134 356
27	0.454 998	8.404 486	0.130 747
28	0.433 040	7.950 490	0.127 252
29	0.412 996	7.517 450	0.123 863
30	0.393 825	7.104 454	0.120 577
31	0.375 488	6.710 629	0.117 387
32	0.357 949	6.335 141	0.114 290
33	0.341 171	5.977 192	0.111 279
34	0.325 123	5.636 021	0.108 353
35	0.309 775	5.310 898	0.105 510
36	0.295 094	5.001 123	0.102 743
37	0.281 053	4.706 029	0.100 052
38	0.267 630	4.424 977	0.097 435
39	0.254 793	4.157 351	0.094 885
40	0.242 502	3.902 584	0.092 408
41	0.230 750	3.660 094	0.089 987
42	0.219 512	3.429 407	0.087 633
43	0.208 764	3.209 778	0.085 333
44	0.198 497	3.000 492	0.083 074
45	0.188 620	2.802 495	0.080 842
46	0.179 161	2.613 865	0.078 627
47	0.170 052	2.434 705	0.076 414
48	0.161 390	2.264 048	0.074 204
49	0.152 890	2.103 312	0.071 992
50	0.144 796	1.950 459	0.069 779
51	0.137 008	1.805 676	0.067 560
52	0.129 513	1.668 618	0.065 334
53	0.122 276	1.539 135	0.063 090
54	0.115 347	1.416 859	0.060 853
55	0.108 658	1.301 492	0.058 601
56	0.102 215	1.192 834	0.056 337
57	0.096 011	1.090 619	0.054 064
58	0.090 040	0.994 403	0.051 786
59	0.084 249	0.901 568	0.049 502
60	0.078 734	0.820 275	0.047 212
61	0.073 134	0.741 511	0.044 925
62	0.068 317	0.668 040	0.042 612
63	0.063 371	0.599 773	0.040 305
64	0.058 639	0.534 899	0.037 999
65	0.054 073	0.477 709	0.035 697
66	0.049 702	0.423 696	0.033 406
67	0.045 715	0.373 994	0.031 131
68	0.041 511	0.326 474	0.028 877
69	0.037 494	0.286 968	0.026 657
70	0.034 067	0.249 271	0.024 480

NOTES.—1. The *Endowment Deposit* indicates the deposit required to secure $1 at the specified age. (Art. 736.)

2. The *Annuity Deposit* indicates the deposit required to secure $1 per annum at and after the specified age. (Art. 737.)

3. The *Assurance Deposit* indicates the deposit required to secure $1 at *death* at or after the specified age.

4. When net premiums have been computed by Art. 746, the usual "office premiums" (Art. 741, 3) may be found approximately by adding 33% for annual life premiums, 25% for life premiums paid up in 10 years, 27% for annual premiums for endowments, and 20% for endowment premiums paid up in 10 years. For most of these premiums see Art. 793, on the next two pages.

793. PREMIUMS FOR ASSURANCE AND ANNUITIES.

NOTES.—1. These are the "office premiums" *usually* charged, allowing dividends, but the rates of different companies vary somewhat from these.

2. For insurance on the "stock plan," that is, *without dividends*, the rates are about 25% below these.

	LIFE ASSURANCE.			SINGLE PREMIUMS FOR A DEFERRED LIFE ANNUITY OF $1, BEGINNING					
Age at which premiums begin.	$1 at death. Single premium.	$1 at death. Annual premium for 10 years.	$1 at death. Annual premium for life.	At age 45.	At age 50.	At age 55.	At age 60.	One year after the premium is paid.	Age at which premiums begin.
15	.24691	.03275	.01492	2.866	1.933	1.24	.773	20.6117	15
16	.26038	.03353	.01544	3.04	2.04	1.32	.813	20.4680	16
17	.26793	.03416	.01597	3.213	2.16	1.40	.853	20.3367	17
18	.27176	.03546	.01647	3.413	2.28	1.48	.903	20.2211	18
19	.28129	.03636	.01697	3.618	2.426	1.573	.96	20.1285	19
20	.28745	.03720	.01745	3.84	2.573	1.666	1.036	20.0445	20
21	.29320	.03797	.01789	4.08	2.733	1.773	1.093	19.9816	21
22	.29910	.03876	.01836	4.346	2.92	1.88	1.16	19.9295	22
23	.30517	.03958	.01884	4.626	3.106	2.00	1.24	19.8819	23
24	.31142	.04041	.01935	4.92	3.306	2.133	1.32	19.8323	24
25	.31786	.04128	.01988	5.24	3.52	2.266	1.40	19.7740	25
26	.32451	.04217	.02043	5.573	3.733	2.413	1.48	19.7032	26
27	.33139	.04310	.02100	5.92	3.973	2.573	1.586	19.6180	27
28	.33850	.04407	.02160	6.306	4.226	2.733	1.68	19.5180	28
29	.34586	.04508	.02223	6.693	4.60	2.893	1.786	19.4015	29
30	.35347	.04611	.02289	7.106	4.706	3.08	1.893	19.2776	30
31	.36136	.04717	.02360	7.546	5.066	3.266	2.013	19.1407	31
32	.36951	.04830	.02435	8.013	5.373	3.466	2.133	18.9955	32
33	.37795	.04945	.02512	8.52	5.706	3.693	2.266	18.84[illegible]1	33
34	.38648	.05066	.02593	9.053	6.066	3.92	2.413	18.6881	34
35	.39571	.05192	.02680	9.613	6.453	4.16	2.56	18.5245	35
36	.40501	.05321	.02771	10.23	6.853	4.426	2.72	18.3675	36
37	.41467	.05456	.02865	10.87	7.293	4.706	2.893	18.2016	37
38	.42462	.05597	.02967	11.57	7.76	5.013	3.08	18.0128	38
39	.43488	.05741	.03073	12.31	8.253	5.333	3.28	17.8620	39
40	.44545	.05892	.03185	13.09	8.786	5.666	3.48	17.6667	40
41	.45635	.06049	.03304	13.95	9.346	6.04	3.706	17.4701	41
42	.46758	.06212	.03429	14.84	9.946	6.426	3.946	17.2620	42
43	.47913	.06382	.03561	15.80	10.586	6.84	4.20	17.0111	43
44	.49101	.06557	.03701	....	11.266	7.28	4.48	16.80[illegible]9	44
45	.50323	.06733	.03849	....	12.00	7.746	4.76	16.5591	45
46	.51578	.06927	.04005	....	12.773	8.253	5.066	16.2985	46
47	.52868	.07125	.04172	....	13.613	8.796	5.40	16.[illegible]268	47
48	.54193	.07330	.04348	....	14.506	9.36	5.76	15.7463	48
49	.55554	.07544	.04535	....	....	9.986	6.133	15.4593	49
50	.56951	.07767	.04733	....	....	10.653	6.546	15.1685	50
51	.58386	.08003	.04947	....	....	11.386	7.00	14.8764	51
52	.59820	.08239	.05167	....	....	12.186	7.48	14.5823	52
53	.61282	.08483	.05402	....	....	12.973	8.01	14.2860	53
54	.62775	.08738	.05649	....	....	....	8.60	13.9867	54
55	.64301	.09007	.05913	....	....	....	9.226	13.6834	55
56	.65830	.09287	.06197	....	....	....	9.92	13.3715	56
58	.69071	.09892	.06824	....	....	....	11.493	12.7224	58
60	.72391	.10567	.07543	....	....	....	....	12.0297	60

EXAMPLE.—The annual premium for a Life Policy of $10000 is $.0268 \times 10000 = \$268$.

793a. PREMIUMS FOR ENDOWMENTS.

Age at which premiums begin.	Annual Premiums for Endowments payable as specified, or at death, if previous.								Age at which premiums begin.
	$1 in 10 yr.	$1 in 15 yr.	$1 at 40 or death.	$1 at 45 or death.	$1 at 50 or death.	$1 at 55 or death.	$1 at 60 or death.	$1 at 65 or death.	
15	10674	.06535	.03307	02711	02276	.01984	01785	01662	15
16	10713	06540	03612	.02857	02376	02067	01857	.01725	16
17	10745	06571	03845	.03014	02402	02184	01931	01788	17
18	10777	06603	.04096	03180	.02612	02273	02017	01852	18
19	10805	06629	04367	03357	02737	02353	02081	01915	19
20	.10818	.06649	04661	.03544	.02865	.02453	02155	01978	20
21	10839	.06670	.01063	03744	.02998	02522	02220	02040	21
22	10850	06678	05344	03949	03142	02624	02307	02103	22
23	10861	.06693	05746	04195	.03266	02750	.02386	02169	23
24	10875	06705	.06204	04456	.03405	02860	02478	02240	24
25	.10888	.06723	.06723	04748	.03647	02986	02570	02314	25
26	10901	.06739	.07325	.05079	.03845	03118	.02672	02393	26
27	10915	06756	08015	05445	04008	03216	02776	02476	27
28	.10930	06775	08820	05847	.04202	03430	02889	02614	28
29	10945	06794	.09797	.06292	04065	03612	.03011	02659	29
30	10962	.06815	.10962	.06815	.04836	03802	03144	02711	30
31	10979	06837		07329	.05168	04000	03285	02868	31
32	10998	.06861		08110	.05546	04216	03437	02991	32
33	11018	06886		.09060	.05976	04472	03602	03161	33
34	11040	06914		09944	06418	04758	03780	03258	34
35	.11063	.06942		.11063	.06942	05017	.03974	03574	35
36	11088	06975			.07546	.05263	.04184	03527	36
37	11110	.07009			.08248	.05541	.04413	03649	37
38	.11145	07049			.09068	05819	.04667	03842	38
39	11178	07087			10040	06125	04943	04046	39
40	11211	07128			11211	07128	.05250	04249	40
41	11346	07177				.07740	.05580	04478	41
42	11285	07229				08422	05958	04734	42
43	11328	07284				09238	06360	04910	43
44	11374	07344				10248	06847	05156	44
45	11425	07409				11425	07409	05580	45
46	.11480	07480					08070	07944	46
47	11540	07566					.08747	06341	47
48	.11606	.07640					.09682	06784	48
49	11679	.07732					10571	07277	49
50	.11758	07830					.11758	07830	50
51	.11843	07914						08467	51
52	11933	.08059						09197	52
53	12027	.08183						10018	53
54	12130	08323						11050	54
55	12244	08476						12244	55
56	12370	.08645							56
58	.12668	09040							58
60	13032	09525							60

NOTE.—*To find the present Value of a Policy* Subtract the *original* annual premium on $1.00, from that due for the *present* age Multiply the remainder by the *amount assured* and divide the product by the *present* premium (on the same amount) plus $3.49 for Life Policies, or $4.99 for Endowments.

EXCHANGE TABLES.

[COMPILED MAINLY FROM TATE'S MODERN CAMBIST, AND THE BANKER'S MAGAZINE.]

793. AMSTERDAM.

MONEY OF ACCOUNT.—6 florins or guilders=600 centimes=120 stivers =240 grotes Flemish=20 schillings Flemish=2⅖ rix dollars.

PAR OF EXCHANGE.—12 florins 9 centimes=£1=$4.86⅔. Or, 1 florin =$0.40. In the United States the quotations of exchange on Amsterdam are so many cents per florin or guilder.

794. AUSTRIA.

MONEY OF ACCOUNT.—1 florin=60 kreutzers. A rix dollar is 1½ florins or 90 kreutzers, and is a nominal money used in exchanges but not in accounts. The value of the money of account is that called Convention, or 20 Guldenfuss, in which the Cologne mark weight of fine silver is supposed to be coined into 20 florins, a standard only $\frac{4}{11}$% above the Wechselzahlung of Frankfort. The currency of Austria is of both paper and metal. The paper money, called Wiener-wahrung, or Vienna value, is at a fixed discount of about 60%: by which 100 florins in cash are equal to 250 florins in W. W. Bills upon Vienna are generally directed to be paid in effective—that is, in cash—sometimes mentioning the kind (as 20 kreutzer-pieces, for example), to guard against their being paid in paper money of the depreciated value.

PAR OF EXCHANGE.—9 florins 50 kreutzers=£1=$4.86⅔. 1 rix-dollar in 20 Guldenfuss=36.56 pence sterling.

795. BAVARIA.

MONEY OF ACCOUNT.—1 gulden=60 kreutzers=240 pfennings. The value of money of account in Bavaria is based upon the South German valuation, the same as that of Frankfort-on-the-Main, making 1 gulden=19.86 d.

PAR OF EXCHANGE.—12.0551 gulden=£1=$4.86⅔.

796. BELGIUM.

MONEY OF ACCOUNT.—The official money of account is kept in francs and centimes the same as in France. But in mercantile accounts and exchange it is generally in florins and centimes, as in Amster-

dam—the denominations of schillings and grotes being sometimes used in London Exchange.

PAR OF EXCHANGE.—The fixed relative value of the franc to the florin is 47¼ centimes of a florin = 1 franc. 25 frcs. 22 centimes = 12 florins 9 centimes = 40 schillings 3 grotes = £1 = \$4.86⅔.

797. BREMEN.

MONEY OF ACCOUNT.—5 schwaren = 1 grote; 72 grotes = 1 rix-dollar. The rix-dollar is valued, in gold, from the old French and German Louis d'or, at the rate of 5 rix-dollars to 1 Louis d'or.

PAR OF EXCHANGE.—1 rix-dollar = 3s. 3.4d. sterling = \$0.79¼. In the U. S. the quotations of exchange on Bremen are so many cents per rix dollar.

798. CANADA.

MONEY OF ACCOUNT.—1 pound = 20 shillings = 240 pence = 4 dollars = 400 cents. The decimal system of dollars and cents has been recently introduced.

PAR OF EXCHANGE.—£1 as represented by the Canadian paper currency, has been considered equivalent to \$4 U. S. currency. The silver coinage, furnished that province by England, is 3⅔% below the silver coinage of the U. S. in value, their 20-cent piece being worth only \$0.1927.

799. CHINA.

MONEY OF ACCOUNT.—1 tael = 10 mace = 100 candareens = 1000 cash.

PAR OF EXCHANGE.—1000 cash = 78¼ pence sterling, or 3067 cash = £1 = \$4.86⅔.

800. DENMARK.

MONEY OF ACCOUNT.—6 marks = 96 skillings = 1 rigsdaler. For some purposes the specie daler = 2 rigsdaler = 48 styver, is used.

PAR OF EXCHANGE.—9 rigsdaler 10 sk. = £1 = \$4.86⅔.

For England and Ireland, see Great Britain.

801. FRANCE.

MONEY OF ACCOUNT.—1 franc = 100 centimes.

PAR OF EXCHANGE.—20 francs gold = 15s 10¼d. sterling = \$3.84. Or, \$1 = 5 frcs. 21 centimes, or £1 = 25 frcs 20 centimes.

810. FOREIGN AND U. S. GOLD COINS.

Their Weight, Fineness and Value as Assayed at the United States Mint.

Note.—The weight is given in Troy-ounces and decimals of the same; the fineness shows how many parts in 1000 are fine gold; the value is the intrinsic relative value, as compared with the amount of fine gold in U. S. coin.

Countries.	Denominations.	Weight.	Fineness.	Value.
Australia	Pound of 1852	0.281	916.5	$5.324
"	Sovereign, 1855 and 1860	0.2565	916	4.857
Austria	Ducat	0.112	986	2.283
"	Sovereign	0.363	900	6.754
"	New Union Coin	0.357	900	6.642
Belgium	25 Francs	0.254	899	4.72
Bolivia	Doubloon	0.867	870	15.593
Brazil	Twenty Milreis	0.575	917.5	10.906
Central America	Two Escudos	0.209	853.5	3.688
" "	Four Reals	0.027	875	0.488
Chili	Old Doubloon	0.867	870	15.593
"	Ten Pesos	0.492	900	9.154
Denmark	Ten Thalers	0.427	895	7.90
Ecuador	Four Escudos	0.433	844	7.555
England	Pound or Sovereign, new	0.2567	916.5	4.863
"	" " average	0.2562	916	4.851
France	Twenty Francs, new	0.2075	899	3.858
"	" " average	0.207	899	3.847
Germany, North	Ten Thalers	0.427	895	7.90
" "	" " Prussian	0.427	903	7.971
" "	Krone (crown)	0.357	900	6.643
" South	Ducat	0.112	986	2.283
Greece	Twenty Drachma	0.185	900	3.443
Hindostan	Mohur	0.374	916	7.082
Italy	Twenty Lire	0.207	898	3.843
Japan	Old Cobang	0.362	568	4.44
"	" "	0.289	572	3.575
"	Yen (new, assumed)	0.535	900	9.966
Mexico	Doubloon, average	0.8675	866	15.53
"	" New	0.8675	870.5	15.611
"	Twenty Pesos (Max.)	1.086	875	19.643
"	" " (Repub.)	1.090	876	19.73
Naples	Six Ducatti	1 245	996	5.044
Netherlands	Ten Guilders	0 215	899	3.997
New Grenada	Old Doubloon (Bogota)	0.868	870	15.611
" "	" " (Popayan)	0.867	858	15.378
" "	Ten Pesos	0.525	891.5	9.675
Peru	Old Doubloon	0.867	868	15.557
"	Twenty Sols	1.056	898	19.213
Portugal	Gold Crown	0.308	912	5.807
Prussia	New Crown (assumed)	0.357	900	6.643
Rome	2½ Scudi (new)	0 140	900	2.605
Russia	Five Rubles	0 210	916	3.976
Spain	100 Reals	0 268	896	4.964
"	80 "	0.215	869.5	3.864
Sweden	Ducat	0.111	975	2.237
"	Carolin, 10 frs	0.104	900	1.935
Tunis	25 Piastres	0.161	900	2.995
Turkey	100 "	0.231	915	4.369
Tuscany	Sequin	0.112	[illegible]	2.313
United States	Dollar	0 05375	900	1.00
" "	Quarter Eagle	0.13437	[illegible]	2.50
" "	Three Dollar	0.16125	[illegible]	3.00
" "	Half Eagle	0.26875	900	5.00
" "	Eagle	0.5375	900	10.00
" "	Double Eagle	1.075	900	20.00

PAR OF EXCHANGE.—13 marks $10\frac{1}{4}$ schil. *banco* =£1=$4.86⅔.
16 " 12 " *current*=£1=$4.86⅔.

Or, 1 mark banco=$35\frac{5}{16}$ cents. In the U. S. the quotations of exchange on Hamburg are so many cents per mark banco.

805. ITALY.

MONEY OF ACCOUNT—100 centesimi=1 lira Italiana. The cities of Milan, Venice, Tuscany, Florence, etc., forming now a part of the kingdom of Italy, the money system is uniform throughout the kingdom. The former lira Austriaca had the same value as the 20 kreutzer piece or one third of an Austrian florin 29 lire 52 centesimi were equal to £1, or 1 lira=$8\frac{1}{8}$d. In Tuscany 30.69 lire were equal to £1, or 1 lira=7.8d.

PAR OF EXCHANGE.—25.30 lire=£1=$4.86⅔.

806. JAPAN.

MONEY OF ACCOUNT.—1 kobang=4 ichibus or itzebus=1500 cash.

PAR OF EXCHANGE.—300 itzebus=100 Mexican dollars, and $4\frac{1}{2}$ dollars =£1=$4.86⅔.

807. PRUSSIA.

MONEY OF ACCOUNT.—1 Prussian dollar 30 silver groschen.

PAR OF EXCHANGE.—1 Cologne mark weight of fine silver is coined into 14 dollars; hence, 6 Prussian dollars 27 silver groschen=£1=$4.86⅔.

808. RUSSIA.

MONEY OF ACCOUNT.—1 ruble=100 copecs. Previous to July, 1839, the Banco or paper ruble was used as the money of account, reckoning 100 silver rubles=350 paper or bank rubles.

PAR OF EXCHANGE.—1 silver ruble=$37\frac{1}{2}$d. sterling. At Odessa the rate of exchange on London is still generally made in paper rubles, in which the par of exchange is 2240 paper rubles=£100 sterling.

809. SPAIN.

MONEY OF ACCOUNT.—20 reals=2 escudos=1 duro. For commercial purposes the Real is principally used, subdivided into 100 centenos.

PAR OF EXCHANGE.—100 Reals=£1=$4.86⅔.

810. FOREIGN AND U. S. GOLD COINS.

Their Weight, Fineness and Value as Assayed at the United States Mint.

Note.—The weight is given in Troy-ounces and decimals of the same; the fineness shows how many parts in 1000 are fine gold the value is the intrinsic relative value, as compared with the amount of fine gold in U. S. coin.

Countries.	Denominations.	Weight.	Fineness.	Value.
Australia	Pound of 1852	0.281	916.5	$5.324
"	Sovereign, 1855 and 1860	0.2565	916	4.857
Austria	Ducat	0.112	986	2.283
"	Sovereign	0.363	900	6.754
"	New Union Coin	0.357	900	6.648
Belgium	25 Francs	0.254	899	4.72
Bolivia	Doubloon	0.867	870	15.593
Brazil	Twenty Milreis	0.575	917.5	10.906
Central America	Two Escudos	0.209	853.5	3.683
" "	Four Reals	0.027	875	0.488
Chili	Old Doubloon	0.867	870	15.593
"	Ten Pesos	0.492	900	9.154
Denmark	Ten Thalers	0.427	895	7.90
Ecuador	Four Escudos	0.433	844	7.555
England	Pound or Sovereign, new	0.2567	916.5	4.863
"	" " average	0.2568	916	4.861
France	Twenty Francs, new	0.2075	899	3.857
"	" " average	0.207	899	3.847
Germany, North	Ten Thalers	0.427	895	7.90
" "	" " Prussian	0.427	903	7.971
" "	Krone (crown)	0.357	900	6.642
" South	Ducat	0.112	986	2.283
Greece	Twenty Drachms	0.185	900	3.442
Hindostan	Mohur	0.374	916	7.082
Italy	Twenty Lire	0.207	898	3.843
Japan	Old Cubang	0.362	568	4.44
"	" "	0.289	572	3.576
"	Yen (new, assumed)	0.535	900	9.966
Mexico	Doubloon, average	0.8675	866	15.53
"	" New	0.8675	870.5	15.611
"	Twenty Pesos (Max.)	1.086	875	19.642
"	" " (Repub.)	1.090	875	19.72
Naples	Six Ducati	1.245	996	5.044
Netherlands	Ten Guilders	0.215	899	3.997
New Grenada	Old Doubloon (Bogota)	0.868	870	15.611
" "	" " (Popayan)	0.867	858	15.379
" "	Ten Pesos	0.525	891.5	9.675
Peru	Old Doubloon	0.867	868	15.557
"	Twenty Sols	1.055	898	19.212
Portugal	Gold Crown	0.308	912	5.807
Prussia	New Crown (assumed)	0.357	900	6.642
Rome	2½ Scudi (new)	0.140	900	2.605
Russia	Five Rubles	0.210	916	3.976
Spain	100 Reals	0.268	896	4.964
"	80 "	0.215	869.5	3.864
Sweden	Ducat	0.111	975	2.237
"	Carolin, 10 frs.	0.104	900	1.935
Tunis	25 Piastres	0.161	900	2.995
Turkey	100 "	0.231	915	4.369
Tuscany	Sequin	0.112	999	2.313
United States	Dollar	0.05375	900	1.00
" "	Quarter Eagle	0.13437	900	2.50
" "	Three Dollar	0.16125	900	3.00
" "	Half Eagle	0.26875	900	5.00
" "	Eagle	0.5375	900	10.00
" "	Double Eagle	1.075	900	20.00

811. FOREIGN AND U. S. SILVER COINS.

As Assayed at the United States Mint, the basis of valuation being $1.22½ per ounce of standard fineness.

NOTE.—Weight in Troy ounces; fineness in thousandths.

COUNTRIES.	DENOMINATIONS.	Weight	Fineness.	Value.
Austria	Old Rix Dollar	0.902	833	$1.023
"	Old Scudo	0.836	902	1.026
"	Florin before 1858	0.451	833	.511
"	New Florin	0.397	900	.486
"	New Union Dollar	0.596	900	.731
"	Maria Theresa Dollar, 1780	0.895	838	1.021
Belgium	Five Francs	0.803	897	.98
Bolivia	New Dollar	0.801	900	.981
Brazil	Double Milreis	0.820	918.5	1.025
Canada	Twenty Cents	0.150	925	.189
"	Twenty-five Cents	0.1875	925	.236
Central America	Dollar	0.866	850	1.002
Chili	Old Dollar	0.864	908	1.068
"	New Dollar	0.801	900.5	.982
China	Dollar (English) assumed	0.866	901	1.063
"	Ten Cents	0.087	901	106
Denmark	Two Rigsdaler	0.927	877	1.107
England	Shilling, new	0.1825	924.5	.23
"	" average	0.178	925	.224
France	Five Francs, average	0.800	900	.98
"	Two Francs	0.320	835	.364
Germany, North	Thaler before 1857	0.712	750	.727
" "	New Thaler	0.595	[illegible]	.729
" South	Florin before 1857	0.340	900	.417
" "	New Florin, assumed	0.340	900	.417
Greece	Five Drachms	0.719	900	.881
Hindostan	Rupee	0.374	916	[illegible]
Japan	Itzabu	0.279	991	[illegible]
"	New Itzabu	0.279	890	[illegible]
"	10 Sen (new coinage)	0.804	800	[illegible]
Mexico	Dollar, new	0.8675	903	1.066
"	" average	0.866	901	1.062
"	Peso of Maximilian	0.861	902.5	1.055
Naples	Scudo	0.844	830	.953
Netherlands	2½ Guilders	0.804	944	1.033
Norway	Specie Daler	0.927	877	1.107
New Grenada	Dollar of 1857	0.803	896	.98
Peru	Old Dollar	0.866	901	1.062
"	Dollar of 1858	0.766	909	.948
"	Half Dollar, 1835 and 1838	0.433	650	.383
"	Sol	0.802	900	.982
Prussia	Thaler before 1857	0.712	750	.727
"	New Thaler	0.595	900	.729
Rome	Scudo	0.864	900	1.058
Russia	Rouble	0.667	875	.794
Sardinia	Five Lire	0.800	900	.98
Spain	New Pistareen	0.166	899	.203
Sweden	Rix Dollar	0.099	750	1.116
Switzerland	Two Francs	[illegible]	899	.366
Tunis	Five Piasters	0.511	898.5	.626
Turkey	Twenty Piasters	0.770	830	.87
Tuscany	Florin	0.220	925	.276
United States	Dollar	0.859⅜	900	
" "	Half Dollar	0.400	900	
" "	Quarter Dollar	0.200	900	
" "	Dime	0.080	900	
" "	Half Dime	0.040	900	
" "	Three Cent	0.024	900	

AMERICAN OR U. S. WEIGHTS, MEASURES AND MONEY.

NOTE.—Most of the following are also used in England and the British Provinces. The Metric System is also authorized in the U. S. (Arts. 807–810, and 426.)

812. LINEAR, OR LONG MEASURE. (Art. 281.)

Used to compute distances in any direction.

TABLE.

12 inches (*in.*)	make	1 foot............*ft.*
3 ft.	"	1 yard............*yd.*
5½ yd.	"	1 rod or pole.....*rd.* or *p.*
40 rd.	"	1 furlong.........*fur.*
8 fur.	"	1 statute mile.....*mi.*

EQUIVALENTS.

mi.		*fur.*		*rd.*		*yd.*		*ft.*		*in.*
1	=	8	=	320	=	1760	=	5280	=	63360
		1	=	40	=	220	=	660	=	7920
				1	=	5½	=	16½	=	198
						1	=	3	=	36
								1	=	12

SCALE OF UNITS:—12, 3, 5½, 40, 8.

ALSO:

3 barleycorns	make	1 inch..............	used by shoemakers.
4 inches	"	1 hand............	" to measure horses.
6 feet	"	1 fathom..........	" " depths at sea.
1.15 statute miles	"	1 geographic mile..	" " distances "
3 geographic miles	"	1 league.	
60 " "	"	1 degree.	
69½ statute "	"	1 degree.	
360 degrees	"	the circumference of the earth.	

813. MARINERS' MEASURE. (Art. 281, 4.)

TABLE.

6 feet	make	1 fathom.
120 fathoms	"	1 cable length.
51 feet (nearly)	"	1 knot of "log line."
1 geographic mile	makes	1 knot of *distance at sea.*

NOTE.—The number of knots of the log line run off in half a minute indicates the number of knots of distance a vessel goes per hour The *log* knot is *commonly* made of about 6¾ fathoms to run off in 28 seconds, to avoid danger near shore by keeping the ship behind her "reckoning."

814. SURVEYORS' LONG MEASURE. (Art. 281, 3.)

Used to compute land distances and areas. A Gunter's chain, which is the measure used by surveyors, is four rods in length, and consists of 100 links.

TABLE OF LINEAR DISTANCES.

7.92 inches (*in.*)	make	1 link....*l.*
25 l.	"	1 rod or pole..*rd.* or *p.*
4 rd., or 66 ft.,	"	1 chain...*ch.*
80 ch.	"	1 mile...*mi.*

EQUIVALENTS.

mi.		*ch.*		*rd.*		*l.*		*in.*
1	=	80	=	320	=	8000	=	63360
		1	=	4	=	100	=	792
				1	=	25	=	198
						1	=	7.92

SCALE OF UNITS:—7.92, 25, 4, 80.

815. CLOTH MEASURE. (Art. 281, 2.)

TABLE.

$2\frac{1}{4}$ in.	make	1 nail	*na.*
4 na.	"	1 quarter	*qr.*
4 qr.	"	1 yard	*yd.*
5 qr.	"	1 Ell English	*E. E.*

EQUIVALENTS.

yd.		*qr.*		*na.*		*in.*
1	=	4	=	16	=	36
		1	=	4	=	9
				1	=	$2\frac{1}{4}$

SCALE OF UNITS:—$2\frac{1}{4}$, 4, 4.

NOTE.—The unit in common use is the *yard*, and this is subdivided into the binary scale of halves, quarters, eighths, and sixteenths. Imported cloths, etc., are estimated by the square yard, to which lineal yards are reduced according to the width of the cloth.

816. CIRCULAR MEASURE. (Art. 282.)

Used to determine localities, by estimating latitude and longitude; also, to measure the motions of the heavenly bodies, and compute differences of time. All circles, of whatever dimensions, are supposed to be divided into the same number of parts—as quadrants, signs, degrees, etc. It will, therefore, be evident, that there can be no "fixed" dimensions of the units named.

TABLE.

60 seconds (″)	make	1 minute	′
60′	"	1 degree	°
30°	"	1 sign	*S.*
12 S., or 360°	"	1 circle	*C.*

EQUIVALENTS.

C.		*S.*		°		′		″
1	=	12	=	360	=	21600	=	1296000
		1	=	30	=	1800	=	108000
				1	=	60	=	3600
						1	=	60

SCALE OF UNITS:—60, 60, 30, 12.

817. SQUARE MEASURE. (Art. 284.)

Used to compute surfaces or areas.

TABLE.

144 square inches (*sq. in.*)	make	1 square foot	*sq. ft.*
9 sq. ft.	"	1 square yard	*sq. yd.*
$30\frac{1}{4}$ sq. yd.	"	1 square rod	*sq. rd.*
40 sq. rd.	"	1 rood	*R.*
4 R.	"	1 acre	*A.*
640 A.	"	1 square mile	*sq. mi.*

EQUIVALENTS.

sq. mi.	*A.*	*R.*	*sq. rd.*	*sq. yd.*	*sq. ft.*	*sq. in.*
1 =	640 =	2560 =	102400 =	3097600 =	27878400 =	4014489600
	1 =	4 =	160 =	4840 =	43560 =	6272640
		1 =	40 =	1210 =	10890 =	1568160
			1 =	30¼ =	272¼ =	39204
				1 =	9 =	1296
					1 =	144

SCALE OF UNITS:—144, 9, 30¼, 40, 4, 640.

818. SURVEYORS' SQUARE MEASURE. (Art. 284, 4.)

TABLE OF AREAS.

625 square links (*sq. l.*)	make	1 pole..........*P.*
16 P.	"	1 square chain..*sq. ch.*
10 sq. ch.	"	1 acre............*A.*
640 A.	"	1 square mile...*sq. mi.*
36 sq. mi. (6 mi. square)	"	1 township......*Tp.*

NOTE.—1 sq. mi. makes a Section (Sec.) of Government land, and the section is subdivided into quarters, each quarter-section containing 160 acres.

EQUIVALENTS.

Tp	*sq. mi.*	*A.*	*sq. ch.*	*P.*	*sq. l.*
1 =	36 =	23040 =	230400 =	3686400 =	2304000000
	1 =	640 =	6400 =	102400 =	64000000
		1 =	10 =	160 =	10000
			1 =	16 =	1000
				1 =	625

SCALE OF UNITS:—625, 16, 10, 640, 36.

819. CUBIC MEASURE. (Art. 285.)

Used to compute the contents of solid substances or volume of any space; it is sometimes called "solid" measure.

TABLE.

1728 cubic inches (*cu. in.*)	make	1 cubic foot......*cu. ft.*
27 cu. ft.	"	1 cubic yard......*cu. yd.*
16 cu. ft.	"	1 cord foot.......*cd. ft.*
8 cd. ft. or 128 cu. ft.	"	1 cord of wood...*cd.*
24¾ cu. ft.	"	1 perch of stone..*Pch.*
40 cu. ft.	"	1 ton of ship cargo.
50 cu. ft. of square timber	"	1 ton.

EQUIVALENTS.

cu. ft.	*cu. in.*
1 =	1728

SCALE OF UNITS:—1, 1728.

820. TIME MEASURE. (Art. 290.)

Used to compute the passage of time.

TABLE.

60 seconds (*sec.*)	make	1 minute......*m.*
60 m.	"	1 hour........*hr.*
24 hr.	"	1 day ... .*da.*
7 da.	"	1 week........*w.*
365¼ da.	"	1 year........*yr.*
10 yr.	"	1 decade......
10 decades, or 100 yr.,	"	1 century.....*C.*

EQUIVALENTS.

yr.		*da.*		*hr.*		*min.*		*sec.*
1	=	365¼	=	8766	=	525960	=	31557600
			=	24	=	1440	=	86400
				1	=	60	=	3600
						1	=	60

SCALE OF UNITS:—60, 60, 24, 365¼.

NOTE.—It is customary to reckon 30 days or 4 weeks to the month, and 12 months to the year, but this is not accurate. Twelve *calendar* months make a year, but these months are not of equal length, as the following table will show:

1. January	has 31 days.	7. July	has 31 days.
2. February	" 28 "	8. August	" 31 "
3. March	" 31 "	9. September	" 30 "
4. April	" 30 "	10. October	" 31 "
5. May	" 31 "	11. November	" 30 "
6. June	" 30 "	12. December	" 31 "

The common year thus consists of 365 days. Once in 4 years, however, one day is added to February, making 366 days; and thus, each year averages 365¼ days. The longest year is called Bissextile, or Leap year. Centuries divisible by 400, and other years divisible by 4, are leap years.

821. LONGITUDE AND TIME. (Art. 291.)

TABLE.

For a difference of	There is a difference of
1° in Long.	4 m. in Time.
1′ "	4 sec. "
1″ "	$\frac{1}{15}$ sec. "
1 hr. in Time	15° in Long.
1 m. "	15′ "
1 sec. "	15″ "

NOTE.—*Add* difference of time for places *east* and subtract it for places *west* of any given place.

822. LIQUID, OR WINE MEASURE. (Art. 286.)

Used for measuring liquids; such as liquors, molasses, water, etc.

TABLE.

4 gills (*gi.*)	make	1 pint	*pt.*
2 pt.	"	1 quart	*qt.*
4 qt.	"	1 gallon	*gal.*
31½ gal.	"	1 barrel	*bbl.*
2 bbl.	"	1 hogshead	*hhd.*

EQUIVALENTS.

hhd.		*bbl.*		*gal.*		*qt.*		*pt.*		*gi.*
1	=	2	=	63	=	252	=	504	=	2016
		1	=	31½	=	126	=	252	=	1008
				1	=	4	=	8	=	32
						1	=	2	=	8
								1	=	4

SCALE OF UNITS:—4, 2, 4, 31½, 2.

ALSO:

36 gallons	make	1 barrel of ale, beer, or milk.
54 "	"	1 hogshead " "
42 "	"	1 tierce.
2 hogsheads	"	1 pipe, or butt.
2 pipes	"	1 tun.

823. APOTHECARIES' FLUID MEASURE. (Art. 286, 3.)

Used in mixing liquid medicines by measure.

TABLE.

60 minims (♏)	make	1 fluid drachm....*f*ʒ.
8 fʒ	"	1 fluid ounce.....*f*℥.
16 f℥	"	1 pint............*O.* (*Octarius.*)
8 O.	"	1 gallon..........*Cong.* (*Congius.*)

EQUIVALENTS

Cong.		*O.*		*f*℥		*f*ʒ		♏
1	=	8	=	128	=	1024	=	61440
		1	=	16	=	128	=	7680
				1	=	8	=	480
						1	=	60

SCALE OF UNITS:—60, 8, 16, 8. NOTE.—One fluid ounce = 455.6944 Troy grains

NOTE.—The minim is a drop of pure water, and is equal to about $\frac{95}{100}$ of a grain Troy.

824. DRY MEASURE. (Art. 287.)

Used for measuring articles not liquid; as grain, fruit, salt, etc.

TABLE.

2 pints (*pt.*)	make	1 quart......*qt.*
8 qt.	"	1 peck......*pk.*
4 pk.	"	1 bushel.....*bu.*
36 bu.	"	1 chaldron...*ch.*

EQUIVALENTS

ch.		*bu.*		*pk.*		*qt.*		*pt.*
1	=	36	=	144	=	1152	=	2304
		1	=	4	=	32	=	64
				1	=	8	=	16
						1	=	2

SCALE OF UNITS:—2, 8, 4, 36.

NOTE.—1 gal. Wine Measure contains 231 cu. in., 1 gal. Ale or Beer Measure (nearly obsolete), 282 cu. in., and 1 bu. 2150$\frac{42}{100}$ cu. in.

825. AVOIRDUPOIS WEIGHT. (Art. 288.)

Used to weigh all coarse articles; as hay, grain, groceries, wares, etc., and all metals, except gold and silver.

TABLE

437½ grains (*gr.*)	make	1 ounce............*oz.*
16 oz.	"	1 pound...........*lb.*
25 lb.	"	1 quarter..........*qr.*
4 qr.	"	1 hundred weight...*cwt.*
20 cwt.	"	1 ton..............*T.*

EQUIVALENTS.

T.		*cwt.*		*qr.*		*lb.*		*oz.*		*gr.*
1	=	20	=	80	=	2000	=	32000	=	14000000
		1	=	4	=	100	=	1600	=	700000
				1	=	25	=	400	=	175000
						1	=	16	=	7000
								1	=	437½

SCALE OF UNITS:—437½, 16, 25, 4, 20.

NOTE.—VARIABLE VALUE OF THE TON.—The Ton, which is used variously as a denomination of weight of 2000 or 2240 pounds, is also sometimes spelled Tun, though this latter orthography usually indicates a liquid measure of 252 gallons. The Ton of 2000 lb. is called the "Short Ton," and that of 2240 lb., the "Long Ton," used in U. S. Customs and in England. The Ton varies in different places, as shown in the following table:

Maryland	Ordinary Ton	2000 lb.
"	Usual Coal Ton	2240 "
"	Miner's "	2470 "
France	Commercial "	2158.4 lb.
"	Metrical "	2204.7 "
Spain	" "	2032 "
Portugal	"	1755.8 "
Holland	(Butter) "	476 "
England	Coal "	140 pecks.

The Ton, of shipping, in the United States is 40 cubic feet. In China and India 50 cubic feet. In Portugal, 73.3 cubic feet. In France, 50.84 cubic feet. In Hamburg, 33.21 cu. feet.

826. TROY WEIGHT. (Art. 289.)

For weighing gold, silver, jewels, and liquors.

TABLE.

24 grains (*gr.*) make 1 pennyweight....*pwt.*
20 pwt. " 1 ounce............*oz.*
12 oz. " 1 pound..........*lb.*

EQUIVALENTS.

lb.		*oz.*		*pwt.*		*gr.*
1	=	12	=	240	=	5760
		1	=	20	=	480
				1	=	24

SCALE OF UNITS:—24, 20, 12.

827. APOTHECARIES' WEIGHT. (Art. 289, 2.)

Used by apothecaries and physicians in mixing medicines by weight.

TABLE.

20 grains (*gr.* xx.) make 1 scruple.... ℈
3 scruples (℈iij) " 1 dram...... ʒ
8 drams (ʒ viij) " 1 ounce..... ℥
12 ounces (℥ xij) " 1 pound..... ℔

EQUIVALENTS

℔		℥		ʒ		℈		*gr.*
1	=	12	=	96	=	288	=	5760
		1	=	8	=	24	=	480
				1	=	3	=	60
						1	=	20

SCALE OF UNITS:—20, 3, 8, 12.

828. U. S. MONEY. (Art. 292.)

TABLE.

10 mills (*M.*)	make	1 cent......*ct.* or ¢.
10¢	"	1 dime.....
10 dimes	"	1 dollar.....*D.* or $.
10 dollars	"	1 eagle.....*E.*

NOTE.—The *mill* is not coined, but is only a unit of computation and account; and the *dime* is a coin, but is not used as a unit of computation or account, the dollar being equal to 100 cts.

829. BOOKS AND PAPER.

Names of different sizes of paper made by machinery.

Double Imperial,	32 by 46 inches.	Imperial,	22 by 32 inches.
Double Super Royal,	28 by 42 "	Super Royal,	21 by 28 "
Double Medium,	23 by 26 "	Royal,	20 by 25 "
"	24 by 37½ "	Medium,	19 by 24 "
"	25 by 38 "	Folio Post,	17 by 22 "
Royal and Half,	25 by 29 "	Foolscap, (about)	12½ by 16 "
Imperial and Half,	26 by 32 "	Crown,	15 by 19 "

A sheet folded in		2	leaves is called	a folio.
"	"	4	"	a quarto, or 4to.
"	"	8	"	an octavo, or 8vo.
"	"	12	"	a 12mo.
"	"	18	"	an 18mo.
"	"	24	"	a 24mo.
"	"	32	"	a 32mo.

NOTE.—In estimating the size of the leaves, as above, the double medium sheet is taken as a standard. Copyists estimate from 75 to 100 words per *folio*, or *single page*.

830. MISCELLANEOUS TABLE.

12	units	make	1 dozen.
12	dozen	"	1 gross.
12	gross	"	1 great gross.
20	things	"	1 score.
24	sheets	"	1 quire of *paper*.
20	quires	"	1 ream.
2	reams	"	1 bundle.
5	bundles	"	1 bale.
100	pounds	"	1 quintal of *fish*.
196	pounds	"	1 barrel of *flour*.
200	pounds	"	1 barrel of *pork* or *beef*.
56	pounds	"	1 firkin of *butter*.
14	pounds	"	1 stone of *iron* or *lead*.
21½	stones	"	1 pig.
8	pigs	"	1 fother.
2	weys (328 lb.)	"	1 sack of *wool*.
12	sacks (39 cwt.)	"	1 last.
3	inches	"	1 palm.
4	inches	"	1 hand.
9	inches	"	1 span.
18	inches	"	1 cubit.
22	inches (nearly)	"	1 sacred cubit.
2½	feet	"	1 military pace.
3	feet	"	1 common pace.

NOTE.—See Art. 287, 2, for weight of grain, etc.

831. RAILROAD FREIGHT.

TABLE OF GROSS WEIGHTS.

The Articles named are Billed at actual weights, if possible, but usually at the weights in the Table below when it is not convenient to weigh them.

Ale and Beer	320	lb. per	bbl.
" "	170	"	½ "
" "	100	"	¼ "
Apples, dried	24	"	bu.
" green	56	"	"
" "	150	"	bbl.
Barley	48	"	bu.
Beans, white	60	"	"
" castor	46	"	"
Beef	320	"	bbl.
Bran	20	"	bu.
Brooms	40	"	doz.
Buckwheat	52	"	bu.
Cider	350	"	bbl.
Charcoal	22	"	bu.
Clover Seed	60	"	"
Corn	56	"	"
" in ear	70	"	"
" Meal	48	"	"
" "	220	"	bbl.
Eggs	200	"	"
Fish	300	"	"
Flax Seed	56	"	bu.
Flour	200	"	bbl.
Hemp Seed	44	"	bu.
Highwines	350	lb. per	bbl.
Hungarian Grass Seed	45	"	bu.
Lime	200	"	bbl.
Malt	38	"	bu.
Millet	45	"	"
Nails	108	"	keg.
Oats	32	"	bu.
Oil	400	"	bbl.
Onions	57	"	bu.
Peaches, dried	33	"	"
Pork	320	"	bbl.
Potatoes, common	150	"	"
" "	60	"	bu.
" sweet	55	"	"
Rye	56	"	"
Salt, fine	56	"	"
" "	300	"	bbl.
" coarse	350	"	"
" in sacks	200	"	sack.
Timothy Seed	45	"	bu.
Turnips	56	"	"
Vinegar	350	"	bbl.
Wheat	60	"	bu.
Whisky	350	"	bbl.
One *ton* weight is	2000 lbs.		

832. ESTIMATED WEIGHTS OF LUMBER AND OTHER ARTICLES.

NOTE.—From 18000 to 20000 lb. is considered a car load in most places, each car itself also weighing about 20000 lb.

	WEIGHT. *lb.*	AM'T FOR CAR LOAD. *Feet.*
LIGHT LUMBER—*Pine*, *Hemlock* and *Poplar*, thoroughly seasoned, per thousand feet	3,000	6,500
" " *Black Walnut*, *Ash*, *Maple* and *Cherry*, per thousand feet	4,000	5,000
MEDIUM LUMBER—*Pine*, *Hemlock* and *Poplar*, green, per thousand feet	4,000	5,000
" " *Black Walnut*, *Maple*, *Ash* and *Cherry*, green, per thousand feet	4,500	4,000
" " *Oak*, *Hickory* and *Elm*, dry, per thousand feet,	4,000	5,000
HEAVY LUMBER—*Oak*, *Hickory* and *Elm*, green, per thousand feet,	5,000	4,000
" " *Oak*, *Hickory* and *Elm*, part seasoned, per thousand feet	4,500	4,500
HOOP POLES, seasoned, (28 feet car)		4 feet high.
" green " "		3 "
STAVES AND HEADING, seasoned, (28 feet car)		4 "
" " green, " "		3 "
OAK BARK, green, per cord	3,500	5 cords.
" dry "	2,500	7 "
SHINGLES, green, per thousand	375	55 M.
" dry "	275	70 M.
LATH, per thousand	500	40 M.
BRICK, common, per car load	4 lb. each.	5,000
FIRE BRICK, "	6 "	3,000
LIME AND COAL, "		250 bu.
COKE, "		500 "
SAND, per cubic yard	3,000	6½ cu. yd.
GRAVEL, "	3,200	6 "
STONE, undressed, per cubic yard	4,000	5 "
" per car load		20,000 lb.
STAGE COACHES	4,000	
TWO-HORSE CARRIAGES	3,000	
ONE-HORSE WAGONS	1,500	
SINGLE SLEIGHS	1,000	
CATTLE	2,000	

TABLES OF MONEY, WEIGHT, AND MEASURE

OF THE

PRINCIPAL COMMERCIAL COUNTRIES IN THE WORLD.

NOTE.—We are indebted originally to the Publishers of "WEBSTER'S COUNTING HOUSE DICTIONARY" for the use of the following admirably arranged Tables, which will be found of great value for reference. Many valuable tables are given in the last edition of that work. The mark a in the following tables signifies *each*; e.g., the Chinese reckon in taels, each tael=10 mace, each mace=10 candareens, each candareen=10 cash. (See Art. 841.)

AMSTERDAM (See NETHERLANDS).

833. AUSTRIA.

(*Chief Commercial City*, VIENNA.)

Money. In Silver.

fl. krt.		£	s.	d.		$	c.	m.
10 0	=	1	0	0	=	4	84	0
0 30	=	0	1	0	=	0	24	2
0 2½	=	0	0	1	=	0	02	0 2/10
7 0	=	0	18	6	=	3	26	7
4 40 or 1 ducat	=	0	9	4	=	2	25	8 3/10
1 0 or 1 silver florin	=	0	2	0	=	0	48	4
2 0 or 1 dollar.. ..	=	0	4	0	=	0	96	8
0 20 or 1 zwanziger.	=	0	0	8	=	0	16	1 4/10

1 florin is equal to 60 kreutzers.

Gold is at a premium as compared with silver, while paper money is at a discount of about 13 per cent.

Weights and Measures.

AUSTRIAN.		ENGLISH.
100 commercial lb	=	123.4½ lb. avoirdp.
1 staro	=	2.31 Winch. bush.
1 polonick	=	0.861 "
1 eimer	=	15 wine gallons.
1 barile..	=	17¾ "
1 ell woolen measure.	=	26.6 in.
1 ell silk	=	25.2 in.

Or more particularly—

Weight.

1 pfund (lb.)	=	4 vierding.
1 vierding.......	=	4 unzen.
1 unzen	=	2 loth.
1 loth...........	=	4 quentchen.
1 stein	=	20 lb.
1 saum	=	275 lb.

Measure.

1 fuss	=	1.03713 U. S. ft.
1 mult	=	4⅓ miles.

Grain.

64 maassel	=	1 metz.
30 metz	=	1 muth.
1 muth	=	52.36 U. S. bu.

834. BAVARIA AND BADEN.

(*Principal Commercial City*, AUGSBURG.)

Money.

fl. krt.		£	s.	d.		$	c.	m.
12 0 at par...........	=	1	0	0	=	4	84	0
0 36	=	0	1	0	=	0	24	2
0 3	=	0	0	1	=	0	02	0 2/10
10 0 gold 10 guild. p.	=	0	16	8	=	4	03	3 1/10
5 0 gold 5 "	=	0	8	4	=	2	01	6 5/10
3 30 silv. 3½ flor. p'ce	=	0	5	10	=	1	41	1 1/10
5 35 or ducat........	=	0	9	3	=	2	23	8 4/10
2 42 or crown thaler	=	0	4	4	=	1	04	8 4/10
1 0	=	0	1	8	=	0	40	3 4/10

1 florin is equal to 60 kreutzers.

Accounts are kept in gulden a 60 kreutzer of the 20 gulden fuss, so called because the Cologne mark of fine silver is worth only 20 fl. Augsburg currency, while all other South German States reckon on the 24 gulden fuss.

COIN.—Gold (old). 1 Caroline=18s. 6d. English=$4.44.
½ caroline=9s. 3d. English=$2.22.
1 double max d'or=24s. 4d. English=$5.84.
1 max d'or=12s. 2d. English=$2.92.
1 ducat (new)=9s. 4d. English=$2.24.

Silver pieces of 3½ gulden, 1 gulden, ½ gulden, 1 kreutzer, 3 kreutzer, all in the 24 gulden fuss.

Weight.

1 pound=560 grammes French=1¼ pound avoirdupois.
1 cwt.=100 pounds=3,200 loth=12,800 quent.
1 Augsburg marc=16 loth=64 quent=256 pfenning=3,643 grains troy English.

Measure.

The foot=11¾ inches English.
1 ruthe=10 feet=120 zoll or inches=1440 lines.
1 ell=2¾ feet=33¾ inches English.
1 klafter=6 feet=5¾ feet English.

FOR CORN.—1 scheffel=6 bushels 1 gallon, English.
1 scheffel=6 metz=12 viertel=48 maas.

FOR LIQUORS.—Wine, 1 eymer=60 maas.
Beer, 1 " =60 "
1 mass=1¾ pints English.

835. BELGIUM.

(*Principal Commercial City*, ANTWERP.)

Money (at par).

fr. cts.		£	s.	d.		$	c.	m.
25 12½	=	1	0	0	=	4	84	0
1 25	=	0	1	0	=	0	24	2
0 10	=	0	0	1	=	0	02	0 1/10
25 0 or 1 gold Leopold	=	0	19	10	=	4	79	9 1/10
10 0 or 10 franc piece	=	0	7	10	=	1	89	5 1/10
5 0 or 5 franc piece	=	0	3	11	=	0	94	7 1/10
1 0	=	0	0	9½	=	0	19	1 1/10

1 franc=100 centimes (cts.)

Weights and measures the same as in France.

1 quintal (old)=103½ lb. av.

836. BRAZIL.

(*Principal Commercial City*, RIO DE JANEIRO.)

Money.

reis.		£	s.	d.		$	c.	m.
6400 or gold piece	=	1	15	9	=	8	65	1 [illegible]
4000 or gold piece of	=	1	0	0		4	84	0
1200 or silver piece of		0	4	2	=	1	00	8 [illegible]
960 "	=	0	4	1	=	0	98	0 [illegible]
640 "	=	0	2	9		0	66	5 [illegible]
320 "	=	0	1	4	=	0	32	2 [illegible]
200 "		0	0	8		0	16	1 [illegible]

1 mil reis is equal to 1000 reis.

The unit is the reis, as in Portugal.

COIN.—Gold dobra a 12,800 reis = $18.00.
Meia dobra a 6,400 reis = $9.00.
Moeda a 4000 reis = $5.75.
Silver pieces of 1200 reis = $1.00; 400 reis = $0.33; 100 reis = $0.08.

Bank Notes are worth less than specie by about one-third.

Exchange on London, 30d. sterling per milreis in bank notes.

Exchange on Paris, fr. 3.15 to fr. 3.30 per 1000 reis.

Weight.

1 quintal = 4 arrobas = 129.54 lb. av.
1 arroba = 32 arratels.
1 arratel (lb.) = 1.012 lb. av.

Gold and silver weight is the arratel a 2 marcos, a 8 onças, a 8 oitavas, a 72 granos.
1 arratel = 7084 troy grains.
1 marco = 7 oz. 7 [illegible] pwt.

Diamonds, emeralds, rubies, pearls, etc., are sold by the quilate. Topazes by the oitava a 3 escrupulos a 3 quilates a 4 granos.
1 oitava = 1 oz. 19 [illegible] dwt. troy.
1 quilate = 4 [illegible] dwt. troy.

Measure.

1 pe (foot) = 1 foot Eng.
1 palmo = 9 [illegible] inches Eng.
1 braça = 2 varas = 3 [illegible] covados = 10 palmos.
1 braça = 2 [illegible] yards Eng.
1 legoa (mile) = 4 [illegible] miles Eng.

CORN, RICE, COFFEE, &c.—1 moyo = 15 fanegas, each fanega = 4 alqueires.
1 moyo = 22 [illegible] bushels Eng.
1 fanega = 11 gal.

WINE. The same as in Portugal.

NOTE.—The metric system of weights and measures was adopted in 1862, and became compulsory in 1873. It was used in official computations and in the Customs Tariff of 1869.

837. BREMEN.

(*One of the three Free Cities of Germany.*)

Money.

rigdl. grosch.		£	s.	d.		$	c.	m.
6 6	=	1	0	0	=	4	84	0
0 24	=	0	1	0		0	24	2
1 0 or gold rigxd.	=	0	3	4		0	40	6 [illegible]
0 36 or 36 groat p'ce	=	0	1	6	=	0	36	3
5 24 or Louis-d'or		0	16	0	=	3	87	2

1 thaler is equal to 72 groten.

838. BRUNSWICK AND HANOVER.

(*Principal Commercial Cities*, BRUNSWICK and HANOVER.)

Money.

th. grs. pfs.		£	s.	d.		$	c.	m.
6 16 0	=	1	0	0	=	4	84	0
0 8 0	=	0	1	0		0	24	2
0 0 10		0	0	1		0	02	0 [illegible]
10 0 0 dble Geo.d'or	=	1	12	4		7	72	4 [illegible]
5 0 0 or single "		0	16	2		3	91	2 [illegible]
1 0 0		0	3	0	=			
0 1 0 or 12 pfen'gs	=	0	0	1 [illegible]		0	03	0 [illegible]

1 thaler is equal to 24 groschen.

839. CADIZ.

1 quintal = 4 arrobas (or 100 lb.) of 2 marcs each.
100 lb. Castilian = 101 [illegible] lb. avoir.
Common Spanish lb. = about 2 lb. avoir.
1 vara (yd.) = .9275 Eng. yd.
1 cahiz (corn) = 1.07 Winch. qr.
1 fanega " = 1 qr. "
1 moyo (wine) = 16 arrobas or [illegible].
1 botta = 30 arrobas of wine = 12 [illegible] gal.

840. CAPE OF GOOD HOPE.

Money.

Same as Holland.

Weights and Measures.

Same as Holland or England.

In 1859 it was officially determined that 1000 "Cape feet" were equal to 1033 English feet.

841. CHINA.

(*Commercial Cities*, CANTON, AMOY, *etc.*)

Money (by weight).

1 tael or *liang* = 583 [illegible] gr. troy
= 10 mace or *tsien*.
1 mace = 10 candareens or *fun*.
1 candareen = 10 cash or *li* = about 1 ct. U. S.

The cash is a copper coin, and it has so depreciated as to require 1400 cash to equal 1 tael.

Pure silver cast into ingots called *shoes*, and known as *sycee*, is used in trade and exchange.

The tael is reckoned according to the price paid per ounce for Spanish dollars in London, and is worth 20 [illegible] more per oz.; thus, if the dollar be valued at 60d. per oz., the tael is valued at 60d. [illegible] per oz. Commonly 717 taels = 1000 Spanish dollars.

The fineness of gold and silver is estimated by weight in 100 *touches* or *touch*. Gold is used as merchandise in regular ingots of a certain weight, the largest being 10 taels, 94 touch fine (6 parts being alloy).

Sycee silver is about 96 touch fine.

Weight.

Gold and silver are weighed by the catty a 16 taels a 10 mace a 100 candareens a 1000 cash.

100 taels = 120 [illegible] troy oz.

FOR MERCHANDISE.—1 tael = 1 [illegible] oz. avoir.
1 catty = 16 taels = 1 [illegible] lb. avoir.
1 picul = 100 catties = 133 [illegible] lb. avoir., or 162 lb. 8 dwt. 13 gr. troy.

Long Measure.

1 covid or *chi* (ft.) = 10 *ts'un* (inches). = 14.625 Eng. inches.

The *chi* or ft. has about 100 variations for different purposes. The commercial *chang* = 10 *chi* = 141 Eng. inches.

The Chinese use five different feet:

For mathematics	13$\frac{1}{8}$	inches English.
For builders ..	= 12$\frac{7}{10}$	"
For engineers	12$\frac{1}{4}$	"
For trade .	= 13$\frac{1}{4}$	"
For tariff .	= 14$\frac{1}{10}$	"

1 li = 180 fathoms of 10 feet of the engineers = $\frac{1}{3}$ of an English mile.

Land Measure.

1 *tsing* = 100 sq. chi = 149.756 sq. ft.
1 mow = 60 tsing.

The British Consulate standard is
1 *pu* = 58 inches Eng.
1 *chang* = 2 pu = 10 chi, hence
1 *mow* = 7,260 sq. ft. Eng.

Capacity.

1 *tow* = 10 shing = about 1$\frac{1}{4}$ gal.

842. DENMARK.

(*Principal Commercial City*, COPENHAGEN.)

Money.

rigsd.	skil.		£ s. d.	$ c. m.
9	16	.. .. ..	1 0 0 =	4 84 0
0	44		= 0 1 0 =	0 24 2
0	3$\frac{2}{3}$	.	0 0 1 =	0 02 0$\frac{1}{10}$
7	80 or 1 Chris'n d'or =		0 16 8	3 93 2$\frac{7}{10}$
2	0 or 1 species silv. =		0 4 4 =	1 04 8$\frac{1}{10}$
1	0		0 2 2	0 52 4$\frac{1}{10}$
0	16 or 1 mark		0 0 4$\frac{1}{2}$	0 09 1

1 rigsbank daler is equal to 96 skillings.
2 rigsbank daler = 1 specie-daler = 3 mark banco in Hamburg.
1 rigsbank daler = 2s. 3d. English.
1 skilling = 1 farthing = half a cent American.

Bank notes in specie daler are freely taken at 100 specie daler for 200 rigsbank daler.

They draw generally on Hamburg at sight or 14 days after date and the exchange on London is 9$\frac{1}{4}$ rigsbank daler for £1 sterling. Exchange on Paris (rarely) from fr. 2.60 to fr. 2.70 per rigsbank daler.

Weights.

1 ort = 7.7 gr.
1 quint = 10 ort = 77 gr.
1 pund = 100 quint = 1.1023 lb. avoir.
1 centner = 100 lb. = 110.2812 lb. avoir.
1 commercelaest = 2 tons.
1 tönde

Of Corn	=	3.948 U. S. bu.
" Beer	=	31.1068 U. S. gal.
" Butter		246.9179 lb. avoir.
" Coal	=	4.825 U. S. bu.

The unit for gold and silver is the Danish-Cologne mark = 3621.156 gr. troy.

The unit for coin is the Hamburg-Cologne mark = 3608 gr. gr. troy.

Measure.

1 pot		0.2126 gal.
1 vleriel (8 pots) ..	=	1.7011 gal.
1 alen (ell) .	=	0.6864 yd.
1 fod (ft.)	=	1.0297 ft.
1 cubikfod .	=	1.0918 cu. ft.

NOTE.—The U. S. Custom House valuations vary somewhat from these.

843. EAST INDIES.

(*Principal Commercial Cities*, BOMBAY, BENGAL, CALCUTTA, and MADRAS.)

Money.

rup's.	ann.	pi.		£ s. d.	$ c. m.
10	8	0	....	1 0 0	4 84 0
0	8	4	.	= 0 1 0 =	0 24 2
0	0	8		0 0 1	0 02 0$\frac{1}{10}$
15	0	0 gold mohur		1 9 0	7 01 8
1	0	0 rupee sicca		= 0 1 10$\frac{1}{2}$	0 45 3$\frac{3}{8}$
0	8	0 half rupee		0 0 11$\frac{1}{4}$ =	0 22 6$\frac{3}{4}$

1 rupee is equal to 16 annas or 50 pice.

More particularly—

1 rupee	=	4 quarters.
1 quarter .	=	100 reas.
1 urdee .	=	2 reas.
1 doreea . .	=	6 reas.
1 dooganey	=	4 reas.
1 fondea . .	=	8 reas.
1 paunchea .	=	5 rupees.
1 gold mohur	=	15 rupees.

NOTE.—The anna and rea are not coined.

Weight.

1 maund (factory maund), = 40 seers, = 16 chattacks.
1 maund = 74 pounds 10 ounces avoirdupois.
1 seer = 30 ounces avoirdupois. The bazaar weight is 10 per ct. heavier.
1 sicca = 10 massa = 38 grains, or 4 punkhos.
1 sicca = 179 grains troy Eng.
1 tola = 180 grains troy.

				lb. avoir.
Bombay maund =	40	seers	. . .. =	28
" "	42	"	.. .	29.4
Surat "	40	"	. .	37$\frac{1}{2}$
" "	42	"	. . . =	39$\frac{1}{2}$
" "	44	"	....	41$\frac{1}{4}$
Bengal factory maund			.. =	74$\frac{2}{3}$
" bazaar	"		.. . =	82$\frac{1}{8}$
Madras	"		. =	25
Bombay candy = 20 maunds			. =	560
Surat " " "				746$\frac{1}{2}$
Madras " " "			.. =	500
Travancore " " "			.. =	640

Measure.

1 cubit = 18 inches Eng. 1 guz = 1 yard Eng.
1 coss = 4,000 cubits = 1$\frac{1}{8}$ mile Eng.

Corn is sold by the khahoon of 40 maunds or 16 soallis = 20 pallies. 1 pallie = 9$\frac{1}{2}$ pounds avoirdupois.

Also in Calcutta,
FOR CORN.—1 garce = 400 mercals = 8 puddys or 64 allocks.
1 garce = 135 bushels.

In Bombay
1 covid = 18 inches English.
FOR CORN.—1 candy = 8 parahs = 16 adowlies.
1 candy = 24$\frac{1}{2}$ bushels.

844. EGYPT.

(*Commercial Cities*, ALEXANDRIA and CAIRO.)

Money (at par).

piast.	par.		£ s. d.	$ c. m.
97	20	. .. .	= 1 0 0 =	4 84 0
5	0	. ...	= 0 1 0	0 24 2
0	17	. .	= 0 0 1	0 02 0$\frac{1}{10}$
50	0 gold new sequin		= 0 10 4	2 50 0$\frac{1}{10}$
12	0 silver new piast.		= 0 3 4	0 60 6$\frac{1}{10}$
4	0 silver grush....		= 0 1 2 =	0 28 2$\frac{1}{10}$
1	0 piastre		= 0 0 2$\frac{1}{2}$ =	0 05 0$\frac{1}{10}$

Large payments are made in purses of 600 current piasters, chiefly in Span. dollars or piasters.

1 Sp. dollar = 20 Egypt. piast.

1 piaster in Alexandria has 40 medinis or paras, or 100 good or 120 current aspers.
In Cairo 1 piaster = 80 aspers or 33 paras.

COIN.—Ducatello a 10, grescio a 30, piaster a 40, mahboub a 90, and zumabo a 120 paras. Also, zenzerd a 107, and mecchini a 110 medinis.

Cotton is sold by cantaros. 1 cantaro = 115 lb. Eng.

Coffee and Cotton are invoiced in Span. dollars, other goods in Egyptian piasters.

Exchange on London, 80 piasters, more or less for £1.

Exchange on Paris, 316 a 320 per fr. 100.

Weight.

The *unit* is the *derhem* or drachme = 47.66 gr. troy.
1 *common oka* = 400 derhem = 2.7235 lb. av.
1 *commercial oka* = 420 derhem = 2.8597 lb. av.
1 " " of Alexandria = 2.8452 lb. av.
1 *common rotolo* = 144 derhem = .98046 lb. av.
1 " " = 12 rekieh or = a 12 derhem.
1 *special* rotolo of gov't = 1.2458 lb. av.

The *kantar* or *kuss* is of 33 different weights, varying from 36½ to 110 common oki.

For *precious metals* and *pharmacy* 1 derhem = 24 kirât (carats) a 4 kömm'hah (gr.) a 3 habb'eh.

Measure.

1 pik or drah (derah) = 4 abdah or rub a 6 kerat.

There are seven different kinds of *pik* in use, of which
1 Turkish pik (for cloth) = 26.5 inches.
1 pik endeseh " = 25.3 inches.

For liquids, 1 guirbeh = 17.611 U. S. gal.
For grain 1 ardebb of Cairo = 5.03 U. S. bu.
" " " of wheat must weigh 180 oki.

ENGLAND (See GREAT BRITAIN.)

845. FRANCE

(*Principal Commercial City*, PARIS).

Metric System (Arts. 307–310.)

Money (at par).

frs.	cts.		£	s.	d.	$	c.	m.
25	0	=	1	0	0	4	84	0
1	25	=	0	1	0	0	24	2
0	10	=	0	0	1	0	02	0$\frac{1}{6}$
20	0 or gold Napoleon	=	0	16	0	3	87	2
5	0 or silver "	=	0	4	0	0	96	8
1	0 "	=	0	0	9$\frac{1}{2}$	0	19	1$\frac{7}{8}$
0	10	=	0	0	1	0	02	0$\frac{1}{6}$

1 franc weighs 5 grammes = 100 centimes.

COIN.—Gold pieces of 100, 40, 20 and 10 fr. Silver pieces of 5, 2, 1, ½ and ¼ fr.
Bank notes of 500 and 1000 francs.

Exchange on London, fr. 25.50 for £1.
Exchange on New York, fr. 5.25 to 5.30 for $1.

Weight.

Milligramme	=	0.0154 gr.
Centigramme	=	0.1543
Décigramme	=	1.5433
Gramme	=	15.4323
Décagramme	=	154.3234

or 5.64 drams avoir.

Hectogramme	=	3.2155 oz. troy,

or 3.5274 oz. avoir.

Kilogramme or kilo.	=	2.204 lb. av.
Myriagramme	=	22.046 lb. av.
Quintal	=	220.46 lb. av.
Mill er tonneau	=	2204.6 lb. av.

The weight of 1 cubic centimetre of pure water is taken as the fundamental unit. It is called a gramme.

1 myriagramme = 10 kilogr. = 100 hectogr. = 1000 decagr. = 10,000 grammes.
1 gramme = 10 decigr. = 100 centigr. = 1000 milligr.

373¼ grammes = 1 lb. troy.
453½ grammes = 1 lb. avoir.

Measure.

Long Measure.

FRENCH		ENGLISH
Millimètre	=	0.03937 in.
Centimètre	=	0.39371 "
Décimètre	=	3.93710 "
Mètre	=	39.37100 "
Décamètre	=	32.8091 feet.
Hectomètre	=	328.09167 "
Kilomètre	=	1093.63890 yd.
Myriamètre	=	10936.38900 "

or 6 miles, 1 furlong, 28 poles.

1 myriamètre = 10 kilomètres = 100 hectomètres = 1000 decam. = 10,000 metres.
1 mètre = 10 décimètres = 100 centimètres = 1000 millimètres.

The *mètre* is the 10,000,000th part of the northern meridian quadrant.

1 mètre = 39.37 in. Eng.
1 lieue = 1 myriamètre = 6¼ Eng. miles.
1 aune = 47¼ in. Eng.

Measures of Capacity.

Millitre	=	0.06103 cu. in.
Centilitre	=	0.61028
Décilitre	=	6.10280
Litre (a cu. decimètre)	=	61.02803

or 2.1135 wine pints.

Décalitre	=	610.28028 cu. in.

or 2.642 wine gallons.

Hectolitre	=	3.5317 cu. ft.

or 26.417 wine gal., 22 imperial gal., or 2.839 Winchester bu.

Kilolitre	=	35.3171 cu. ft.

or 1 tun and 12 wine gallons.

Myriolitre	=	353.1710 cu. ft.

FOR WINE, &c.—1 litre = 1 cubic décimètre.
1 myrialitre = 10 kilol. = 100 hecto. = 1000 decal. = 10,000 litres.
1 litre = 10 decil. = 100 centil. = 1000 millil.

Superficial Measure.

Centiare	=	1.1960 sq. yd.
Are (a sq. décamètre)	=	119.6046 "
Décare	=	1196.0460 "
Hectare	=	11960.4604 "

or 2 acres, 1 rood, 35 perches.

Solid Measure.

Décistère	=	3.5317 cu. ft.
Stère (a cubic mètre)	=	35.3165
Décastère	=	353.1655

846. FRANKFORT ON THE MAIN.

AND THE SOUTHERN PARTS OF GERMANY.

Money

1 gulden a 60 kreutzers a 4 pfennige.
1 gulden or florin = $.4117.

COINS. *Gold.* Union crown = $6.6461, Prussian Frederick-d'or = $4.000, Pistole = $3.959, [illegible] = $4.016, Napoleon = $3.859, the Ducat = $2.28[illegible], and [illegible].
Silver. [illegible] $.350 equal to 1 fl. 45 kreutzer, the [illegible] thaler = 1.5 fl., the 2-fl. [illegible], half [illegible] fl., 6 kreutzer, 3 kreutzer, and the kreutzer piece.

Weight.

1 cwt. = 100 great or heavy pds. = 108 small or light pds.
1 lb. heavy = 17[illegible] oz. avoir.
1 lb. [illegible] = 2 mark = 32 loth = 128 quent = 514 [illegible] = 15 [illegible] oz. troy.
1 mark = 7 oz. 14 dwt. troy.
1 cwt. of 100 heavy or 108 light lb. = 111 lb. avoir.
1 carat of jewels = 1 dwt. 7½ gr. troy.

Measure.

1 foot = 11¼ in. Eng.
1 foot = 12 zoll = 144 lines.
1 ell = 21[illegible] in. Eng.
1 Frankfort Brabant ell = 27¼ in. Eng.
FOR CORN.—1 malter = 4 simmer = 4 sechter = 4 gescheide.
1 malter = [illegible] bush. 1½ gal. Eng.
1 simmer = [illegible] gal. Eng.
FOR LIQUIDS.—1 ohm = 80 mass = 4 schoppen.
1 mass = 1 gescheid = 3[illegible] pints Eng.
1 ohm = 31 [illegible] gal.
1 fuder = 6 ohms; 1 stuck = 8 ohm.

847. GERMANY.

See Baden, Bavaria, Bremen, Hamburg, Prussia and Frankfort on the Main for old denominations of money, weight and measure.

Money.

The gold coinage adopted for the German States and Austria is the *union crown* and half crown. [illegible] union crown contains 10 grammes of fine gold and is valued at $6.6461.

The standard of silver coinage is not uniform, as appears under different states.

Weight.

The unit of weight common to all the states for commercial purposes is the *zollpfund*, equal to the half-kilogramme = 1.10231 lb. av.

1 zollpfund = 30 zolloth
1 zollcentner = 100 zollpfund.

Most of the states have adopted as the unit of weight for *coinage* the *münzpfund* of 500 grammes (same as zollpfund) in place of the Prussian mark, and this is subdivided into 10 000 [illegible]. The standard of coinage for the different states is as follows: the *münzpfund* of fine silver is coined into 30 Vereins thalers (for Northern Germany), into 45 Austrian florins or gulden, or into 52½ South German fl. or guld.

848. GREAT BRITAIN.

(*Principal Commercial City*, LONDON.)

Money.

The National Currency of Great Britain is called *Sterling Money*. The Pound Sterling is represented by a gold coin called a *Sovereign*, and its custom house value in the United States is fixed by law at $4 84. The *intrinsic* value of the *Sovereign* varies somewhat, depending on the date of the coinage. Victoria sovereigns are worth the most, as being of the latest coinage; those of William IV. or George III. less, as more worn. The intrinsic value of the legal standard sovereign is $4 86. The commercial value of the pound sterling varies, like merchandise, according to demand. $4.84 is that on which duties are charged. Thus if you buy a bill of goods in London of £100, on which the duty in this country is 25 per cent., and import them, you pay at the custom-house 25 per cent. on $484, or $121. What is called the *par* value of the pound sterling in the United States is $4 44. The *par* value of the pound in London, in American currency, is $4 86. The difference between the *par* value of the pound sterling in this country ($4 44) and the actual value to us here, at the time, of a pound sterling in London, is called the *Exchange*. Thus, if exchange on London in New York is 8 per cent., a pound sterling is worth $4 44 and 9 per cent. added, or $4 84. If 7 per cent., of course less; if 10 per cent., more.

Freight bills for goods by ship are payable at $4 80 the pound, which is 8 per cent. on $4.44. Exchange on London is usually 7 to 10 per cent. in New York, i.e. a pound sterling in London is worth $4 44 and 7 to 10 per cent. additional in New York, nearly.

In the preceding and following Tables the Pound Sterling is estimated at $4 84, it being understood that its commercial value is sometimes higher and sometimes lower.

$1 = £ .2054838 if £1 = $4 86⅔.

4 farthings, qr.	=	1 penny, d.
12 pence	=	1 shilling, s.
20 shillings	=	1 pound, £
A sovereign	=	20 shillings.
A guinea	=	21 "
A crown	=	5 "
A florin	=	2 "
A groat	=	4 pence.

The farthing is not coined; the penny is copper; the sixpence, shilling, and crown, silver; sovereign and guinea, gold.

The English Tables of Weights, Measures, Time, &c., are the same essentially as the American.

The English *Imperial gallon* contains 277.274 cu. in., thus making 6 wine gallons (American) = about 5 imp. gal.

The *Standard Imperial Bushel* contains 2218.192 cu. in., thus making 33 Winchester bushels (American) = about 32 imp. bushels.

849. GREECE.

(*Principal Commercial Cities*, ATHENS, PATROS, NAUPLIA, *etc.*)

Money.

drachm	lept		£ s. d.	$ c. m.
26	55		1 0 0	4 84 0
1	30		0 1 0	0 24 2
0	11		0 0 1	0 02 0[illegible]
40	0	or gold piece	1 10 0	7 [illegible] 1
5	0	or silv. piece	0 3 9	0 90 7[illegible]
1	0		0 0 [illegible]	0 17 6[illegible]

1 drachme is equal to 100 leptae.

Weight.

The metric system has been adopted, but old names are retained with the term *royal* prefixed to distinguish the new units.

The unit of weight is one and a half kilogramme.

The centigramme is called a *royal kokkos*.
10 royal kokkoi = 1 royal obolos.
10 royal oboloi = 1 royal drachme (gramme).
1500 royal drachmae = 1 mina.
1 mina = 400 old drachmae = 3.30693 lb. av.
100 royal minai = 1 royal talenton
10 royal talenta = 1 royal tonos = 3306.93 lb. avoir.
1 oka (old) = 400 drachmae = 1280 royal drachmae = 2.8219 lb. av.
9 oki = 1 pinaki
44 oki = 1 cantaro.

1 cantaro	{	= 118.07 libre-grosse (heavy Venetian lb.)
		= 124.16 lb. av.
1 migliar of 1000 libre-grosse	{	= 8.47 cantari
		374.855 oki.

Measure.

Volume.—(1 millimeter) = 1 kybos.
10 kyboi = 1 mystron
10 mystra = 1 kotyle
10 kotylai = 1 litra (liter)

100 litra	{	= 1 koilon or kilo.
		= 26.417 U. S. gal.
1 old kilo = 8.485 " "		

Grain is estimated by weight at 468¾ old drachma for 1 hectoliter.

Length.—10 royal grammata = 1 royal dactylos.
10 royal dactyloi = 1 royal palamé.
10 " palamé = 1 royal pechus, or piki (meter).

1000 royal pecheon	{	1 royal stadion (kilometer)
		3280.867 U. S. ft.
10 " stadia	{	1 royal skoinis
		6.213 U. S. miles.

Surface.—1000 royal square-piki	{	1 royal stremma.
		10 ares.
		.2471 U. S. acres.

850. HAMBURG AND LUBECK.

(*Commercial Cities of* GERMANY.)

Money.

mk.	c.	schil.	pfen.		£ s. d.	$ c. m.
16		8	0	... . ..	= 1 0 0	= 4 84 0
0		13¼	0	.	= 0 1 0	= 0 24 2
0		1	3	.	= 0 0 1	= 0 02 0[illegible]
8		0	0 or 1 ducat		= 0 9 3	= 2 23 6[illegible]
3		0	0 or 1 dol. cur.		= 0 4 4	= 1 04 8[illegible]
1		0	0		= 0 1 2¼	= 0 29 2[illegible]
0		1	0	.	= 0 0 0	= 0 01 5[illegible]

1 mark current is equal to 16 schillings.

1 thaler = 3 marks = 48 schillings; but they have two different values.
1st. According to the coin, called current;
2d. Estimated, used in trade, and called banco, generally 25 per cent. better than current.

1 mark currency = $0.28.

Exchange on London, 14 marks banco, more or less, for £1.

Exchange on Paris, fr. 150 to fr 1.70 per mark banco.

Weight.

1 pound = 16¾ oz. avoir. Eng.
1 pound = 32 loth a 4 quent.
1 centner = 111 lb. = 119¼ lb. Eng.
1 ship pound = 2¾ cwt. = 20 lies pound.
1 lies pound for shipping = 14 lb.
1 " " land carriage = 10 lb.
1 stone of flax, " " = 20 "
1 " wool, etc., " " = 10 "
100 new lb., or 1 centner = 110.232 lb. av.
100 marcs, bank weight = 62.6664 lb. troy.

For jewels the weight is the same as Berlin.

Measure.

HAMBURG.			ENGLISH.
1 foot	.. .., .	=	11.289 in.
100 feet	. . ....	=	94.021 feet.
100 sq. ft.	. .. .	=	88.4 sq. ft.
100 cu. ft.	.	=	83.115 cu. ft.
100 ells	. .. .	=	62.681 yd.
100 viertels			159.47 imp. gal.
100 fass of corn (bbl.)			18.901 imp. qr.
1 corn last		=	11¾ imp. qr.
1 commercial last			3 tons
1 old ship last..	.	=	2 tons.
1 coal last		=	2 tons

1 foot = 12 zoll = 96 achtelzoll.
1 Rhineland foot in Hambro' = 12⅜ in. Eng.
1 Hambro' ell = 22[illegible] in. Eng.
1 Brabant ell in Hambro' = 27 in. Eng.
1 Hambro' mile = 4¾ Eng. miles.

Grain.

CORN.—Is sold by the last a 3 wispel a 10 scheffel a 2 wispel a 10 scheffel a 2 fass.

BARLEY.—Is sold by the stock a 3 wispel a 10 scheffel a 4 fass.
1 fass = 1 bush. 3 gal. 4¼ pints Eng.
1 scheffel = 2 bush. 7 gal. 1½ pint.
1 wispel = 29 bush.
1 last = 10 quarters 7¼ bush.

When grain is sold by *weight*, 1 last contains—

	lb. avoir		lb. avoir
Wheat . ..	5,400	Oats	3,600
Rye ...	5,100	Malt or Peas..	3,900
Barley....	4,800	Beans ...	5,580

HOLLAND (See NETHERLANDS).

851. ITALY.

The money, weights and measures are similar to the French, different names being used. Instead of the *franc* the *lira* of 100 centesimi is used; instead of the *kilogramme* the *chilogrammo*; instead of the *metre* the *metro*; instead of the *hectare* the *ettaro*, etc. The former Papal States partly retain the old Roman money, the scudo a 10 paoli a 10 bajocchi; 1 scudo = $1.08.

852. JAPAN.

(*Principal Commercial Cities*, NAGASAKI, YEDDO, YOKOHAMA, *etc.*)

Money.

The *old* currency, which is still largely used, is the kobang a 4 itzebue a 1000 sicu.

1 kobang = $1.38.

Also 1 rice or tael = $.6752, = 10 moume a 10 pun a 10 rin a 10 mon.

The *new* currency is based on the decimal system of the United States:

1 yen = $1.00.
1 sen = .01.
1 rin = .001.

The *gold* coins are the 1 *yen*, the 2, 5, 10 and 20 *yens*, and these are legal tender for any amount.

The *silver* coins are the 5, 10, 20, and 50 *sen*, and these are legal tender only for an amount not exceeding 10 yens.

NOTE.—Another silver coin is used in trade, called the *ye gin*, which has about the value of an old Mexican dollar, but is not legal tender. The *copper* coins are the *sen*, *half-sen* and *rin*; these are not legal tender.

Weight.

1 picul=133⅓ lb avoir.
1 kin=160 monme=.617 lb. avoir.
1 monme=10 pun=27.01 grains.
1 pun=10 rin.
1 rjoo=116¼ grains.

Measure.

1 sasi=10 sung.
1 sasi=11.928 inches.
1 ri=36 tsjoo=2.562 miles.
1 sjoo, or masa=0.459128 gallon.

853. LIBERIA.

Weights and measures the same as throughout the interior of Africa, viz.:

1 Gondar....	=	7.74 pints.
1 Massuah ardeb............	=	2.32 gal.
1 kuba (chief liquid measure)	=	1.78 pints.

854. MEXICO.

(*Chief Commercial Cities*, MEXICO, VERA CRUZ and TAMPICO.)

Money (old).

dols.	reals.		£	s.	d.		$	c.	m.
16	0	or gold doubloon	=3	5	0	=	15	78	0
8	0	or ½ "	=1	12	6	=	7	86	5
4	0	or ¼ "	=0	16	3	=	3	93	$2\frac{9}{16}$
1	0	or $\frac{1}{16}$ "	=0	4	0	=	0	96	8
1	0	sil. dol. (8 reals)..	=0	4	2	=	1	00	$8\frac{1}{8}$
0	4	" ½ dol.........	=0	2	1	=	0	50	$4\frac{1}{16}$
0	2	" ¼ dol...	=0	1	0½	=	0	25	$2\frac{1}{32}$
0	1	" ⅛ dol.........	=0	0	6¼	=	0	12	$6\frac{1}{64}$

1 dollar is equal to 8 reals.
1 peso a 8 reals de plata a 4 cuartos.
1 peso=1 dollar U. S. currency.
The piaster or duros of 1833 and 1834 are about 6 per cent. less value.

COIN.—Gold doblones a 16 duros.
½, ¼ and ⅛ do.
Silver duros or dollars, ½, ¼ and ⅛.
Reales and ½ reales.

The unit of the *new* coinage is the *silver dollar*, *peso*, or *piaster*=100 cents=$1.056.
The new silver coins are 1 dollar, 50 cts., 25 cts., 10 cts., and 5 cts. The new gold coins are pieces of 20, 10, 5, 2½ pesos or dollars, and the 1 dollar or peso.
20 pesos=$19.68 U. S. gold.

NOTE.—The fineness of each piece is stamped on it: the legal fineness of silver coins being 902¾ thousandths, and of gold coins 875 thousandths.

Weight.

The Spanish-Castilian weights and measures are used with some modifications.
25 libras (pounds)=1 arroba.
4 arrobas $\begin{cases} =1 \text{ quintal.} \\ =101 \text{ lb. av.} \end{cases}$
1 tercio (of indigo or tobacco)=150 libras.
1 carga (of tobacco)=from 300 to 425 libras.
1 monton (of ore at Mexico)=3200 libras.

Measure.

109¼ varas=100 yd. U. S.
50 sq. varas=1 estajo or almud.
5000 sq. varas $\begin{cases} =1 \text{ morgen.} \\ =0.8654 \text{ acres U. S.} \\ =85.025 \text{ ares Fr.} \end{cases}$
1 cavalleria=105.48 acres.
1 sillio=6.763 sq. miles.
8 almudes=1 cuartilla.
4 cuartillas=1 fanega.
12 fanegas=1 carga=18.9 U. S. bu.
The U. S. bu. is also used.
1 *barrel* of flour=196 lb. net.
" wine=19 or 20 wine gal.
1 fasco of " =5 pints U. S.

855. NAPLES AND SICILY.

(*Principal Commercial Cities*, NAPLES, PALERMO and MESSINA.)

(For Money, see Italy.)

Weight.

1 cantaro=100 rotoli a 33⅓ oncie.
1 rotolo=1.9643 lb. avoir.

The libbra for gold, silver, &c., has 12 oz. 360 trappesie, 7200 acini.
1 libbra=10 oz. 1¼ dwt. troy.

Measure.

1 palmo=12 oncie=60 minuti=120 punti.
1 palmo=10.415 in. Eng.
1 canna=8 palmi=2.89321 yards U. S.

CORN.—1 carro a 36 tomoli a 2 mezzetti a 2 quarte 56.75 U. S. bu.

WINE.—1 carro=2 botti=24 barrili=1440 caraffi, in the country 1584 caraffi.
1 barile=11.524 gal.. 1 caraffo=1.$\frac{1}{4}$ pints.
1 carro=276.59 U. S. gal.

Oil is sold by the salma a 16 staji a 256 quarti or 1536 misurelle, and weighs about 325 lb. avoir.
1 quarto in measure=$\frac{5}{8}$ pint.
1 staja in measure=2.9 gal.

856. THE NETHERLANDS.

(*Principal Commercial City*, AMSTERDAM.)

Money.

1 gulden=100 cents=1s. 8d. Eng.=$0.4084.
5 cents=1 stuiver=1d. Eng.=0.020\frac{3}{16}$.
2½ guilders=$1.00.

COIN.—Gold pieces of 10 and 5 gulden.
Silver pieces of 3 and 1 gulden, 50, 25, 10 and 5 cents.
Old gold coin.—Ducats weighing 52¾ grains Eng., double ducats, ryders=14 gulden.

Butter is sold by the ton, which differs from the common ton=336 pounds Holl. 1 pound=1$\frac{5}{16}$ avoirdupois. 1 ship-pound=300 pounds.

Exchange on London, 12 g. 15 cts., more or less, for £1.
Exchange on Paris, 2 fr. 10 cts., more or less, per gulden.

Weight and Measure.

The Netherlands adopted the French metric System of Weights and Measures in 1820, retaining, however, their old designations for the same. Much confusion having re-

sulted, an Act was passed April 7th, 1869, establishing from January 1st, 1870, a series of new international names of weights and measures, with permissible use of the old denominations during the first ten years. Thus the principal new and (old) names are the

Kilogram	(Pond) . ..	= 2.205	lb. av
Metre	(El or) ...	39.37	inches.
Kilometre	(Myl)	. = 1093	yards.
Are	(Vierkante roede)	119.6	sq. yd.
Hektare	(Bunder) ...	— 2.47	acres.
Stere	(Wisser).... ..	35.31	cu. ft.
Litre	(Kan)	— 1.76	pints.
Hektolitre	(Vat)............	= 22	gal

100 lb. (old) 108.923 lb. avoir.
1 ahm (liquid) 4 ankers = 24 viertels = 128 mingles.
1 ahm 41 gal.
100 mingles = 32 wine gal. or 26¾ imp. gal.

They sell
French wine by hhd. of 180 mingles.
Spanish and Portuguese wine by pipe of 349 mingles.
French brandy by hhd. of 30 viertels.
Beer by bbl. (ahm) of 128 mingles.
Vegetable oils by ahm of 120 mingles.
Rum by anker of 10¼ Eng. gal.

857. NORWAY.

(*Principal Commercial City*, CHRISTIANA.)

Money.

sp. dol.	skil.		£	s.	d.	$	c.	m.
4	75	... . =	1	0	0	= 4	84	0
0	23	. =	0	1	0	= 0	24	2
0	2	. .. =	0	0	1	= 0	02	0$\frac{1}{10}$
0	24 or 1 mark	. =	0	0	9	= 0	19	1$\frac{7}{10}$
1	0 specie dollar	=	0	4	4	= 1	01	8$\frac{1}{10}$
0	60 or 1 rigsb. dol	=	0	2	2	= 0	53	4$\frac{1}{10}$
0	1 nearly	.	0	0	0¹	0	01	0$\frac{1}{10}$

1 specie dollar is equal to 120 skillings.

Measure.

1 fod = 12.353 inches.
1 mil = 4.68 miles.

Weights and other Measures the same as in Denmark.

858. PERSIA.

(*Commercial Cities* ISPAHAN, TAURIS, TEHERAN, *etc.*)

Money.

			$ c. m.
1 keran	= 1000 dinars	=	0 22 7¼
1 toman	10 kerans	=	2 27 8

Weight and Measure.

1 miscal	.... =	74.7 gr. troy.
1 batman	. . =	13 lb. avoir.
1 guz shah, or zer	. =	40 U. S. inches
6000 guz	}	1 farsang 4¼ miles U. S.

NOTE.—The Russian weights and measures are largely used in commerce.

PERU (See SOUTH AMERICA).

859. POLAND.

(*Principal Commercial City*, WARSAW.)

Money.

flor.	grosch.		£	s.	d.	$	c.	m.
42	0	.... =	1	0	0	= 4	84	0
2	3	. . .. . =	0	1	0	= 0	24	2
0	5..	.	0	0	1	= 0	02	0$\frac{1}{10}$
18	15 or 1 gold ducat	=	0	9	3	= 2	23	8$\frac{1}{10}$
8	0 or 1 rix dollar	. =	0	4	0	= 0	96	8
1	0 or 1 silver florin		0	0	5¾	0	11	5$\frac{1}{10}$

1 florin is equal to 30 groschen

Formerly, the gulden a 30 graschen Polish.
1 gulden $0 11¼ cents.

At present the Russian coin is the only legal tender.

Bank notes of the Polish National Bank of 5, 50 and 100 gulden.

Exchange on London, 32 Polish gulden more or less, for £1.

Exchange on Paris, fr. 60.50 @ fr. 60.75 per 100 gulden.

Weight.

1 funt (lb.) = 14 $\frac{1}{2}$ ounces avoir.
1 funt (lb.) 13, ounces troy
1 lb. 16 oz. = 32 loth 128 drams a 3 scruples a 24 grains.
1 centner = 8 stones 400 lb. = 87½ lb. avoir.
Wool is sold by the stone of 32 lb.

Measure.

1 foot (stopa) 11¼ in. Eng.
1 ell (lokiec) 25 in. Eng.
1 mile 8 wersts 5 miles Eng.

CORN.—1 kwart 2 litre 1¾ pint Eng.
1 korzek = 128 kwarts 28 gal. Eng.

860. PORTUGAL.

(*Chief Commercial Cities*, LISBON and OPORTO.)

Money.

The *rei* is the unit of account.
1000 reis 1 milreis = $1 0847
1000 milreis = 1 conto (de reis).
1000 contos 1 conto de conto.

The new gold coins are the corôa (crown) of 10,000 reis, the ½, ⅕, and $\frac{1}{10}$ crown.

The new silver coins are the 5 tostões, or 500 reis, the 2 tostões, 1, and ½ tostão.
5 tostões = $ .1053.

The former denominations were the crusado, a 4 tostões, a 2½ reals, a 2 vinteins, a 20 reis, a 6 ceitis.
1¼ old crusado = 1 new crusado.

Weight.

The *metric* system of weights and measures has been adopted; the *old* system, still used, is as follows:

1 tonelada 54 arrobas; 4 arrobas 1 quintal, a 128 arratels or libra (lb.), a 2 meios arratels, a 2 quartos, a 4 onças, a 8 oitavas, a 3 escrupulos, a 24 grões.
1 libra 1.012 lb. avoir.
1 arroba of the custom-house = 100 arratels.
1 marco, gold or silver ½ arratel.
1 arratel, Apothecaries' = 12 onças.

Measure.

1 pe (foot) = 12 pollegadas, a 12 linhas, a 12 pontos.
1 pe = 1.0827 ft. U. S.

1 vara or ell { = 40 pollegadas. 1.203 yd., U. S.

2 varas = 1 braça.

117 1/3 braças { = 1 estadio. = 1604 U. S. mile.

8 estadios { = 1 milha (small mile). 1.2835 U. S. miles.

3 milhas { = 1 legoa (large mile). 3.8504 U. S. miles.

1 sq. vara = 1.21 sq. meter.

4 sq. varas = 1 sq. braça.

4840 sq. varas { = 1 geira. 1.447 U. S. acre.

DRY MEASURE. 1 moio = 15 fangas, a 4 alqueires, a 2 meios alqueires, a 2 quartas, a 2 oitavas, a 2 salamins.

1 fanga of *Lisbon* = 1.5111 U. S. bu.

100 *Lisbon* measures = 79 1/2 like measures of *Oporto*.

LIQUID MEASURE. 1 almude = 2 potes, a 6 canadas, a 4 quartilhos.

1 almude in Lisbon = 4.422 U. S. gal.

1 tonelada of wine = 2 pipes, a 30 or 32 almudes = 18 almudes = 1 baril.

100 almudes of Lisbon = 66 almudes of Oporto.

100 almudes of Faro = 47 1/2 almudes of Lisbon.

861. PRUSSIA.

(*Principal Commercial City*, BERLIN.)

Money.

thal.	sg.	pf.		£	s.	d.	$	c.	m.
6	20	0	.... =	1	0	0	4	84	0
0	9	9	.... =	0	1	0	0	24	2
0	0	10	=	0	0	1	= 0	02	0
5	20	0	gold Freder'k =	0	16	9	4	05	3
1	0	0	silver thaler =	0	3	1	0	74	6
0	1	0	silbergrosch'n	0	0	1	0	02	5

1 thaler = 30 silver groschen a 12 pfenning.

COINS. Friedrichs d'or = 16s. 6d. English = $3.96. Double do. = 33s. = $7.92. Half do. 8s. 3d. = $1.98. In silver pieces of 2, 1, 1/3, 1/6, 1/12, 1/30 thaler. Do. of 2, 1, 1/2 groschen.

Bank notes of 1, 5, 50, 100, 500 thaler freely taken in the whole of Germany for their nominal value.

Exchange on London, 6 thalers 25 gr., more or less, for £1. Do. Paris, fr. 3.75, more or less, per thaler.

Weight.

1 pound = 467 7/10 grammes French = 1 1/33 pound avoir.

1 cwt. = 110 pounds Pr. = 113 1/4 lb. avoir.

1 last (shipping) is 4,000 pounds.

Wool is sold by the stein of 22 pounds = 22 3/4 pounds avoir.

Gold and silver are sold by the mark = 1/2 pound = 7 oz. 10 dwt. troy English.

The mark is = 288 grains.

For assay of silver the mark is divided into 16 loth a 18 gr.; and of gold into 24 carats a 12 gr. 1 carat of jewels is = 1/16 quent = 1 dwt. 7 grains troy.

Measure.

The foot = 12 1/3 inches English.

1 ruthe = 12 feet = 144 zoll = 1728 linien.

1 ell = 25 1/2 zoll = 26 inches English.

1 faden = 6 feet. 1 mile = 4 1/2 miles English.

FOR CORN. 1 scheffel = 1 1/2 bushel.

1 scheffel = 16 metz; 24 scheffel = 1 wispel.

862. ROME.

The *metric system* is used, but the following are also used to some extent.

Money.

5 quattrini = 1 bajocco.

10 bajocci = 1 paolo.

10 paoli = 1 Roman scudo (dollar or piaster.

5 scudi = 1 sequin (gold) = $4.19.

1 scudo (silver) = $1.00.

1 scudo = ... (new coinage).

1 lira = 100 centesimi (new coinage).

1 gold lira = 1 gold franc of France.

Weight.

1 decina = 10 libbre, a 12 once, a 24 denari, a 24 grani.

1 libbra { .74758 lb. avoir. .90855 lb. troy.

For shipping weight the unit is the rubbio which for grain = 640 libbre, for salt 600 libbre, for peas, beans, etc. = 750 libbre.

Measure.

1 piede (ft.) = .9708 U. S. ft.

5 piede = 1 passo.

1 canna mercantile = 78.45 U. S. inches.

1 braccio de mercante (or commercial ell) = 26.4 inches.

1 braccio per le tele (ell for linen) = 25 inches.

1 post = 2 miglia, a 1,000 passi.

1 miglio = .9245 U. S. mile.

1 rubbio of land = 4.59 acres.

1 rubbio of grain = 8.26 bu.

1 barile of wine = 32 boccali = 15.419 U. S. gal.

1 soma of oil = 80 boccali = 43.38 U. S. gal.

863. RUSSIA.

(*Chief Commercial Cities*, ST. PETERSBURG, MOSCOW, RIGA and ODESSA.)

Money.

1 rouble or rubel = 100 copecks, a 2 diengas, a 2 dienushka.

1 *standard* silver-coin rouble = $.7732.

1 old pistole or half-imperial (gold) = $3.9972.

1 new pistole or half-imperial (since 1886) = $3.8614.

1 imperial ducat (gold) = 20 Polish florins = $2.392.

The silver coins are the rouble and 50, 25, 20, 15, 10 and 5 copecks.

The gold coins are the half-imperial, pol-imperial, or pistole, and the imperial ducat, the latter = 3 silver roubles.

Silver roubles ... credit notes. (In place of former bank assignates of 50, 25, 5, 3 and 1 silver roubles, now constitute the current money, although they are at a discount against silver coin.

This currency is referred to in exchange and commercial quotations, as "silver roubles."

Weight and Measure.

RUSSIAN.		ENGLISH.
1 arsheen	=	28 in.
1 sashen = 3 arsheen	=	7 feet.
100 feet	=	114 1/2 feet.
1 verst	=	5 fur. 12 poles.
150 1/2 versts	=	100 U. S. miles.
1 dessiatine	=	2.85 acres.
1 lb.	=	6315.5 gr.
100 lb.	=	90.282 lb. avoir.
1 pood	=	36 lb. 1 oz. 11 dr.

1 chetviort............ = 5.956 U. S. bu.
100 " .... = 74.4 quar. Eng.
1 wedro .... = 3¼ wine gal.

More particularly—

Weight.

1 pound (funt) = 0.9028 lb. av.
1 pood = 40 " = 36.113 lb. av.
1 bercowitz = 10 poods = 361.13 lb. av.
63 poods = 1 ton. 1 ship last = 2 tons.
1 brutto last = 6 chetwerts.
(The funt = 96 zolotnick. 1 zol. = 96 doll.)

Measure.

The *Rhenish fuss* (foot) = 1.03 Eng. ft., is generally used in estimating export duties on timber.

CORN, &c.—1 chetviert = 4 pajok.
8 tschetwerick = 32 tschetwerks = 64 garner.
1 tschetwerick = 6.8612 gal. U. S.
1 kuh or sark = 10 tschetwericki.
1 wedro = 3.249 gal. U. S.
1 fass = 40 wedroja.
1 kilo of Odessa = 15.708 U. S. bu.

864. SARDINIA.

(*Principal Commercial Cities*, GENOA and TURIN.)

Metric system and also old denominations.

Money.

The lira nuova a 100 centesimi = 1 franc = 9½d. Eng. = $0.193.

COIN.—Gold pieces a 20, 40, 80, and 100 lire nuove or $3.75, $7.50, $15.00, and $18.75. Silver scudi d'argento a 5 lire nuove. Pieces of 2 and 1 lire and 50 and 25 centesimi.

Bank notes of 5, 10 and 20 scudi.

Exchange on London, 25.50 lire, more or less, for £1 sterling.

Exchange on Paris, 21 lire per fr. 20.

Weight.

IN GENOA. 1 libbra = .6994 lb. av.
1 cantaro = 6 rubbia = 100 rotoli = 150 libbre = 1800 once = 104.76 lb. av.

IN TURIN. 1 libbra = 13 oz. avoir.

The Customs use the French kilogramme.

Gold and silver weight is the marco = 8 uncie a 24 denari a 24 grani.

1 marco = .5421 lb. avoir.

Measure.

IN GENOA. 1 palmo = 9.87 in. Eng.

FOR CORN.—1 mina = 3.291 U. S. bu.
1 mondino = 8 mina, a 4 quartino, a 4 quarte.

FOR WINE.—1 barile = 20.870 U. S. gal.
1 mezzarola = 2 barili = 100 pinti.

FOR OIL.—1 barile = 7½ rubbia.

IN TURIN. 1 piede liprando = 1 foot 8¼ in. Eng.
1 piede manelle = 13½ in. Eng.
1 raso ell = 23.598 U. S. in.

FOR CORN.—1 sacco = 5 emine a 8 copi a 24 cucchiari.
1 sacco = 29.388 U. S. gal.

FOR WINE.—1 brenta = 13.013 gal.
1 carro = 10 brenta a 36 pinti a 2 boccali.

865. SAXONY (See GERMANY).

866. SMYRNA AND THE LEVANT

(See TURKEY).

867. SOUTH AMERICAN STATES.

COLOMBIA.

Weights and measures same as France.
Measures of length same as England.

In ordinary commerce the arroba of 25 lb., the quintal of 100 lb., and the carga of 250 lb. are used.

COSTA RICA.

Weights and measures same as Spain.

ECUADOR.

Metrical system of France, but coins principally of France, Great Britain and the United States.

PARAGUAY.

Same as Brazil, and also
1 lino .... = 69¼ sq. yd.
1 legua madra = 12¾ sq. miles.

PERU.

French metric system adopted in 1860, but weights and measures in ordinary use same as Brazil.

URUGUAY. Same as Brazil.

VENEZUELA. Same as Colombia.

868. SPAIN.

(*Chief Commercial Cities*, MADRID, CADIZ, and VALENCIA.)

Money.

1 duro, piaster, or dollar (of 8 reales plata Mexicana) = 2 escudos, a 10 reales, a 10 decimos, a 10 centimos.
1 escudo = $.50 U. S. gold.
1 duro = $.997 or $1.00 gold.
1 doblon de Isabel = 5 duros = $5.0165.

The *Spanish dollar* is current in nearly all countries.

The *escudo* is the highest unit of account.

Formerly they used eight different sorts of money:—1. Castilian, 2. Mexican, 3. Catalonian, 4. Majorcan, 5. Valencian, 6. Arragon, 7. Navarre, and 8. The Canarian money.

The Castilian is the chief, and is 1 real de plate antigua = 1⅞ real de vellon = 16 cuartos = 34 maravedis de plata antigua = 64 marav. de vellon = 640 Castil dineros.

10⅔ reales de plata antigua = 1 piaster.

1 real de plata = 5d. Eng. = $0.100.

COIN.—Gold, 1 quadruple pistole = 8 escudos = $15.60 to $16; doblon or onza de Oro = $16 subdivided into ½, ¼, ⅛, and 1/16.

Weights and Measures.

Since Jan. 1, 1859, the metric system of weights and measures has been used in Spain, with only a slight change in the names of the denominations, but the old weights and measures are still largely used, and are as follows:

Weight.

1 Castilian marca=8$\frac{1}{8}$ oz. avoir. or 7 oz. 8$\frac{2}{15}$ dwt. troy. Eng.
1 marca=8 onzas=64 ochaves=4608 granos.
1 quintal macho=6 arrobas=150 libras.
300 marcas=152$\frac{1}{4}$ lb. avoir.
1 quintal=4 arrobas=100 libras=101.438 lb. avoir.

Jewels and pearls are weighed by the Castilian ounce a 140 quilates, a 4 granos.
1 oz.=431$\frac{1}{4}$ grains troy.

Measure.

1 pie=11$\frac{1}{8}$ in. Eng.
1 estado=2 varas=6 pies=5 ft. 6$\frac{3}{4}$ in. Eng.
1 league=4$\frac{1}{4}$ miles Eng.

FOR CORN.—1 cahiz=12 fanegas a 12 celemines or almudos a 4 quartillos=1.575 bu.

FOR LIQUIDS.—1 cantaro or arroba major 8 azumbres=32 quartillos=4.2618 gal.
1 arroba menor for oil=3.319 gal.
1 moyo=16 cantaros. 1 pipa=27 cantaros.
1 bota=30 cantaros.

869. SWEDEN.

(*Principal Commercial City*, STOCKHOLM.)

Money (new).

100 öre $\begin{cases} =1 \text{ riks-daler riks-mynt.} \\ =\$.2756. \end{cases}$

Weight (new).

1 centner=100 pund, a 100 ort, a 100 korn=93.697 lb. avoir.

Measure (new).

Length.—1 mil=360 ref, a 10 stanger, a 10 fot, a 10 tum, a 10 linie.
1 fot = .9741 U. S. ft.
1 mil=6.642 U. S. miles.

Area.—100 sq. fot=1 sq. stang.
100 sq. stanger=1 sq. ref. (sq. chain)=.2178 U. S. acre.

Volume.—1 cu. fot=1000 cu. tum (or 10 kannor), a 1000 cu. linie $\begin{cases} = .74268 \text{ bu. U. S.} \\ =6.9139 \text{ gal. U. S.} \end{cases}$

The principal *old* denominations are as follows:

Money.

rd.	skil.		£	s.	d.	$	c.	m.
12	0	in banco	=1	0	0	=4	84	0
0	23	....	=0	1	0	=0	24	2
0	2$\frac{1}{4}$	....	=0	0	1	=0	02	0$\frac{3}{16}$
5	25	or 1 gold ducat..	=0	9	2	=2	21	8$\frac{6}{14}$
2	25	or 1 specie silver	=0	4	4	=1	04	8$\frac{8}{13}$
1	0	banco....	=0	1	8	=0	40	3$\frac{4}{13}$
1	12$\frac{1}{2}$	or hf. specie silv.	=0	2	2	=0	52	4$\frac{4}{13}$

1 rd. banco is equal to 48 skillings.
1 silver specie is equal to 96 skillings.
1 riksdaler specie a 48 skillings=$1.05.
Banco=1 riksdaler specie.

Weight.

1 skal pound....	=	15 oz. avoir.
1 schip pound...	=	400 skal lb.
1 cwt............	=	120 lb.
1 scale of spelter	=	165 lb.
1 stone wool....	=	32 lb.
1 mark (for gold)	=	6 oz. 16 dwt. troy.

Measure.

1 foot=1 foot Eng.
1 faam=3 alnar=6 feet=17 verthum.
1 alnar=2 feet Eng.

CORN.—1 tonn=4 bu. Eng.
1 tonn=8 quarts=32 kappar=56 cans=448 quarrtiera.

WINE.—2 pipes=1 fuder=4 oxhoofts=12 eimer=720 stop.

870. SWITZERLAND.

(*Principal Commercial Cities*, GENEVA, BERN, BASLE and ZURICH.)

Money. Old System.

fr.	batz.	rap.	£	s.	d.	$	c.	m.
17	7	5	=1	0	0	=4	84	0
0	8	7	=0	1	0	=0	24	2
0	0	7	=0	0	1	=0	02	0$\frac{1}{16}$
4	0	0 piece of ...	=0	4	8	=1	12	9$\frac{1}{12}$
1	0	0 or 10 batz ..	=0	1	1$\frac{1}{2}$	=0	27	2$\frac{1}{12}$
0	1	0	=0	0	1$\frac{1}{2}$	=0	02	6$\frac{1}{12}$

1 franc is equal to 10 batzen.

New System—as in France.

1 franc=10 batzen a 10 rappen or 1 livre a 20 sols a 12 deniers.
1 franc=1 livre (old)=$0.27.

COIN.—Gold pistoles a 32 francs=$8.65.
" $\frac{1}{2}$ pistoles a 16 francs=4.32\frac{1}{2}$.
" Ducats=$2.22.
Silver pieces of 40, 20, 10, and 5 batzen.

N.B.—Each Canton has besides these its own currency.

Exchange of Basle on London, 17 francs 5 rappes, more or less, for £1.
Exchange on Paris, fr. 1.50 per fr. 1, or 50 per cent. premium, more or less, in favor of Basle.

Weight.

1 cwt.=100 lb.=50 kilogrammes=110$\frac{1}{4}$ lb. avoir.
1 lb.=$\frac{1}{2}$ kilogramme=1 lb. 1$\frac{5}{8}$ oz. avoir.

Measure.

The basis is the Helvetian foot.
1 foot=$\frac{3}{10}$ French meter=11$\frac{4}{5}$ in. Eng.
2 feet=1 ell; 4 feet=1 stab or staff.
16,000 feet=1 hour (mile)=2.9826 miles.

FOR CORN.—1 malter=10 viertel=100 immi.
1 malter=4$\frac{1}{4}$ bu. U. S.
1 immi=3$\frac{1}{4}$ pints.

FOR WINE.—1 ohm=100 maas (or measures).
1 ohm=39.626 U. S. gal.
1 maas=3$\frac{1}{4}$ pints U. S.

871. TURKEY.

(*Principal Commercial Cities*, CONSTANTINOPLE and SMYRNA.)

Money.

pias.	par.		£	s.	d.	$	c.	m.
96	32	....	=1	0	0	=4	84	0
4	33$\frac{3}{5}$	....	=0	1	0	=0	24	2
0	16	....	=0	0	1	=0	02	0$\frac{1}{16}$
200	0	gold new dble.seq.	=1	11	0	=7	50	2
100	0	" 1 sequin....	=0	18	0	=3	35	6
1	0	....	=0	0	2$\frac{1}{4}$	=0	04	5$\frac{3}{8}$
23	0	or 1 Span. dollar.	=0	4	2	=1	00	8$\frac{1}{16}$

Piaster a 40 paras a 3 aspers.
Also piaster (grush) a 100 aspers.
1 medjidie, or lira Turca=100 piasters.
1 purse silver is 500 piasters.
1 purse gold is 30,000 piasters.
1 juk is 100,000 coined aspers.

The government or bank notes bear 8 per cent. interest, but are at a discount.

Exchange on London, 140 piasters, more or less, for £1.

Exchange on Paris, from 400 to 410 piasters for 100 francs.

Weight.

1 pound, chequi = 11¼ oz. avoir.
1 oka = 2 lb. 14 oz. avoir.
1 oka = 4 chequi = 400 drachmas.
1 taffee = 600 drachmas.
1 batman = 6 okas.
1 cantaro = 44 okas.

Gold and silver weight like Alexandria.
1 chequi opium = 250 drachmas.
1 chequi goat-hair = 800 drachmas.

PIECE GOODS. 1 mazzee = 50 pieces.

Measure.

The large pik halebi, archim = 27 in. Eng.
The small pik endaesa = 27 in. Eng.

FOR CORN. The killow = 1.0245 bu. U. S.
1 fortin = 4 killows = 30 gallons English.
1 killow of rice should weigh 10 okas.
1 kilô of Smyrna = 1¼ killow (legal).

FOR LIQUORS.—1 almud = 1.375 gal. U. S.
1 almud of oil should weigh 22¾ pounds avoir.
1 oka = .3386 gal. U. S.

872. TUSCANY.

(*Principal Commercial Cities*, FLORENCE and LEGHORN.)

Money.

Since becoming a part of the kingdom of Italy, Tuscany has adopted the Italian lira or lira nuova = 1 franc = $ 1.8, but the former system is still used to some extent.

1 lira Toscana = 100 centesimi = 7½d. Eng. = $0.15.
1 lira Toscana = 20 soldi = 240 denari.
25 lire Toscane = 21 francs.

COIN.—Gold:	Ruspont a 3 zecchini.	= $6.96
	Zecchini gigliati	2.32
	Half "	1.16
Silver:	Francesconi a Leopoldini	= 1.08
	Half "	0.54
	Tallari	1.04
	Testoni	= 0.33
	Lira a 12 crazie, about	0.15

Weights and Measures.

LEGHORN		ENGLISH.
1 braccio	=	22.98 in.
155 bracci	=	100 yards
1 sacco	=	2.0739 Winchester bushels.
4 sacci	=	1 imperial quarter nearly.
100 lb.	=	74.864 lb. avoir.
1 centinajo	=	100 lb.
1 rottolo	=	3 lb.

More particularly—

Weight.

1 quintal = 100 lb. = 1200 uncie a 24 denari.
1 lb. = 12 oz. avoir.
1 quintal = 74¾ lb. avoir.

FOR GOLD.—1 lb. = 10 ½ oz. troy and is divided into 24 carati a 8 ottavi.
FOR SILVER, into 12 uncie a 24 denari.
Jewels are weighed by the carat a 4 grani.

Measure.

1 braccio = 1.9149 U. S. ft.
1 miglio = 1.0275 U. S. mile.
The braccio used by builders = 21¾ in. Eng.

FOR CORN.—1 sacco = 3 staja = 6 mines;
100 sacchi = 207.375 U. S. bu.

FOR WINE.—1 barile = 20 fiaschi = 80 mezzette = 160 quartucci = 10⅛ gal. Eng.
1 barile of oil = 8.8539 U. S. gal.

873. SHIPPING MEASUREMENT.

FOR GRAIN.—42 cubic feet = 1 ton shipping measurement.

1 bushel = 60 lb.
1 bushel = 2218¼ cubic inches.
8 bushels = 1 quarter.
1 quarter = 17745 cub. in. or 10.27 ft.

Therefore 1 ton will take 4⅒ quarters.

1 bushel being equal to 60 lb.,
1 quarter will be equal to 480 lb.,
1 ton = 1968 lb. or 17 cwt. 2 qr. 8 lb. fully.

1 ship of 200 tons measurement can therefore carry 820 quarters, but it generally can carry much more.

874. LONDON EXCHANGE.

The greater part of all the foreign trade of the United States is settled through England and France.

London, the great clearing-house of the world, receives from, or gives to, according to variation in the exchanges—

Amsterdam	12 flor. and 8 stivers	for	£1
Bremen	600, rix-thalers	"	£100
Berlin	6 thl. & 25 sil. gros.	"	£1
Christiania	4 specie-daler 30 skil	"	"
Copenhagen	9 rigsb. daler, 10 "	"	"
Constantinople	110 piasters	"	"
Frankfort	121 silver-gulden florins	"	£10
Genoa	25 lira, 35 centesimi	"	£1
Hamburg	13 marks 12 schil	"	"
Milan	25 lire, 40 centisimi	"	"
Leghorn	25 " 50 "	"	"
Paris	25 fr., 21 centimes	"	"
Rome	46 Paoli	"	"
Stockholm	12 dal. in banco, 1 sk.	"	"
Vienna	13 flor., 70 kreutzers	"	"

Calcutta	23d. stg.	for	1 Com. rupee.
Gibraltar	48½d.	"	1 Span. dollar.
Lisbon	53½d.	"	1 milreis.
Madrid	50½d.	"	1 hard dollar.
Naples	39½d.	"	1 ducat.
New York	49½d.	"	1 U. S. dollar.
Palermo	118½d.	"	1 onza.
Pekin	76½d.	"	1 thousand cash.
Rio Janeiro	20d.	"	1 milreis.
St. Petersburg	38½d.	"	1 silver rouble.
Venice	47d.	"	6 Austrian lire.

875. COMPARISON OF COMMERCIAL WEIGHTS.

Country.	*Weight.*	*U. S. lb. av.*
France	1 kilogram.	= 2.20
Germany	1 pfund	= 1.10
Austria	1 pfund	= 1.23
Russia	1 funt	= .90
Sweden	1 pund	= .93
Denmark	1 pund	= 1.10
Turkey	1 oka	= 2.83
Prussia	1 zolpf'd	= 1.10
Netherlands	1 pond	= 2.20
East Indies	1 seer	= 2.06
China	1 catty	= 1.33
Japan	1 kin	= .03
Mexico	1 libra	= 1.02
Brazil	1 arratel	= 1.03

876. COMPARISON OF GRAIN MEASURES.

Country.	*Measure.*	*U. S. bush.*
England.......	1 bushel.......	= .973
France.........	1 hectoliter....	=2.84
Prussia........	1 scheffel	=1.56
Austria........	1 metze........	=1.75
Russia	1 chetverik	= .74
Germany......	1 scheffel......	=1.5 to 3
Persia	1 artaba.......	=1.85
Turkey........	1 kilô	=1.03
Brazil.........	1 Fan..........	=1.5
Mexico	1 alque	=1.13

877. COMPARISON OF LIQUID MEASURES.

Country.	*Measure.*	*U. S. gal.*
England.........	1 gal....... ...	=1.2
France	1 decalitre.....	=2.64
Prussia	1 quart	= .30
Austria..........	1 maas.	= .37
Sweden	1 kanna........	= .69
Denmark	1 kande........	= .51
Switzerland. ...	1 pot	= .40
Turkey..........	1 Almud.......	=1.38
Mexico.....	1 Farco........	= .68
Brazil........ ...	1 medida......	= .74
Cuba	1 arroba.......	=4.01
South Spain	1 arroba.......	=4.25

878. MISCELLANEOUS TABLE OF FOREIGN WEIGHTS AND MEASURES.

Ahm of Hanover...........	=41.48 gal. U. S.
" of Leipsic	=40 "
Amir, or Emir, of Stuttgard	=78 "
Arroba of Brazil...	=32.38 lb.
Arroba of Buenos Ayres...	=25.36 "
Balsam Copaiva, 8 lb......	=1 gal.
Butt of wine	=130 gal.
Canado of Balsam Copaiva.	=30 lb.
Chaldron coal, Brit. Prov..	=36 bu.
" " Cumberland	=53 "
Cheki of opium (fr. Smyrna)	=1¾ lb.
Coal, a rail. wag. load, Pictou	=62 cwt.
Flax, head of, about.......	=6¾ lb.
Foot, 100 feet St. Domingo.	=106.60 ft.
Honey, 1 gallon............	=12 lb.
Linseed, one bushel......	=47 "
Mudd, or Maud, of Rotter'm)	=148 lb.
Moyo of salt (Spain).... ..	=70 bu.
Modius of salt (fr. Ivica, Sp.)	=40 "
" "(Oporto & St. Ubes)	=28 "
Mass (of Antw'p) ¼th of ohm	=10 gal.
Ohm "	=40 "
Picul (of hemp) of Manilla.	=139.5 lb.
" " of Siam ...	=133⅓ "
Pounds of Austria.... 100 lb.	=123.50 lb.
Pounds of Antwerp ...... 100 lb.	=103.35
" Bavaria........ "	=123.50
" Belgium....... "	=103.35
" Brussels....... "	=103.35
" Bremen .... "	=110.12
" Berlin......... "	=103.11
" Denmark...... "	=110.60
" Ger. Zoll. States "	=110.25
" Hamburg ..... "	=110.04
" Malaga..... .. "	=101.44
" Netherlands... "	=108.93
" Portugal ...... "	=101.19
" Prussia .... "	=110. 5
" Rotterdam .... "	=108.93
" Russia .... "	= 90.00
" Spain .... "	=101.44
" St. Domingo... "	=107.93
" Trieste........ "	=123.60
" Vienna........ "	=123.50
Palm of Italy, of marble.......	=6 inches.
Quintal of France.............	=220$\frac{46}{100}$ lb.
Skippond of Gottenburg.......	=300 lb.
Salma of oil	=82.16 gal.
Vara, Spanish.................	=8 feet.
Vara of Baracoa...............	=20 "

879. RATES OF FOREIGN MONEY OR CURRENCY, FIXED BY LAW.

NOTE.—This table, prepared by Mr. E. B. Elliott, is taken by permission from "U. S. Duties on Imports," 1871, by Lewis Heyl, Esq., of the Treasury Department, W. H. & O. H. Morrison, publishers. The same may also be found in U. S. Tariff, 1869, by E. D. Ogden, Esq., Entry Clerk, N. Y. Custom House, Bogert & Nixon, publishers. Both these manuals are authorized standards in the various Departments of Government.

This Table is, to a considerable extent, a repetition of what may be found in the foregoing Tables. It is thus given for greater convenience, and as showing the specific values established by law in the United States, while the foregoing show the values as recognized in London, in Sterling Currency, and that reduced to Federal Currency. The slight discrepancies between the two are thus accounted for, the following being the popular values or rates at which these foreign coins are receivable in the U. S.

CURRENCY.	FRACTIONAL PARTS OF THE CURRENCY.			VALUE. $ cts.
Dollar, rix, of Bremen.................. =	72 grotes	of	5 swares	78¾
Dollar, thaler, of Bremen.............. =	72 "	of	5 "	71
Dollar, thaler, of Prussia and the Northern States of Germany, and Saxony... =	30 groschen	of	12 pfennings....	69
Ducat of Naples .... =	100 grani.....		....	80
Florin of the city of Augsburgh, of Austria, of Bohemia, and of Trieste .... =	60 kreutzers	of	4 pfennings....	48½

CURRENCY.		FRACTIONAL PARTS OF THE CURRENCY.			VALUE. $ cts.
Florin of Frankfort, of Nuremburg, and of the Southern States of Germany	=	60 kreutzers	of	4 pfennings	40
Florin of the Netherlands	=	100 centimes			40
Franc of France and Belgium		100 "	of		18$\frac{6}{10}$
Guilder of Netherlands and other places same as the Florin					
Lira of the Lombardo and Venetian Kingdom	=	100 centesimi	of	100 millessemi	16
Lira of Sardinia	=	4 reali	of	20 soldi	18$\frac{6}{10}$
Lira of Tuscany	=	20 soldi	of	12 denari	16
Livre of Genoa		20 "	of	12 "	18$\frac{6}{10}$
Livre of Leghorn	=	20 "	of	12 "	16
Livre Tournois of France					18½
Marc Banco of Hamburg	=	16 shillings	of	12 pfennings	35
Milrea of Azores		1000 reas			83⅓
Milrea of Madeira	=	1000 "			1 00
Milrea of Portugal	=	1000 "			1 12
Ounce of Sicily	=	30 tari	of	20 grani	2 40
Pagoda of India	=	36 fanams	of	48 jittas	1 94
Pagoda, Star, of Madras	=	36 "	of	48 "	1 84
Pound Sterling of British Provinces of Nova Scotia, New Brunswick, Newfoundland, and Canada	=	20 shillings	of	12 pence	4 00
Pound Sterling of Great Britain		20 "	of	12 "	4 84
Pound Sterling of Jamaica					4 84
Real Plate of Spain	=	34 Maravedis			10
Real Vellon of Spain		34 "			5
Rouble, silver, of Russia	=	100 kopecks			75
Rupee, British India		16 annas	of	12 pice	44½
Rupee Company	=	16 "	of	12 "	44½
Specie Dollar of Denmark		6 marks	of	16 skillings	1 05
Specie Dollar of Norway	=	6 "	of	16 "	1 06
Specie Dollar of Sweden	=	48 skillings	of	12 'ore	1 06
Tael of Shanghai	=	10 mace	of	100 candareens	1 33
Tael of China		10 "	of	100 "	1 48

880. *Currencies by Usage, in the absence of a Consular Certificate of the actual value of Exchange.*

Banco Rix dollar, Denmark					53
" " Norway and Sweden					39¼
Crown of Tuscany	=	20 soldi	of	12 denari	1 05
Current Mark					28
Dollar Rix of Batavia	=	48 stivers			75
Dollar or Pezzo of Leghorn	=	20 soldi	of	12 denari	90$\frac{9}{10}$
Dollar Rix Mynth of Sweden					99¼
Dollar (Rix-tal Thaler) of Gottenburg					27¼
Dollar Roman					1 05
Francisconi					1 06
Florin of Basle					41
Florin of Prussia					22¾
Guilder, Brabant					38½
Guilder of Curacoa	=	20 stivers	of	12 pfennings	40
Guilder St. Gall	=	60 kreutzers	of	4 "	40$\frac{5}{10}$
Kobang of Japan	=	4 itzebou	of	1000 seni	1 38
Livre of Catalonia	=	20 sueldos	of	12 dineros	53⅓
Livre of Genoa					21
Livre of Neufchatel	=	20 sols	of	12 deniers	26¼
Livre of Swiss	=	100 centimes			27
Livre Tournois of France					18½
Piastre, Turkish	=	100 aspers			5
Rouble paper, of Russia	=	100 kopecks		Varies from 4 roubles 65 copecks to 4 roubles 84 copecks to the dollar.	
Scudi of Malta	=	12 tari	of	20 grani	40
Scudi, Roman					99 a 99½
Tical of Siam					61

TABLES OF THE CHIEF COMMERCIAL

WEIGHTS AND MEASURES OF DIFFERENT COUNTRIES,

REDUCED TO THE LEGAL STANDARDS OF THE UNITED STATES, BOTH COMMON AND METRIC.

[These Tables were prepared for the U. S. Treasury Department by Mr. E. B. Elliott, and are copied, by permission, from "U. S. Duties on Imports, 1871, by Lewis Heyl, of the Treasury Department."]

881. ABYSSINIA.

Rottel (rotolo, or liter) of 12 wakihs, each of 10 derimes = 4800 (troy) grains = 311.03 grammes.
Mocha of 12 derimes = 480 gr. = 31.10 grammes
Pik (Turkish) = 27 inches = 0.686 meter.
Ardeb (in Gondar) of 10 madegas = 0.125 bu. = 4.40 liters.
Ardeb (in Massuah) of 24 madegas = about 0.300 bu. = 10.57 liters.
Kuba....... { =62 inches.... / =0.276 gallon. } = 1.016 liter.

882. ARGENTINE CONFEDERAT'N.

Quintal of 100 libra = 101.27 lb. av. = 45.9367 kilogrammes.
Arroba of 25 libra = 25.32 lb. av. = 11.4842 kilogrammes.
Libra = 1.0127 lb. av. = 459.367 grammes.
Marco (for gold and silver) = 3544.4 grains = 229 684 grammes.
Vara of 3 pies = 0.9478 yard = 0.8667 meter.
Pie = 0.9478 foot = 0.2889 meter.
Lastre (last) of 2 tonneladas (tons) or 15 fanegas = 58.404 bushels = 205.80 liters.
Fanega = 3.8936 bushels = 137.20 liters.
Frasco = 0.6274 gallon = 2.375 liters.
Baril of 32 frascos = 20.0787 gal. = 76 liters.

883. AUSTRIA.

Pfund = 8642.209 gr. = 560.012 grammes.
Zoll-pfund (customs-pound) = 7716.174 gr. = 500 grammes.
Münzpfund (coin-pound) = 7716.174 grains = 500 grammes.
Centner = 123.4615 lb. av. = 56 0012 kilogr.
Saum = 275 lb. av. = 154.003 kilogrammes.
Metze = 1.7454 bu. = 61.5045 liters.
Eimer of 40 maass = 14.95 gal = 56.605 liters.
Maass = 0.37 gal. = 1.415 liter.
Fuss of 12 zoll = 1.03713 ft = 0.31611 meter.
Elle (imperial) = 0.85217 yd. = 0.77921 meter.

884. AZORES or WESTERN ISLANDS.

(See Portugal.)

Alqueire of 2 meios. = 0.334 bu. = 11.95 liters.
Fanga of 4 alqueires = 1.336 bu. = 47 80 liters.

885. BADEN.

Pfund.. { =1.0123 lb. av.. / =1.3396 lb. troy } = 500 grammes.
Fuss .... = 0.98428 ft .... = 0 3 meter.
Elle..... = 0.65618 yd... = 6 decimeters.
Zuber .. = 42.5732 bu..... = 1500 liters.
Malter... = 4.25752 bu .... = 150 liters.
Fuder... = 39.6262 gal..... = 1500 liters.
Stütze... = 3.9626 gal...... = 15 liters.

886. BAVARIA.

Centner..... = 123.459 lb. av. = 56 kilogr.
Pfund....... = 1.23459 lb. av. = 500 grammes.
Zoll-pfund and münz-pfund..... } = 1.0123 lb. av.. = 500 grammes.
Mark........ = 0.6268 lb. troy = 233.950 gram.
Fuss......... = 0.95757 ft..... = 0.291859 met.
Elle.......... = 0.9110 yd.. .. = 0.833015 met.
Schäffel...... = 6.3103 bu... . = 222.337 liters.
Maass....... = 0.2824 gal... = 1.06908 liter.
Schenkeimer. = 16.944 gal.... = 64.1416 liters.

887. BELGIUM.

French system.

888. BRAZIL (Like Portugal).

Metric system obligatory from 1st January, 1873.

Tonelada (ton for shipping) = 2240 lb. avoir. = 1016.1 kilogrammes.
Medida = 0.73306 gallon = 2.7748 liters.
Arratel = 1.0192 lb. avoir. = 459 grammes.

889. BREMEN.

Pfund...... = 1.99 lb. av... = 498.5 grammes.
Fuss....... = 0.9493 ft..... = 0.28935 meter.
Elle....... = 0.6330 yd.... = 0.5787 meter.
Scheffel.. .. = 2 103 bu..... = 74.10987 liters.
Stübchen... = 0.85108 gal... = 33.21318 liters.

890. BRUNSWICK.

Pfund.... = 1.02958 lb. av. = 467.11 grammes.
Fuss..... = 0.93635 ft.... = 0.28536 meter.
Elle...... = 0.6242 yd.... = 0.570725 meter.
Wispel. = 35.3544 bu.... = 1245.7904 liters.
Stübchen. = 0.85108 gal.... = 32.21318 liters.

891. CANADA (Like England).

Ell........ = 1.25 yd........ = 1.14296 meter.
Minot..... = 1.10749 bu..... = 39.025 liters.

892. CHILI.

Libra....... = 1.01412 lb. av. = 460 grammes.
Fanega..... = 2.838 bu...... = 100 liters.
Quartillo ... = 0.2906 gal .... = 1.1 liter.
Vara........ = 2.7493 ft...... = 0.836 meter.

893. CHINA.

Pecul.. { =133.333 lb. av . / =162 0.63 lb. tr. } = 60.4787 kilogr.
Catty.... = 1.3333 lb. av..... = 604.7896 gram.
Chih (custom-house) } = 14.1 in ... . = 0.35813 meter.
Sei...... = 3.4716 bu......... = 122.43 liters.

894. COCHIN-CHINA (Like China).

Tael . = 590.75 gr. troy = 38.28 grammes.
Covid . = 0.4106 yd ..... = 0.351 meter.

895. CEYLON or SELAN.

(English Measure.)

Candy.... = 545 lb. av. . = 247.2 kilogr.
Amomam = 5.7757 bu . . = 203.52 liters.

896. CURACAO (Like Netherlands).

Vara (yard) .. = 33.375 inches = 0.8477 meter.

897. CYPRUS.

Pik . = 0.7347 yd . . = 0.6718 meter.
Medinno = 2.1312 bu .... = 75.097 liters.
Cass = 1.25 gal = 4.73 liters.
Kantar = 524.30 lb. av = 237.77 kilogr.
Oka { [illegible] gr. = 2.7957 lb. av } = 1.2831 kilogr.
Rotolo . 1¼ oka.

898. DENMARK.

Pund = 1.1025 lb. av = 500 grammes.
Mark = 0.630134 lb. t'y = 235.2941 gram.
Fod = 1.03 ft. = 0.31385 meter.
Alen = 0.68648 yd. .. = 0.62771 meter.
Tönde(ton) = 3.9478 bu = 139.1213 liters.
Pott. .. . = 0.2552 gal. = 0.96612 liter.

899. ECUADOR (Like Spain).

900. EGYPT.

Derhem (drachm) = 47.6512 gr. troy = 3.0884 grammes.
Oka = 2.7235 lb. av. = 1.23536 kilogramme.
Rotolo = 0.9804173 lb. av. = 444.73 grammes.
Government rotolo = 1.2236 lb. av. = 554.91 grammes.
Pik (Istambuli) = 0.[illegible] = 0.677 meter.
Ardeb (Alexandria) = 7.6907 bu. = 271 liters.

901. ENGLAND.

Pound av. = 1.215278 lb. troy = 453.5926 gram.
Pound troy = 0.822857 lb. av. = 373.2418 gram.
Imperial quarter = 8.25212 U. S. bu. = 290.7813 liters.
Imperial bu. = 1.03152 Winch. bu. = 36.34766 liters.
Imperial gallon = 1.20032 gal. = 4.543458 liters.
Ale and beer gallon = 1.2204 gal. = 4.6209 lit.
Yard = 3 feet = 0.914383 meters.

902. FRANCE.

Mètre =
39.37040 inches, or
9.84260 hands, or
3.280867 feet, or
1.093622 yard, or
0.0049709 hf-ch'n
= 1 meter, or 10 decimeters, or 100 centimet., or 1000 millimeters.

Kilomètre =
1093.633 yards, or
99.4202 h'f-ch'ns, or
49.7101 chains, or
0.621376 mile
= 1/10 myriamet., or 1 kilomet., or 10 hectomet., or 100 dekamet., or 1000 meters.

Litre =
0.26417 gallon .
1.05667 quart, liq. meas.
= 1 cub. decimeter.

Hectoliter =
2.8378 bushels .. .. ..
90.8 quarts, dry measure
= 100 liters.

Each of the French measures of volume has its half and its double measure.

Gramme = 15.4323488 gr. = 10 decigr., or 100 centigr., or 1000 milligr.

Kilogramme =
15432.3488 grains, or
2.204621 lb. av. (of 7000 gr.), or
2.679227 lb. t'y (of 5760 gr.), or
0.0787365 av. qr. (of 28 lb.), or
0.0196841 cwt. (of 112 lb.), or
0.02204621 centals (of 100 lb.)
= 10 hectogram, 100 dekagram, 1000 gra.

Metric quintal =
7.873647 av. qr. (of 28 lb.), or
1.968412 cwt. (of 112 lb.) or
2.204621 centals of 100 lb.
= 10 myriagr. 100 kilogr.

Millier, or metric tonne =
19.68412 cwt. (of 112 lb.), or
0.984206 long T. (of 2240 lb.), or
1.1023162 short T. (of 2000 lb.)
22.046212 centals (of 100 lb.)
= 10 quintals, or 1000 kilogram.

903. GERMANY.

Zollverein (Customs' Union) an important commercial Union originating in 1828; embracing in 1853 all the German States, except Austria, Lichtenstein, Holstein, the two Duchies of Mecklenburg, and the three free cities of Hamburg, Lübeck and Bremen; dissolved with the close of the year 1865, and held together only by temporary agreements until Nov. 1, 1867, when a new Commercial Treaty was concluded between the North German Confederation and the South German States.

The unit of weight is the zollpfund (customs' pound) of 500 grammes.
Zollpfund of 30 zoll-loth = 7716.1744 U. S. gr. = ½ kilogramme.
Zollcentner of 100 zollpfund = 110.23106 lb. avoir. = 50 kilogrammes.
Zollstein of 20 zollpfund = 22.04621 lb. avoir. = 100 kilogrammes.
Münzpfund of 10000 ass = 7716.1744 U. S. gr. = 500 grammes.

904. NORTH GERMAN CONFEDERATION.

(Established since the war with Austria, in 1866.)

(*New System.*)

By a decree of the 17th of August 1868, the metrical (French) system of weights and and measures has been adopted, commencing with the 1st of January 1870, and will be compulsory from the 1st of January, 1872.

The *base* of the new system is the *meter* or *stab*, the same as the French mètre.

The *unit of length* is also the meter or stab.
0.01 meter.. = 1 zentimeter or neu-zoll.
0.001 meter .. .. = 1 millimeter or strich.

The *unit of surface* is the quadrat-meter (square meter), or quadrat-stab.
0.01 of a quadrat-meter . = 1 ar.
0.001 of a quadrat-meter ... = 1 hektar.

The *unit of volume* is the 0.001 of a kubikmeter or kubik-stab, and is called a liter or kanne.
½ liter=1 schoppen.
100 liter=0.1 kubik-met.=1 hectolit. or fass.
50 liter=1 scheffel.

The *unit of weight* is the kilogramme (equal to 2 pfund).
10 grammes=1 dekagramme or neu-loth.
0.1 gramme.=1 dezigramme.
0.01 gramme.=1 zentigramme.
0.001 gramme.=1 milligramme.
½ kilogramme { = 1 pfund } . { =1.1023106 lb. avoir.
{ =50 neu-loth }
50 kilogramme, or 100 pfund...=1 zentner.
1000 kilogramme, or 2000 pfund=1 tonne.

The *unit of money-weight* continues to be the münzpfund (of 500 grammes) divided into 10,000 as.

The *unit of weight for purposes of assay*, or for trying gold and silver, is the 0.001 part of the münzpfund (=½ gramme or 500 milligrammes), which unit is again divided into 1000 parts.

905. GREECE.

French system since 1836.

Former. *Metric.*
Mina (kilogr.)=2.20462 lb. av.=1.00 kilogr.
Royal mina..=3.30693 lb av.=1.5 kilogr.
Talanton.....=330 697 lb. av.=150.0 kilogr.
Piki ....=1.09363 yd....=1.0 meter.
Litra.........=1.0567 qt......=1.0 liter.
Kailon.......=2.83782 bu....=1.0 hectolit.

906. GUIANA.

BRITISH GUIANA. See LONDON.

FRENCH GUIANA.

Livre.... { =1 079176 lb av } =489.5058 gram.
{ =1.31110 lb.troy }
Pied de roi=1.06575 ft......=0.3248394 met.
Aune ...=1 .2972 yd.....=1.188146 met.
Muid .....=70.85525 gal.....=268.2195 liters.
Boisseau.=0.35915 bu......=13.0083 liters.

DUTCH GUIANA (See NETHERLANDS).

907. HAMBURG.

Pfund. { =1.0683 lb. av.. } =481.60945 gram.
{ =1.2984 lb. troy }
Mark....=0.628551 lb. troy=233.85489 gram.
Fuss ....=0.94021 ft.......=0.28657 meter.
Elle ....=0 62681 yd .....=0.57314 meter.
Brabantine elle..=0.75615 yd..=0.69141 met.
Fass.............=1.5597 bu ..=54 9615 lit.
Ohm (=4 ankers)=38.2782 gal.=144.8906 lit.

908. HANOVER (Like PRUSSIA).

909. HAVANA (ISLAND OF CUBA).

Castilian weight (See SPAIN).

Varra (Cubana)....=33.375 in..=0.8477 met.
Fanega............=3.12367 bu.=110.66 lit.
Arroba (former Castilian cantara) } =4.10 gal...=15.44 lit.

910. HUNGARY (Like AUSTRIA).

Oka ....=3.0817 lb. av.=1.400 kilogr.
Arsin.........=0.68919 yd...=0.63240 met.
Stab..........=5.18565 ft....=1.5805 meter.
Metzen ......=1.77354 bu...=62.4934 liters.
Urna or eimer=14.305889 gal.=54.1597 liters.
Fass..........=52.545 gal...=198.8848 lit.

911. INDIA (EAST).

(English Measures.)

BENGAL.

Tola...........=180 gr. troy..=11 66375 gra.
Man or maund=82.2855 lb.av.=37.324 kilogr.
Fact'y maund=74.667 lb. av.=33.868 kilogr.
Guz ....=1.00 yd......=0.91433 met.
Kahoon { =2986.666 lb. av...=1354.72 kilog.
(gram) { =42.44 bu.

BOMBAY.

Candy.......=560.00 lb. av...=254.00 kilogr.
Covid (haut).=1.50 ft.........=0.4572 meter.
Candy (gr'n).=358.4 lb. av....=162.567 kilog.
Rice candy (n'r 25 bu.) } =215.9375 lb. av.=97.947 kilogr.
Maund......=28.00 lb. av....=12.70 kilogr.

MADRAS.

Candy.......=500 lb av....=226.8 kilogr.
Maund.....=25.00 lb av..=11.339 kilogr.
Garce.......=139 512 bu....=4.916 kilolit.
Parah.......=1.7489 bu....=61 47 liters.
Covid (cubit)=0.50 yd.......=0.45719 meter.

912. IONIAN ISLANDS.

(Like ENGLAND.)

Libra sotille Ionia..... } =1 lb. troy..=373.2466 gram.
Libra grossa..=1 lb. av....=453.5922 gram.
Jarda Ionia...=1 yd... ...=0.91438 meter.
Gallone. ... { =0.12894 bu. } =4.54358 lit.
{ =1.20032 gal. }
Chilo..........=1.03152 bu ...=36.34766 lit.
Barilla (16 imp. gal.). } =19.24807 gal..=72.72537 lit.

913. ITALY.

(Metrical and decimal system of France.)—Formerly:

Libbra.......=0.81463 lb. av..=369 508 gram.
Piede (Liprando).. } =1 68561 ft......=0.513737 met.
Sacco.......=3.27179 bu.....=115.0278 lit.
Brenta......=12.99317 gal ...=49.285 liters.

914. JAMAICA (Like ENGLAND).

915. JAPAN.

Monme.......=27 0037 gr troy=1.75 gram.
Rjoo ....=116.1288 gr. t'y=7 525 gram.
Sak ....=11 9301 in......=0 303 met.
Sjoo, or masa.=0.459128 gal...=1.738 liter.

916. LUBECK

Pfund.......=1.07249 lb. av.=486.474 gram.
Mark ....=1 2522 lb troy=467.3642 gram.
Fuss........=0.91865 ft ....=0 2876 meter.
Elle ....=0.6291 yd ....=0 5752 meter.
Scheffel (rye, barley) ... } =0.98349 bu.=34.694 liters.
Scheffel (oats, fruit) } =1.12138 bu.=39.514 liters.
Ohm..........=38 4374 gal.=145.501 liters.
Quartico......=0.24023 gal.=0.90938 liter.

917. MADEIRA (See PORTUGAL).

Arratel (libra)	1.01083 lb. av / 1.2286 lb. troy	= 436.547 gram.
Alqueire	0.399 bu	14.097 liters.
Almude	= 4.68007 gal.	.. = 17.718 liters.

918. MALTA.

Foot .	= 11.1666 in.	0.28363 meter.
Canna	2.2944 yd	= 2.0980 meters.
Salma.	= 8.1974 bu.	= 288.51 liters.
Barile. ..	11.9208 gal.	= 42.57 liters.
Libbra .	= 1.7 lb. av / 0.85076 troy	= 317.5 grammes.

919. MARTINIQUE.

(French metric system.)

Livre	1.079178 lb. av / = 1.311429 lb. t'y	= 489.5058 gram
Aune	1.3048 yd	.. 1.191 meters.
Barique	50 gal ..	.. = 189.25 liters.

920. MAURITIUS (ISLE DE FRANCE.)

French metric system.—Still used:

Quintal	108 lb. av	48.989 kilogr.
Ton (shipping)	= 2100 lb. av	970.576 kilogr.
Aune .. .	1.3018 yd	1.191 meter.
Velt	2.00 gal	= 7.57 liters.
Cask . . .	= 60.05 gal. .	= 227.14 liters.

921. MEXICO.

Weight like HAVANA.

Tercio of tobac.	160 lb. av	72.576 kilogr
Baril .	20 gal.	= 75.71 liters.
Vara	= 0.9139 yd. .	= 0.83800 met.
Fanega . ..	1.54728 bu.	55.501 liters
Frasco.	= 2.5 qt. . .	= 2.3659.

922. MOLDAVIA.

(DANUBIAN PRINCIPALITIES.)

Oka (Jassay)	2.8505 lb. av	1.29298 gram
Palma	0.9174 ft	. = 0.27953 met.
Khalebi (wool goods)	0.7344 yd.	. = 0.6718 meter.
Kot (silk and linen goods)	0.6906 yd	. 0.6314 meter
Kilo	. = 19.25 bu	4.851 hectolit.

923. MOROCCO.

Artal .	= 1.12 lb. av	= 508 grammes.
Codo (dhra'a)	0.62446 yd	0.571 meter.
Mudd .	about 0.407 bu	= 14.00 liters.

924. NETHERLANDS.

Former Weights and Measures.

Troyish pond	1.3186 lb. t'y	492.1677 gram
Old pond	1.0891 lb. av	494.1004 gram.
Voet	= 0.9289 ft	= 0.283133 met.
Old Amsterdam el	0.7522 yd.	0.6878 meter

(The Netherlands adopted the French metric system in 1816.)

Last 30 mud	. = 85.13 bu	30 hectolit.
Ell	= 1.099 yd .	1 meter
Vah 100 kannen	26.41 gal	100 liters
Pond { 10 onsen, or 100 looden, or 1000 wigtjes }	=	{ 2.2046 lb. av, or 2.67923 lb t'y, or 1 kilogramme. }

Apothecary's pond = 373 wigtjes (grammes).

In the Netherlands' Colonies (Batavia, etc.), the former weights and measures are still in use, also—

Koyang (rice)	3662.042 lb. t'y = 1661.066 kilogr.
Old kan	= 0.3963 gal = 1.50 liter

1 pikol = 100 catties = 10 gantang (coffee) = 135.6312 lb. av = 61.5210 kilogramme.

On the Molucca Islands, the New Netherlands measures have been introduced since 1839.

925. NEW GRANADA, OR UNITED STATES OF COLOMBIA.

(Recently adopted French measures and weights.)

926. PAPAL STATES.

The French metric system, although adopted in 1848, to commence with 1st of January, 1850, has not been enforced.

Hitherto,

Piedi	= 0.9766 U. S. ft.	= 0.2976 meter.
Canna mercantile	2.179 yd .	= 1.9926 meter.
Rubbio .	0.8396 bu	294.46 liters.
Barile (wine)	15.112 gal	= 58.3216 lit.
Barile (oil).	= 15.185 gal....	= 57.4800 lit.
Libbra	= 0.74733 lb. av / 0.90815 lb. troy	= 339.073 kilog.

927. PERSIA.*

Guz shah (gere or arkin) 3.333 feet = 1.0160 meter.

1 artaba = 8 collothun = 25 capichas = 50 chenicas = 200 sextarios = 1.8514 bushel = 65.298 liters.

The only weight common to all provinces is the miskal = 174.7045 troy grains = 4.8408 grammes.

In Tauris, Meshid, Herat:
40 seers = 40 miskal = 5.880047 lb. avoir. = 3.038 kilogrammes.

In Ispahan:
1 mahnd shah = 1280 miskal = 13.6601 lb. avoir = 6.196 kilogrammes.

In Reshl:
2 royal mahnd = 2500 miskal = 27.8202 lb. avoir = 12.892 kilogrammes.

In Shiraz, Bushir, and Gamri:
Mahnd (or maund) = 2500 miskal.

In Teheran:
1 rik = 1000 miskal.

928. PERU.

Spanish (Castilian) weights.

Vara	= 33.367 inches	= 0.8475 meter
Fanega = 140 Castilian lb.		= 64.41302 kilogr.

929. PORTUGAL.

Arratel or libra = 1.01192 lb. av. = 459 gram.

Palemo de craveiro .	= 8.661 in....	= 0.22 meter.
Vara	1.2030 yd	1.1 meter.
Pe	= 1.0827 ft	= 0.33 meter.
Alqueire .	0.3928 bu	13.841 lit.
Almude	...... = 4.4224 gal.	... = 16.74 liters.

930. PRUSSIA.

(For metric weights and measures, see GERMANY.)

Former

Pfund .... =1.03114 lb. av. =467.72 grammes.
Fuss...... =1.0297 ft...... =0.31385 meter.
Elle....... =0.7294 yd..... =0.66694 meter.
Scheffel ... =1.5597 bu.... =54.9615 liters.
Eimer..... =18.149 gal.... =68.702 liters.

931. RUSSIA.

Funt (pound). { =1.0972 lb. troy / =0.9028 lb. av.. } =409.5116 grammes.
Pud of 40 funti=36.113 lb. av.=16.3805 kilogrammes.
Berkovetz of 10 pudi=361.13 lb. av.=163.805 kilogrammes.
Stopa=14 inches=0.35559 meter.
Arsheen=28 inches=0.71119 meter.
Sasheen of 3 arsheens=7 feet=2.13357 meter.
Chetviert=5.9560 bushels=2.099 hectoliters.
Vedro=3.2490 gal.=12.2989 liters.

932. SANDWICH ISLANDS.

Weights, etc., as in United States.

Barrel of whale oil=31.5 gal.=119.2427 liters.

933. SAXONY (See NORTH GERMANY).

Old pfund (Leipzig). } =1.03093 lb.av.=467.6246 gram.
Fuss......... =0.9291 ft..... =0.28319 meter.
Elle.......... =1.8582 ft..... =0 56638 met.
Sheffel .... =2.9510 bu.... =103.988 liters.
Eimer........ =17.79519 gal.. =97.3026 liters.

934. SPAIN.

French metrical system. In the Spanish Colonies the old *weights and measures* are still in use, principally Castilian.

Castilian :

Libra... =7100.21 gr. troy=460.093 grammes.
Pié ..... =0.91407 ft...... =0.278635 meter.
Vara.... =0.914117 yd ... =0.835905 meter.
Fanega. =1.57527 bu..... =55.501 liters.
Cantara (arroba mayor, for wine)=4.2618 gal. =16.133 liters.
Cuartillo=0.13322 gal. =0.5043 liter.

935. SOUTH AMERICA.

UNITED STATES OF COLOMBIA

(Or NEW GRANADA).

French kilogrammes in custom-house practice; other measures as in Venezuela.

VENEZUELA.

Castilian weights and measures (see SPAIN).

BOLIVIA.

Kilogrammes at custom-house.

936. SWEDEN AND NORWAY.

Skalpund { =0.936991 lb. av. / =1.137049 lb. t'y } =425.010 gra.
Fot........ =0.974102 ft....... =0.296901 met.
Kanna..... =0.69156 gal.. ... =2 617188 lit.
Am........ =41.4934 gal..... =157.0313 lit.

937. SWITZERLAND.

French system since 1851.

Pfund......... =1.10236 lb. av.=500 gram.
Centner=100 pfund ..... } =110.236 lb. av.=50 kilogr.
Fuss... ..... =11.81 in....... =0.3 meter.
Quarter....... =0.4257 bu..... =15 liters.
Pot .... =1.58504 qt..... =1.5 liter.
Muid.......... =39 626 gal.... =150 liters.

938. TRIPOLI.

Kantar=40 oke (lb.)=107.606 lb. av.=48.833 kilogrammes.
Oka=40 uckie=2.6916 lb. av.=1.2208 kilogr.
Pik=26.42 inches=0 671 meter.
Pik or dra arabic=19.03 inches=0.483 meter.
Hueba=16 orbah=3.0152 bu.=107.3 liters.

939. TUNIS.

Uckia=488.90 gr. troy=31.680 grammes.
Rotoli (pound) (rug)=1.1175 lb. av.=506.88 grammes.
Rotoli sucky (meat, etc.)=1.2532 lb. av.= 568 445 grammes.
Rotoli ghredari (vegetables)=1.4098 lb. av.= 639.453 grammes.
Drad hendaseh (woolen goods)=0.6728 met.
Turkish pik=0.6370 meter.
Arabian pik=0.4883 meter.
Cafiz=14.0753 bushels=4 96 hectoliters.
Metter=2.6417 gallons=10 liters.

940. TURKEY.

Cantar=44 oke=100 rotoli=124.7036 lb. av.= 56.565 kilogrammes.
Oka=2.83418 lb. av.=1285.56 grammes.
Chequi (for gold, etc.)=0.86108 lb. troy= 321.39 grammes.
Pik=27.9 inches=0.6858 meter.
Endaseh=25.7 inches=0.6528 meter.
Kiló=1.00075 bushel=35.266 liters.

941. TABLE OF SPECIFIC GRAVITIES.

Specific Gravity is the ratio of the weight of a body to that of an equal volume of some other substance, adopted as a standard of reference. For solids and liquids the standard is pure water at 60° F., the barometer being at 30 in. Aeriform bodies are referred to the air. A cubic foot of pure water weighs 1000 ounces, and the following is a table of the relative weights of a number of the most familiar solids and liquids. By removing the decimal point three places toward the right, that is, multiplying by 1000, the result will indicate the number of ounces in a cubic foot of the substance named.

Acid, acetic	1.009	Iron ore	4.900
" arsenic	3.391	Ivory	1.917
" nitric	1.217	Juniper	.556
" sulphuric	1.841 to 2.125	Lard	.947
Air	.001227	Lead, cast	11.350
Alabaster	1.874	" white	7.235
Alcohol, of commerce	.835	Lead, ore	7.250
" pure	.794	Lemon tree	.703
Alder wood	.800	Lignum vitae	1.333
Ale	1.035	Lime	.804
Alum	1.714	" stone	2.386
Aluminum	2.560	Mahogany	1.063
Amber	1.064	Malachite	3.500
Ambergris	.780	Manganese	8.000
Apple tree	.793	Maple	.750
Amethyst	2.750	Marble	2.716
Ammonia	.875	Medlar	.944
Ash	.800	Mercury, common	13.568
Blood, human	1.054	" pure	14.000
Bone of ox	1.656	Mica	2.750
Brass (about)	8.000	Milk	1.032
Brick	2.000	Naphtha	.700
Butter	.942	Nickel	8.279
Cedar	.457 to .561	Nitre	1.900
Cherry	.715	Oak	1.170
Cider	1.018	Oil, castor	.970
Coal, bituminous (about)	1.250	" linseed	.940
" anthracite	1.500	" whale	.923
Copper	8.788	Opal	2.114
Coral	2.540	Opium	1.337
Cork	.240	Pearl	2.510
Diamond	3.530	Pewter	7.471
Dolomite	2.540	Phosphorus	1.770
Earth (mean of the globe)	5.210	Pine	.540 to .698
Elm	.671	Platinum (native)	17.000
Emerald	2.678	" wire	21.042
Ether	.632	Poplar	.383
Fat of beef	.923	Porcelain	2.385
Feldspar	2.600	Potassium	.865
Filbert	.600	Plum	.785
Fir	.550	Quartz	2.600
Glass, bottle	2.733	Rosin	1.100
" green	2.642	Salt	2.130
" flint	2.760	Sand	1.500 to 1.800
" plate	2.760	Silver, cast	10.474
Gold, native	15.600 to 19.500	" coin	10.534
" pure, cast	19.258	Slate	2.110
" hammered	19.362	Steel	7.816
" coin	17.647	Stone	2.000 to 2.700
" 22 carats fine	17.486	Sugar	1.606
" 20 "	15.709	Sulphur, fused	1.990
Granite	2.652	Tallow	.941
Graphite	1.987	Tar	1.015
Gunpowder	.900	Tin	7.291
Gum Arabic	1.452	Turpentine, spirits of	.870
Gypsum	2.288	Vinegar	1.013
Hazel	.600	Walnut	.671
Hematite ore	4.507	Water, distilled	1.000
Honey	1.456	" sea	1.028
Ice	.930	" Dead Sea	1.240
Iodine	4.948	Wax	.897
Iridium	23.000	Willow	.585
Iron	7.645	Wine	.992
" cast	7.207	Zinc, cast	7.190

942. CAPACITY OF BOXES.

The following table will often be found convenient, taking *inside* dimensions:

A box 24 inches by 16 inches, and 28 inches deep, will contain *a barrel*.
A box 26 inches by 15⅕ inches, and 8 inches deep, will contain *a bushel*.
A box 13⅘ inches square and 11¼ inches deep, will contain *a bushel*.
A box 12 inches by 11⅕ inches, and 9 inches deep, will contain *a half bushel*.
A box 10 inches square and 10⅘ inches deep, will contain *a half bushel*.
A box 8⅖ inches by 8 inches square, and 8 inches deep, will contain *a peck*.
A box 8 inches square and 4⅕ inches deep, will contain *a gallon*.
A box 7 inches square and 4⅘ inches deep, will contain *a half gallon*.
A box 4 inches square and 4⅕ inches deep, will contain *a quart*.
A box 4 inches square and 4⅛ inches deep, will contain *a pint*.

943. CONTENTS OF FIELDS AND LOTS.

The following table will assist farmers in making an accurate estimate of the amount of land in different fields under cultivation.

10 rods	×	16 rods	=	1 A.		
8 "	×	20 "	=	1 "		
5 "	×	32 "	=	1 "		
4 "	×	40 "	=	1 "		
5 yards	×	968 yards	=	1 "		
10 "	×	484 "	=	1 "		
20 "	×	242 "	=	1 "		
40 "	×	121 "	=	1 "		
80 "	×	60½ "	=	1 "		
70 "	×	69⅐ "	=	1 "		
220 feet	×	198 feet	=	1 "		
440 "	×	99 "	=	1 "		
110 "	×	396 "	=	1 "		
60 "	×	726 "	=	1 "		
120 "	×	363 "	=	1 "		
240 "	×	181½ "	=	1 "		
200 "	×	108$\frac{9}{10}$ "	=	½ "		
100 "	×	145$\frac{2}{10}$ "	=	⅓ "		
160 "	×	108$\frac{9}{10}$ "	=	⅖ "		
25 "	×	100 "	=	.0574 "		
25 "	×	110 "	=	.0631 "		
25 "	×	120 "	=	.0688 "		
25 "	×	125 "	=	.0717 "		
25 "	×	150 "	=	109 "		
2178 square feet			=	.05 "		
4356 " "			=	.10 "		
6534 " "			=	.15 "		
8712 " "			=	.20 "		
10890 " "			=	.25 "		
13068 " "			=	.30 "		
15246 " "			=	.35 "		
17424 " "			=	.40 "		
19602 " "			=	.45 "		
21780 " "			=	.50 "		
32670 " "			=	.75 "		
34848 " "			=	.80 "		

944. QUOTATIONS OF STOCKS AND BONDS.

(See Arts. 623–642.)

DESCRIPTION.	Bid.	Ask'd	EXPLANATION.
	%	%	
6s, fund, 1881, cp.	110	110¼	U. S. Bonds of the funded debt, payable in 1881, bearing 6% interest, with coupon certificates, for which $110 was bid per $100 of the Bonds.
6s, 1881, reg	114½	114¾	U. S. Regist'd Bonds, payable in 1881, bearing 6%.
Call Bds. '72, c. 2d s.	113		Second Series of U. S. Bonds payable in 1872, called in by the Treasurer for payment, with coupons.
5–20's 1868	118	118¼	U. S. Bonds issued in 1868, redeemable after July 1, 1873, payable July 1, 1888, bearing 6% int. in gold.
Northwest. pref.	93¾	94¼	Preferred Stock of the Northwestern R. R.
Hud. R. 7s, 2d M. S. F., 1885.	106		Bonds of Sinking Fund of Hudson River R. R., secured by a "Second Mortgage," payable in 1885, bearing 7%.
Harlem Con. M. & S'kg F'd 6's.	94½		Bonds of the Consolidated Mortgages & Sinking Fund of Harlem R. R. bearing 6%.
Kansas Pacific 1st M. (gld.) 6, J. & D	86	88	1st Mortgage Bonds of Kansas Pacific R. R., bearing 6% interest payable in June & Dec. in gold.
Columbus & Xenia Stock ex d	107	108	Stock of Columbus & Xenia R. R. *without* the dividend due.
Dayton & Michigan 8 p. c. st'k guar.	104	105	Stock of Dayton & Mich. R. R., on which a dividend of 8 per cent. is guaranteed.
	SALES.		
25000 U. S. 5–20s, 1864, c.	112⅜		U. S. 5–20 Coupon Bonds of 1864, par value $25000, sold at 12⅜% premium.
100 Reading, b. 10 flat.	117		100 shares Reading R. R. Stock, buyer to take it within 10 days, *without* accumulated dividends.

945. ANSWERS TO PROBLEMS.

NOTE.—Answers to mental problems, and to some of the simpler written problems, also some verbal answers and detailed statements of accounts required, are omitted.

Art. 90.

3. 1,179.
5. 11,252.
7. $1,417.56.

Art. 92.

1. 390,105,273.
2. 97,844,449,983.
3. 182,674,396.
4. 184,980,489,264.
5. 1,907,935.
6. 3,837,917.
7. 597,538.
8. 22,587,757.
9. 144.70.
10. 3,986.95.
11. $3,308,352,706.
12. 3,697,633.
13. $3,109,788,234.
14. 295,567,000.
15. 4,129,686.
16. 48,860 miles. $2,212,413,000.
17. 8,885,040 acres. 3,800,900 bales.
18. 2,923,213.
19. 1,520,449,949.
20. 38,858,371 Pop. of U. S.

Art. 93.

1. $94.27.
2. $225 68.
3. $11,165.27.
4. $178.99.

Art. 106.

3. 13,795,486,068.
4. 1,163,329,140,-480.
5. 2,191,197,946,-000.
6. 2,752,220,031,-000.
7. 25,809,405,584.
8. 3,327,600,000.
9. 20,143,200 feet.
10. $99,060.
12. 6,744,834,096.
13. 9,740,801,856.
14. 20,879,277,804.
15. 31,153,538,394.
16. 885,461,654,322.
17. 2,659,326,810.
18. 5,077,007,400.
19. 365,280.
20. 1,043,680.
21. 164,698,240.
22. 38,903,616,000,-000.
23. 264,586,448.
24. 41,781,923.
25. 656,234,909.
26. 535,752.
27. 1,264,617.
28. 3,843.
29. 360,000.
30. 360,000.

Art. 109.

6. 255,276.
7. $87,605.
8. 1,254,400 lb.
9. 1,826,100,000.
10. 20,160,000.

Art. 110.

3. $3,801,624.
4. 5,127,231,200.
5. 5,464,435,200.
6. 26,190,693.
7. 490,046,060.
8. 99,006,900.
9. 7,300,049,022.
10. 761,861,556.

Art. 111.

4. 3,421,978.
5. 87,932,560.
6. 595,696.
7. 89,712,000.
8. 712,776.
9. 25,349 280.
10. 214,444,800.

Art. 112.

2. 374,512.
3. 7,354,676.
4. 5,544.
5. 1,726,272.
6. 629,937.
7. 7,643,814,761,-088.
8. 888,030.
9. 25,839,000.
10. 7,592,400,000.

Art. 121.

2. 59,031.
3. 9,199.
4. 4,913,618.
5. 998,999,999.
6. 28,803,306.
7. 25,100,063.
8. 3,750.
9. 1,786.
10. 1,100.
11. 1,998,000.
12. 85.
13. 95,400.
14. 2.
15. 2,820.
16. 43,047.
17. 252,874,044.
18. 8,879,991.
19. 1,965,414.
20. 26,946.

Art. 136.

1. 108,692[illegible].
2. 609,111 [illegible].
3. 12,962[illegible].
4. 209,511[illegible].

5. 6,092,649 $\frac{930940}{1592740}$.
6. 80.
7. 1,002.
8. 1,087 $\frac{3807}{8200}$.
9. 52,037,988 $\frac{5}{15}$.
10. 433,635,007 $\frac{2}{14}$.
11. $49,626 $\frac{4}{25}$.
12. $2.36.
13. 75 cts.
14. 404 $\frac{9}{16}$.
15. 178 $\frac{4928}{5280}$.
16. 8,601 $\frac{32}{173}$.
17. 2,769 $\frac{160}{650}$.
18. 1 mile.
19. $\frac{1}{3}$.
20. 0.
21. 359,809 $\frac{18}{71}$.
22. $827,088,176 $\frac{1}{2}$.
23. 225 $\frac{19111779}{37443321}$.
24. 271 months.
25. 30 and 188 $\frac{2088}{3540}$.
26. 4 $\frac{313178}{1565175}$.
27. $46,220 $\frac{1469}{1767}$.
28. $510 $\frac{3440}{20150}$.
29. 7 days.
30. 70 tons.

Art. 139.

4. 16,386. Rem. 43.
5. 1,112,311.
Rem. 105.

Art. 152.

2. 2,273 $\frac{4}{37}$.
3. 434 lb.
4. $13.28 $\frac{4}{7}$.
5. $\frac{4}{3}$ or 1 $\frac{1}{3}$.

Art. 157.

1. 216,000.
2. 1,243,670,000.
3. 2,833,333 $\frac{1}{3}$.
4. 718. Rem. 33.
5. $35,112 $\frac{6}{12}$.
6. 20,833 $\frac{1}{3}$.
7. 4752.
8. 437 $\frac{80}{100}$.
9. 336.
10. 148.
11. 25,485 $\frac{4}{7}$.
12. 281 $\frac{188}{300}$.
13. 5,169,150.
14. 31,201 $\frac{5}{76}$.
15. 7,833,333 $\frac{4}{15}$.

Art. 158.

1. $229.142.
2. 72 sheep.
3. 893,010 bu.
4. 2,106 miles.
5. 1,841 miles.
6. Cal., $46,650,000. Oregon, $5,700,-000.
7. 3,464.
8. 990 miles.
9. 50 men.
10. 6th power.
11. 80 $\frac{7}{36}$ hr.
12. 7602 bu.
13. 23 lots.
14. 21,504 pop.
15. 14.

Art. 161.

1. $2,918.07.
2. 32,400,000 bu.
3. $7,660.
4. 56,207 lb.
5. $101.
6. $18,581.77.
7. $268,271 $\frac{1083}{1627}$ c.p.
$184,221 $\frac{1433}{1627}$ cir.
8. 68,350,000 bu.
9. Chi. 445,283,482.
Jap. 34,959,366.
10. 790,058 depositors.
$218,378,685 deposits.
11. $625,311.96.
12. $295 $\frac{84117}{712183}$.
13. $11,159,000 gain.
14. $12.998,942 $\frac{1}{19}$.
15. $19,683.13 $\frac{168}{365}$.
16. $20,090.
17. $20,768.
18. 6 $\frac{12}{35}$ days.
19. $31,453.
20. 211 $\frac{13}{58}$ mos.

Art. 175.

1. 1,080.
2. 5,040.
3. 2,520.
4. 25,200.
5. 12,600.
6. 1,596.
7. 720.
8. $480.
9. 120 inches.
10. 15,939.

Art. 180.

1. 16.
2. 4.
3. 14.
4. 23.
5. 56.
6. 42.
7. 148.
8. 2.
9. 23.
10. 512.

Art. 182.

1. 123,332,544.
2. 89,702,525,906.
3. 72,055,480.
4. 1,449,894,400.
5. 146,808,853,191.
6. 5,148,000.

Art. 185.

1. 261 $\frac{1}{3}$.
2. 3,456.
3. 404,910.
4. 8,640.
5. 197 $\frac{1}{3}$.

Art. 187.

1. 31 $\frac{2}{3}$ miles.
2. $50.
3. 111 $\frac{1}{2}$.
4. 83°

Art. 189.

1. 4.80.
2. 20 feet.
3. 12 feet.
4. $30\frac{5}{13}$ days.
5. 1,000.

Art. 207.

1. $\frac{135}{7}$.
2. $\frac{134}{3}$.
3. $\frac{375}{105}$.
4. $\frac{227}{4}$.
5. $\frac{24729}{30}$.
6. $\frac{3605}{720}$.
7. $\frac{377}{16}$.
8. $\frac{2008}{15}$.
9. $\frac{2812}{5}$.
10. $\frac{136115}{17}$.
11. $\frac{96}{4}$.
12. $\frac{709}{20}$.
13. $\frac{3744}{12}$.
14. $\frac{475}{25}$.
15. $\frac{9072}{9}$.

Art. 209.

1. $3\frac{4}{5}$, $6\frac{1}{4}$, 3.
2. $10\frac{5}{12}$, $7\frac{2}{17}$, $3\frac{30}{95}$.
3. 25, $5\frac{12}{17}$, $17\frac{8}{9}$.
4. $\frac{37}{10}$, $\frac{14}{100}$, $\frac{3}{100}$.
5. $88\frac{28}{84}$, $64\frac{91}{125}$, $9\frac{1095}{1250}$.

Art. 210.

1. $\frac{12}{27}$.
2. $\frac{20}{60}$.
3. $\frac{25}{80}$.
4. $\frac{16}{24}$, $\frac{15}{24}$, $\frac{28}{24}$, $\frac{22}{24}$.
5. $\frac{51}{85}$, $\frac{170}{85}$, $\frac{10}{85}$, $\frac{17}{85}$.
6. $\frac{27}{63}$.
7. $\frac{168}{27}$.
8. $\frac{20}{24}$, $\frac{21}{24}$, $\frac{16}{24}$.
9. $\frac{50}{70}$, $\frac{42}{70}$, $\frac{15}{70}$.
10. $\frac{28}{48}$, $\frac{30}{48}$, $\frac{36}{48}$, $\frac{27}{48}$.

Art. 211.

1. $\frac{3}{4}$.
2. $\frac{1}{8}$.
3. $\frac{2}{13}$.
4. $\frac{3}{7}$.
5. $\frac{3}{7}$.
6. $\frac{5}{24}$.
7. $\frac{2}{3}$.
8. $\frac{15}{43}$.
9. $\frac{3}{7}$.
10. $\frac{4}{11}$.
11. $\frac{2}{14}$.
12. $\frac{4}{15}$.
13. $\frac{2}{5}$.
14. $\frac{7}{9}$.
15. $\frac{7}{10}$.

Art. 213.

1. $\frac{1}{8}$.
2. $\frac{5}{32}$.
3. $\frac{5}{2}$.
4. $\frac{8}{3}$.
5. $\frac{7}{4}$.
6. 1.
7. $\frac{12}{5}$.
8. 5.
9. $\frac{63}{80}$.
10. $\frac{1184}{15}$.
11. $\frac{4}{3}$.
12. $\frac{1}{3}$.
13. $\frac{63}{80}$.
14. $\frac{7}{15}$.
15. $\frac{24}{7}$.

Art. 215.

1. $\frac{18}{24}$, $\frac{15}{24}$, $\frac{20}{24}$, $\frac{16}{24}$, $\frac{14}{24}$.
2. $\frac{42}{56}$, $\frac{35}{56}$, $\frac{48}{56}$, $\frac{20}{56}$, $\frac{18}{56}$.
3. $\frac{40}{48}$, $\frac{21}{48}$.
4. $\frac{30}{120}$, $\frac{72}{120}$, $\frac{40}{120}$, $\frac{75}{120}$.
5. $\frac{28}{42}$, $\frac{18}{42}$, $\frac{35}{42}$, $\frac{21}{42}$.
6. $\frac{2}{6}$, $\frac{15}{6}$, $\frac{5}{6}$.
7. $\frac{8}{24}$, $\frac{9}{24}$, $\frac{56}{24}$.
8. $\frac{120}{360}$, $\frac{90}{360}$, $\frac{72}{360}$, $\frac{60}{360}$, $\frac{45}{360}$, $\frac{30}{360}$.
9. $\frac{80}{120}$, $\frac{96}{120}$, $\frac{100}{120}$, $\frac{105}{120}$.
10. $\frac{378}{504}$, $\frac{441}{504}$, $\frac{432}{504}$, $\frac{280}{504}$.

Art. 219.

1. $3\frac{11}{24}$.
2. $3\frac{47}{105}$.
3. $2\frac{19}{60}$.
4. $1\frac{3}{20}$.
5. $1\frac{145}{168}$.
6. $14\frac{13}{24}$.
7. 2.
8. $8536\frac{134}{180}$.
9. $170\frac{5}{6}$.
10. $568\frac{1}{72}$.
11. $1\frac{9}{20}$.
12. $2\frac{21}{40}$.
13. $12\frac{9}{10}$.
14. $203\frac{3}{4}$.

Art. 221.

1. $1\frac{8}{12}$.
2. $1\frac{3}{4}$.
3. $2\frac{2}{7}$.
4. $6\frac{2}{9}$.
5. $\frac{7}{10}$.
6. 6.
7. $7\frac{47}{55}$.
8. $3\frac{57}{121}$.
9. $93\frac{1}{3}$.
10. $265\frac{13}{14}$.

Art. 223.

3. $316\frac{4}{5}$.
4. $1809\frac{1}{2}$.
5. 7250.
6. 16.
7. 30,105.
8. 2,000.
9. $1,283\frac{1}{3}$.
10. $946\frac{52}{73}$.

Art. 224.

2. $\frac{1}{8}$.
3. $\frac{2}{9}$.
4. $\frac{35}{72}$.
5. 49.
6. $13\frac{1}{8}$.
7. $65\frac{1}{3}$.
8. 1,073.
9. $\frac{5}{18}$.
10. $\frac{3}{85}$.
11. 36.
12. $19\frac{5}{8}$.
13. 3.152.
14. $208\frac{1}{3}$.
15. $2128\frac{23}{35}$.

16. $5,274,339\frac{7}{9}$.
17. $3,811,058\frac{2}{11}$.
18. $200,922\frac{16}{27}$.
19. $776,914\frac{1}{8}$.
20. $13,946,832\frac{9}{17}$.

Art. 225.

3. $\frac{1}{12}$.
4. $\frac{8}{77}$.
5. $\frac{39}{220}$.
6. 0.
7. $\frac{17}{80}$.
8. $763\frac{11}{12}$.
9. $24\frac{22}{35}$.
10. $446\frac{7}{8}$.
11. $\frac{47}{72}$.
12. $1\frac{1}{4}$.
13. $843\frac{1}{3}$.
14. $8\frac{5}{8}$.
15. $59\frac{23}{240}$.
16. $\frac{1}{10}$.
17. $1\frac{1}{8}$.
18. $16\frac{2}{7}$.
19. $9\frac{14}{15}$.
20. $275\frac{19}{24}$.

Art. 227.

1. $\frac{5}{51}$, $\frac{7}{80}$, $\frac{4}{70}$.
2. 12, 25, $33\frac{1}{3}$.
3. 2, $1\frac{5}{9}$, $\frac{18}{25}$.
4. $\frac{7}{54}$.
5. $64\frac{4}{5}$.
6. 2.

Art. 228.

4. $25\frac{13}{18}$, $3,556\frac{14}{15}$.
5. $200\frac{1}{15}$, $144\frac{1}{18}$.
6. 48, 336, 18.
7. 46.296, 8, $14\frac{2}{5}$.
8. 1,136.
9. $1\frac{1}{5}$, $1\frac{1}{2}$, $\frac{9}{10}$.
10. $\frac{18}{25}$, $1\frac{71}{304}$.
11. $\frac{2}{11}$.
12. $\frac{54}{88}$.
13. $1\frac{1}{35}$.
14. $2\frac{1}{4}$.
15. $5\frac{1}{23}$.
16. $\frac{4}{15}$.
17. $1\frac{509}{750}$.
18. $24\frac{17}{30}$.
19. $1\frac{17}{51}$.
20. $82\frac{113}{160}$.

Art. 231.

31. $2\frac{5}{8}$.
32. $\frac{5}{24}$.
33. $2\frac{1}{3}$.
34. $\frac{4}{21}$.
35. $5\frac{5}{6}$ sum.
$1\frac{1}{6}$ difference.
$8\frac{1}{6}$ product.
$1\frac{1}{2}$ quotient.
36. $2.58\frac{1}{3}$.
37. $2\frac{4}{7}$ tubs.
38. $\frac{7}{12}$ bu.
39. $1,614\frac{13}{40}$ lb.
40. $4,765\frac{439}{1260}$ lb.
41. $986\frac{7}{72}$ acres.
42. $355.61\frac{77}{120}$.
43. $226\frac{23}{330}$ acres.
44. $3.17\frac{3}{16}$.
45. $19,200.
46. $\frac{1}{8}$ of the orange.
47. $12.
48. $8.02\frac{1}{12}$.
49. $3.28\frac{1}{8}$.
50. $13.78\frac{1}{8}$.
51. $\frac{5}{32}$ left.
Worth $15,000.
52. $72,000.
53. $\frac{1}{24}$ acre more than B.
$\frac{1}{6}$ times more.
Both own $\frac{19}{24}$ A.
54. $5,561\frac{97}{423}$ lb.
55. $1,681\frac{3}{7}$ bu. wheat.
$1,255\frac{7}{15}$ bu. rye.
$564\frac{24}{35}$ bu. oats.
56. $1.56\frac{1}{4}$.
57. $83\frac{37}{60}$ acres.
58. $\frac{5}{83}$.
59. $33,600 from B.
$11,200 from C.
$\frac{7}{40}$ remaining.
60. 4 of each.
$\frac{3}{16}$ uninjured.
61. $33,918\frac{12}{16}$ loss.
62. $9\frac{919}{1260}$.
63. $364\frac{1}{4}$.
64. 5 sons.
$19,284 each son's share.
65. $3,213\frac{105}{123}$ in land.
$2,410\frac{16}{41}$ in cotton.
$1,483\frac{169}{333}$ in grain.
$9,641\frac{23}{41}$ in all.

Art. 247.

1. $\frac{1}{4}$, $\frac{1}{4}$, $\frac{10253}{50000}$, $\frac{3}{4}$.
2. $\frac{1}{8}$, $\frac{3}{400}$, $\frac{1}{80}$, $62\frac{1}{4}$.
3. $6\frac{9}{40}$, $80\frac{1}{40}$, $8\frac{3}{50}$, $15\frac{2}{100}$.
4. $120\frac{1}{80}$, $\frac{13}{80}$, $\frac{9}{16}$, $\frac{627}{50000}$.
5. $\frac{141}{400}$, $3\frac{21}{40}$, $37\frac{3}{4}$, $62\frac{1}{40}$.
6. $\frac{14}{75}$, $\frac{1}{240}$, $2\frac{421}{3500}$, $\frac{31}{300}$.

Art. 249.

1. .675, .125, .28, .024, .15.
2. .0003, $12.75, 25.075, 300.0075, 32.
3. 6.3775, .07125, 1.406, 21.016.
4. 2.5625, .04, 7.8125, .512, $.066\frac{2}{3}$.
5. .16, .04, .025, .064, $.185185\frac{5}{27}$.
6. .325, .0175, $.153\frac{1}{8}$, .0032, $.0013\frac{1}{3}$.
7. .04375, $12.15, 25.032, 37.1625.
8. $.00083\frac{1}{3}$, $.076\frac{2}{3}$, 56.0875, 19.0125.
9. $.190476\frac{4}{21}$, $.4571428\frac{5}{7}$, .54.
10. $49.8333\frac{1}{3}$, 6.5, $11.4117647\frac{1}{17}$, 17.034.

Art. 250.

1. $.41\dot{6}, .4\dot{6}, .\dot{6}1538\dot{4}$, $.\dot{6}4285\dot{7}\dot{1}$.
2. $2.\dot{2}1052631578947368\dot{4}$.
   $.\dot{5}, 1.\dot{1}, \dot{4}4\dot{0}, 1.\dot{1}$.
3. $\frac{17}{99}, \frac{2}{1}, \frac{4}{45}, \frac{1}{9}$.
4. $\frac{23}{180}, \frac{378}{45}, \frac{115}{9}, \frac{407}{99}$.

Art. 252.

3. .370192.
4. 29.43078.
5. 26.5684483875.
6. 416.6625.
7. 124 2331 gal.
8. 460 31945 acres.
9. 61,152.3806185.
10. 53 4028.
11. \$388 15$\frac{1}{4}$.
12. \$20 82$\frac{1}{2}$.
13. 259.4 sq. mi.
14. 189,380.152453.
15. 2,820.301286.

Art. 255.

1. .1407.
2. 16 825.
   \$.49025.
3. \$12 0180.
   .0001875.
4. 1.7723265$\frac{10}{21}$.
5. .1532+.
   240.413.719$\frac{7}{11}$.
6. 23,594,860.86 persons.
7. 1.941 6325 ft.
8. \$.148628.
9. \$2,277.50.
10. 1,443 75 cu. in.

Art. 256.

2. 134.02.
   13.402.
   134 020.
3. 76,025.
   87.5.
4. \$125.
   \$137.50.
6. \$2,070.
7. \$5,912.50.
8. \$3,638.75.
9. \$6,918.75.
10. 702,666.6.

Art. 257.

1. .739,246.
2. 1,329.10875.
3. .3804803.

Art. 258.

3. 2.04925.
4. 2.72375.
5. .035144.
6. .8749938.
7. 7.00105.
8. 11 94503125 yd.
9. .6999993.
10. .9990999.
11. 1.9125.
    8.19241.
12. 0.
    .003.
13. \$15 gain.
14. 1741 515.
15. 7648.3205.

Art. 261.

2. 400.
   2 5.
   250.
3. .025.
   1600.
   1600.
4. .0016.
   .00005.
   .00001.
5. .1344.
   64.
   2.4.
6. .0125.
   .006.
   .08.
7. .8.
   80.
   500.
8. $150.46\dot{4}$ lb.
9. 29$\frac{1}{247}$ mi.
10. 8.86$\frac{1025}{60625}$ da.

Art. 262.

1. 7.042.
   1.09125.
   .4.
2. 1.1072.
   .400235.
   12.345.
3. .005.
   .082.
   .0115.
   .42875.
4. .1875.
   .0000473.
   .000035.
   .0000006.
6. \$.06$\frac{1}{4}$.
7. \$.01$\frac{1}{4}$ per foot.
8. \$1.14$\frac{2}{5}$.
9. \$.14$\frac{2}{3}$.
10. \$37.50.

Art. 263.

3. .403258.
4. 1.20088.
5. 11.3526477.
6. .16039.
7. 4.2903.
8. 365.329.

Art. 264.

1. \$3571.776.
2. 90,000 ft.
3. \$50,834,935.80.
4. \$16.875,000.
5. 62.720 lb.
6. \$70,257.61$\frac{63}{427}$.
7. 178 lb.
8. Save 50 cts. per week at the latter rate.
9. 17.42335$\frac{45}{254}$ lb.
10. \$.88$\frac{5}{9}$.
11. $153\,16\dot{4}$ total wt. $51.0\dot{5}48\dot{1}$ lb. av. weight.
12. 17$\frac{1}{4}$ inches.
13. 4$\frac{4}{11}$ miles.
14. \$3,057$\frac{1}{7}$.
    \$857$\frac{1}{7}$.

16. $5,274,339\frac{7}{9}$.
17. $3,811,058\frac{2}{11}$.
18. $260,922\frac{16}{27}$.
19. $776,914\frac{1}{8}$.
20. $13,946,832\frac{9}{17}$.

Art. 225.

3. $\frac{1}{12}$.
4. $\frac{8}{77}$.
5. $\frac{39}{220}$.
6. 0.
7. $\frac{17}{80}$.
8. $763\frac{11}{12}$.
9. $24\frac{22}{35}$.
10. $446\frac{7}{8}$.
11. $\frac{47}{72}$.
12. $1\frac{1}{4}$.
13. $843\frac{1}{3}$.
14. $8\frac{5}{8}$.
15. $59\frac{23}{240}$.
16. $\frac{1}{10}$.
17. $1\frac{1}{6}$.
18. $16\frac{2}{7}$.
19. $9\frac{14}{15}$.
20. $275\frac{19}{24}$.

Art. 227.

1. $\frac{5}{51}$, $\frac{7}{80}$, $\frac{4}{75}$.
2. 12, 25, $33\frac{1}{3}$.
3. 2, $1\frac{5}{9}$, $\frac{18}{25}$.
4. $\frac{7}{51}$.
5. $64\frac{4}{5}$.
6. 2.

Art. 228.

4. $25\frac{13}{16}$, $3,556\frac{14}{15}$.
5. $200\frac{1}{12}$, $144\frac{1}{18}$.
6. 48, 336, 18.
7. 46,296, 8, $14\frac{2}{5}$.
8. 1,136.
9. $1\frac{1}{5}$, $1\frac{1}{2}$, $\frac{9}{10}$.
10. $\frac{18}{25}$, $1\frac{71}{304}$.
11. $\frac{2}{11}$.
12. $\frac{54}{88}$.
13. $1\frac{1}{35}$.
14. $2\frac{1}{4}$.
15. $5\frac{11}{23}$.
16. $\frac{4}{15}$.
17. $1\frac{309}{750}$.

18. $24\frac{17}{30}$.
19. $1\frac{17}{51}$.
20. $82\frac{113}{160}$.

Art. 231.

31. $2\frac{5}{6}$.
32. $\frac{5}{24}$.
33. $2\frac{1}{3}$.
34. $\frac{4}{21}$.
35. $5\frac{5}{6}$ sum.
$1\frac{1}{6}$ difference.
$8\frac{1}{6}$ product.
$1\frac{1}{2}$ quotient.
36. 2.58\frac{1}{3}$.
37. $2\frac{4}{7}$ tubs.
38. $\frac{7}{12}$ bu.
39. $1,614\frac{13}{40}$ lb.
40. $4,765\frac{439}{1260}$ lb.
41. $986\frac{7}{72}$ acres.
42. 355.61\frac{77}{120}$.
43. $226\frac{23}{330}$ acres.
44. 3.17\frac{3}{16}$.
45. $19,200.
46. $\frac{1}{8}$ of the orange.
47. $12.
48. 8.02\frac{1}{12}$.
49. 3.28\frac{1}{8}$.
50. 13.78\frac{1}{8}$.
51. $\frac{5}{32}$ left.
Worth $15,000.
52. $72,000.
53. $\frac{1}{24}$ acre more than B.
$\frac{1}{9}$ times more.
Both own $\frac{19}{24}$ A.
54. $5,561\frac{97}{423}$ lb.
55. $1,681\frac{3}{7}$ bu. wheat.
$1,255\frac{7}{15}$ bu. rye.
$564\frac{34}{35}$ bu. oats.
56. 1.56\frac{1}{4}$.
57. $83\frac{37}{60}$ acres.
58. $$\frac{5}{83}$.
59. $33,600 from B.
$11,200 from C.
$\frac{7}{40}$ remaining.
60. 4 of each.
$\frac{3}{70}$ uninjured.
61. $33,918$\frac{13}{16}$ loss.
62. $9\frac{919}{1260}$.

63. 364\frac{1}{4}$.
64. 5 sons.
$19,284 each son's share.
65. $3,213$\frac{105}{123}$ in land.
$2,410$\frac{16}{41}$ in cotton.
$1,483$\frac{169}{333}$ in grain.
$9,641$\frac{23}{41}$ in all.

Art. 247.

1. $\frac{1}{4}$, $\frac{1}{4}$, $\frac{10253}{50000}$, $\frac{3}{4}$.
2. $\frac{1}{8}$, $\frac{3}{400}$, $\frac{1}{80}$, $62\frac{1}{4}$.
3. $6\frac{9}{40}$, $80\frac{1}{40}$, $8\frac{3}{30}$, $15\frac{2}{105}$.
4. $120\frac{1}{80}$, $\frac{13}{80}$, $\frac{9}{16}$, $\frac{627}{50000}$.
5. $\frac{141}{400}$, $3\frac{21}{40}$, $37\frac{3}{4}$, $62\frac{1}{40}$.
6. $\frac{14}{75}$, $\frac{1}{240}$, $2\frac{421}{3500}$, $\frac{31}{300}$.

Art. 249.

1. .675, .125, .28, .024, .15.
2. .0003, $12.75, 25.075, 300.0075, 32.
3. 6.3775, .07125, 1.406, 21.016.
4. 2.5625, .04, 7.8125, .512, .066$\frac{2}{3}$.
5. .16, .04, .025, .064, .185185$\frac{5}{27}$.
6. .325, .0175, .153$\frac{1}{3}$, .0032, .0013$\frac{1}{3}$.
7. .04375, $12.15, 25.032, 37.1625.
8. .00083$\frac{1}{3}$, .076$\frac{2}{3}$, 56.0875, 19.0125.
9. .190476$\frac{4}{21}$, .4571428$\frac{4}{7}$, .54.
10. 49.8333\frac{1}{3}$, 6.5, 11.4117647$\frac{1}{17}$, 17.034.

***Art.* 250.**

1. $.41\dot{6}, .\dot{4}\dot{6}, .\dot{6}1538\dot{4}$,
$.6\dot{4}2857\dot{1}$.
2. $2.\dot{2}10526315789$-
$473684\dot{1}$.
$.\dot{5}$, $1.\dot{1}$, $\dot{4}4\dot{0}$, $1.\dot{1}$.
3. $\frac{17}{99}, \frac{2}{3}, \frac{2}{45}, \frac{1}{3}$.
4. $\frac{23}{180}, \frac{379}{45}, \frac{115}{9}, \frac{407}{99}$.

***Art.* 252.**

3. .370192.
4. 29.43078.
5. 26.568483875.
6. 416.6625.
7. 124.2331 gal.
8. 460.31945 acres.
9. 61,152.3806185.
10. 53.4028.
11. \$388.15$\frac{1}{2}$.
12. \$20.82$\frac{1}{2}$.
13. 259.4 sq. mi.
14. 189,380.152453.
15. 2,820.301286.

***Art.* 255.**

1. .1407.
2. 16.825.
\$.49025.
3. \$12.0180.
.0001875.
4. 1.7723265$\frac{10}{21}$.
5. .1532$\frac{3}{7}$.
240,413.719$\frac{7}{33}$.
6. 23,594,860.86 persons.
7. 1,941.6325 ft.
8. \$.148628.
9. \$2,277.50.
10. 1,443.75 cu. in.

***Art.* 256.**

2. 134.02.
13,402.
134,020.
3. 76,025.
87.5.
4. \$125.
\$137.50.
6. \$2,070.
7. \$5,912.50.
8. \$3,638.75.
9. \$6,918.75.
10. 702,666.6.

***Art.* 257.**

1. .739,246.
2. 1,329.10875.
3. .3804803.

***Art.* 258.**

3. 2.04925.
4. 2.72375.
5. .035144.
6. .8749938.
7. 7.00105.
8. 11.94503125 yd.
9. .6999993.
10. .9990999.
11. 1.9125.
8.19241.
12. 0.
.003.
13. \$15 gain.
14. 1741 515.
15. 7648.3205.

***Art.* 261.**

2. 400.
2.5.
250.
3. .025.
1600.
1600.
4. .0016.
.00005.
.00001.
5. .1344.
64.
2.4.
6. .0125.
.006.
.08.
7. .8.
80.
500.
8. $150.46\dot{4}$ lb.
9. 29$\frac{1}{124}$ mi.
10. 8.86$\frac{106}{80525}$ da.

***Art.* 262.**

1. 7.042.
1.09125.
.4.
2. 1.1072.
.400235.
12.345.
3. .005.
.082.
.0115.
.42875.
4. .1875.
.0000473.
.000035.
.0000006.
6. \$.06$\frac{1}{4}$.
7. \$.01$\frac{1}{4}$ per foot.
8. \$1.14$\frac{5}{8}$.
9. \$.14$\frac{3}{8}$.
10. \$37.50.

***Art.* 263.**

3. .403258.
4. 1.20088.
5. 11.3526477.
6. .16039.
7. 4.2903.
8. 365.329.

***Art.* 264.**

1. \$3571.776.
2. 90,000 ft.
3. \$50,834,935.80.
4. \$16,875,000.
5. 62.720 lb.
6. \$70,257.61$\frac{53}{427}$.
7. 178 lb.
8. Save 50 cts. per week at the latter rate.
9. 17.42335$\frac{35}{254}$ lb.
10. \$.88$\frac{8}{9}$.
11. $153\ 16\dot{4}$ total wt.
$51.05\dot{4}8\dot{1}$ lb. av. weight.
12. 17$\frac{1}{2}$ inches.
13. 4$\frac{2}{11}$ miles.
14. \$3,057$\frac{1}{7}$.
\$857$\frac{1}{7}$.

15. $1,384,194,348.308$\frac{1}{3}$.
16. 3,765$\frac{285}{311}$.
17. 2,280,022$\frac{2}{9}$ bu.
18. $122.35 gain.
19. 363,142$\frac{5}{7}$ bu.
20. 42.

Art. 277.

1. 4.704\frac{3}{8}$.
2. $400.
3. 3$\frac{1}{8}$ hr.
4. 14 sheep.
5. 84 cattle.
6. 1428 ft.
7. 51 cts.
8. 224.273.
9. 496.62$\frac{11}{12}$.
10. 46,833.43$\frac{211}{323}$.
11. 30,398.8.
12. 2.0020326250.
13. $1400 A.
    $480 B.
    $453.60 C.
    $260 D.
14. $180 from his wife.
    $300 from speculation.
    $450 from rise of property.
    $375 from uncle.
    $195 from father.
15. $698.50 left.
16. 24%.
17. 4%.
18. 30%.
19. 15%.
20. 5%.
21. 26$\frac{2}{3}$%.
22. 375%.
23. 88% copper.
    12% nickel.
24. 25%.
25. 27$\frac{7}{9}$% for board.
    31$\frac{1}{9}$% for clothing.
    5$\frac{5}{9}$% for books.
    6$\frac{2}{3}$%.
    28$\frac{8}{9}$% left.
26. 60%.
28. 450.
29. $24.
30. $45.
31. $5.
32. 100 cents.
33. $75,000.
34. $28,000.
35. $8,750.
37. 2,500 sheep.
38. 140.
39. $75.
40. 20,000.
41. 4.88\frac{8}{9}$.
    4.22\frac{2}{9}$.
    3.77\frac{7}{9}$.
42. $40,000.
43. $71,300.
44. 28%.
45. 14%.
46. $116.25.
47. $57,200 older son's.
    46,800 younger.
48. $110.
49. 900.
50. 3$\frac{1}{13}$%.
51. 25 yd.
52. 68 lb.
53. 1,711,953.
54. 467,418.
55. 305$\frac{1}{4}$ +.
56. 92,599 nearly.
57. 1,457,364 nearly.
58. 4$\frac{657}{1000}$% nearly.
59. Nearly 61$\frac{48}{125}$% greater.
60. Real estate, $3800.

Art. 293.

5. 800 pt.
6. 140 perches.
7. 44,400 sec.
8. 378 pt.
9. 750 lb.
10. 44.8 pt.
11. 140 nails.
12. 3,840 min.
13. 736 solid feet.
14. 89.1 in.
15. 20,736 cu. in.
16. 2,916 qt.
17. 1,332 pwt.
18. 83.84 oz.
19. 40,320 gr.
20. 2.6 qr.
21. 115.2 sheets.
22. 78$\frac{1}{5}$ lb.
23. 1930$\frac{2}{3}$ dr.
24. $\frac{12}{125}$ oz.

Art. 294.

3. 23 gal. 2 qt. 1 pt.
4. 10 oz. 16 pwt.
5. 4 yd. 1 ft. 9 in.
6. 2 qr. 6 lb. 4 oz.
7. 30 hhd. 27 gal. 2 qt. 2 gi.
8. 4 fur. 17 rd. 4 yd. 10 in.
9. 250 lb. 4 oz. 4 pwt.
10. 6 fur. 16 rd.
11. 13 s. 4 d.
12. 3 qt. 1.2 pt.
13. 2 da. 19 h. 12 min.
14. 3 pk.
15. 13 hr. 30 min.

Art. 295.

5. 6 hhd. 36 gal.
6. 10 lb. 3 oz. 1 pwt.
7. 21 bu. 6 qt. 1 pt.
8. 12 mi. 2 fur.
9. 3 sq. mi.
10. 2 hhd.
11. $\frac{1}{210}$ hhd.
12. $\frac{7}{360}$ da.
13. $\frac{1}{10}$ bu.
14. 17 yd. 2 ft. 9 in.
15. 17 cd.
16. 4 hhd. 32.2 gal.
17. 38 yd. 3 qr. 3 na.
18. .09 lb.
19. 1.1 wk.
20. 25 A.

Art. 296.

2. $\frac{65}{81}$ w.
3. $\frac{29}{64}$.

4. $13\frac{1}{4}$.
5. .175.
6. $\frac{7}{40}$.
7. .375.
8. $\frac{1}{12}$.
9. $\frac{9}{128}$.
10. .2.
11. $\frac{4}{5}$.
12. .85.
13. .875.
14. $\frac{8}{15}$.

Art. 297.

1. 119 gal. 2 qt. $3\frac{5}{17}$ gi.
2. 260 $\frac{4791}{5773}$ qt.
3. Gains.
4. $539\frac{7}{12}$ dr.
5. 13 lb. $12\frac{93}{175}$ oz.
6. 10 prescriptions.
7. 16 lb. $11\frac{1}{36}$ oz.
8. $3\frac{67}{130}$ oz.
9. 77 lb. $12\frac{197}{431}$ oz.

Art. 299.

2. 122 bu. 3 pk. 1 qt.
3. 90 gal. 2 qt. 1 pt. 1 gi.
4. 502 A. 1 R. 15 P.
5. 62 yd. 3 qr. 1 na.
6. 67 m. 7 fur. 37 rd. 3 yd. 4 in.
7. 49 w. 15 h. 13 mi.
8. 16 s. 12° 46′ 34 ″.
9. 48 m. 4 fur. 6 ch. 3 p. 9 l.
10. 170 C. 72 cu. ft. 231 cu. in.
11. 211° 41 mi. 6 fur. 10 rd. 2 ft. 8 in.
12. 2 sq. mi. 324 A. 3 R. 2 P. 45 sq. ft. 114 sq. in.
13. 1 bu. 2 pk. 6 qt. $1\frac{4}{5}$ pt.
14. 34 yd. $\frac{8}{9}$ na.
15. 9 mi. 33 rd. 2 yd. 2 ft. $10\frac{34}{99}$ in.
16. 23 A. 144 sq. rd. 17 sq. yd. 5 sq. ft. 117 sq. in.
17. 7 T. 7 cwt. 54 lb. 11 oz. $297\frac{5}{7}$ gr.
18. 33 mi. 4 fur. 11 rd. 4 yd. 1 ft. $2\frac{4}{5}$ in.
19. 9 sq. rd. 29 sq. yd. 8 sq. ft. 130 sq. in.
20. 7 d. 21 h. 14 m. $44\frac{4}{7}$ s.

Art. 301.

2. 3 fur. 24 rd. 1 yd. 2 ft.
3. 6 oz. 17 pwt. 17 gr.
4. 209 bu. 3 pk. 4 qt. 1 pt.
5. 378 yd. 3 qr. 1 na.
6. 531 gal. 1 pt.
7. 8 m. 5 fur. 3. rd. 10 ft. 2 in.
8. 20 A. 17 P. 94 sq. ft. 36 sq. in.
9. 232 bu. 1 pk. 2 qt.
10. 3 T. 8 cwt. 59 lb. 2 oz.
11. 71 lb. 7 oz. 7 pwt. 12 gr.
12. 21 A. 3 R. 16 sq. rd.
13. 5 d. 21 hr. 22 m. 30 s.
14. 107 yd.
15. 9 mi. 1 fur. 24 rd. 12 ft.

Art. 303.

3. 3 fur. 36 rd. 2 ft.
4. 1 T. 6 cwt. 1 qr. 17 lb.
6. 38 yr. 6 m. 4 d.
7. 84 yr. 17 d.
8. 36 yr. 12 d.
9. 398 A. 8 P.
10. 1 m. 2 fur. 3 rd. 7 ft. 6 in.
11. 1° [illegible]′ 20″.
12. 71° 45′ 12″.
13. 8″
14. 33° 4′.
15. 55° 57′.

Art. 305.

2. 225 A. 1 R. $28\frac{4}{7}$ P.
3. 7 yd. 1 qr. $1\frac{3}{5}$ na.
4. 1 cwt. 1 qr. 2 lb. $13\frac{5}{7}$ oz.
5. 1 cwt. 3 qr. 5 lb.
6. $301\frac{5}{7}$ rotations.
7. $146\frac{2}{3}$ curbstones.
8. 21 baskets.
9. 1.841 dollars. 11.98 gr. remain.
10. 12 d. 4 h. 11 m. $5\frac{77}{306}$ s.
11. $298\frac{2}{3}$.
12. $6\frac{30}{91}$.
13. $7\frac{41}{45}$.
14. $\frac{20}{171}$.
15. $\frac{1}{13200}$.

Art. 306.

1. 11 m. $44\frac{4}{7}$ s.
2. 4 hr. 53 m. $37\frac{11}{30}$ s.
3. 87° 25′ 35″ W.
4. 6° 19′ 33″ W.
5. 15 h. 18 m. 20 s. later, or 3 hr. 18 m. 20 s. next A. M.
6. 11 hr. 53 m. $16\frac{2}{15}$ s.
7. 12° 31′ 45″ E.
8. 6 hr. 34 m. 12 s. A. M.
9. 10 hr. 44 m. 4 s. A. M.
10. 8 hr. 7 m. 48 s. P. M.
11. P. 7 hr. 45 m. 48 s. P. M.
J. 2 hr. 20 m. 52 s. P. M.
L. 11 hr. 59 m. $36\frac{4}{7}$ s. A. M.
P. 12 hr. 9 m. $21\frac{1}{4}$ s. P. M.

B. 12 hr. 54 m. P. M.
N. Y. 7 hr. 3 m. 59$\frac{4}{5}$ s. A. M.
C. 6 hr. 9 m. 28 s. A. M.
St. A. 6 hr. 33 m. 40 s. A. M.
H. 12 hr. 39 m. 54$\frac{1}{5}$ s. P. M.

Art. 312.

1. 439.135.
2. 148 A. 3 R. 7 sq. rd. 9 sq. yd. 5 sq. ft. 122.4 sq. in.
3. 4,213 lb. 7$\frac{139}{175}$ oz.
4. 360 sq. meters.
5. 19$\frac{1229}{3937}$ da.
6. 922 bu. 6 qt.
7\. 238.47+ liters.
8. 88.905$\frac{185}{11023}$ kilograms.
9. 2,067 C. 10 cu. ft. 1,465.344 cu. in.
10. 2,267 coins. 4,805 miligrams remain.

Art. 315.

9. 1,583 ft. 7″ 4‴.
10. 1,132 ft. 4″.

Art. 316.

1. 13 ft. 7′ 8″.
2. 12 ft. 5′.
3. 9 ft. 1′ 10″ 4‴ 10$\frac{11}{47}$⁗.
4. 37 yd. 1′ 9″ 2‴ 1$\frac{7}{17}$⁗.
5. 2 ft. 10′ 9″ 8‴ 6$\frac{30}{131}$⁗.
6. 68 ft. 10′ 2″ 4‴ 3$\frac{39}{163}$⁗.
7\. 37 ft. 11′ 1″ 11‴ 6$\frac{222}{373}$⁗
8. 27 ft. 7′ 2″ 2‴.
9. 131 ft. 18′ 8″ 3‴ 3⁗.
10. 11′ 8‴ 3+⁗.

Art. 317.

1. \$2.60.
2. \$2.25.
3. \$4.68$\frac{3}{4}$.
4. 3$\frac{1}{21}$ ft.
5. \$19.11$\frac{2}{3}$.
6. 9$\frac{3}{8}$ C.
7\. \$283.50.
8. \$16.08.
9. \$704.
10. \$8.22$\frac{2}{9}$.
11. \$67.62.
12. \$46.44.
13. 21,168 brick. \$84.67.
14. 4,375 bu. \$546.87$\frac{1}{2}$.
15. \$9.37$\frac{1}{2}$.
16. \$60.40.
17. \$3,210.
18. \$94.50.
19. \$9.33$\frac{1}{3}$.
20. \$110.
21. 10,800 bu.
22. 880,000 yd.

Art. 325.

1. 200. 1,104.
2. 3.69+.
3. 4.509+.
4. 1.99+.
5. 1.134+. 13.4+%.
6. 2.846+.
7\. 7.3+. 630.19+%.
8. 1.464+.
9. 1.728+.
10. 1.227+.
11. 1,111.102+.
12. 755.15+.
13. 524.49+.
14. .0835+.
15. .087+.
16. \$192.28.
17. 39$\frac{1}{3}$ yd.
18. 645.
19. \$332.50.
20. 20.

Art. 328.

1. \$6,000.
2. 22,250 inhabit'ts.
3. \$10,000.
4. \$8,000.
5. \$2,600 − \$1,800 = \$1,800 − \$1,000, etc.

Art. 329.

1. 12.
2. 72.
3. 14.
4. 3.
5. $\frac{15}{16}$.
6. 25.
7\. $\frac{20}{37}$.
8. 375.
9. 4.7.
10. $\frac{3}{4}$.
11. $\frac{4}{5}$.
12. $\frac{1}{8}$.

Art. 331.

2. 49 Apples.
3. \$7.65.
4. 11 cows.
5\. 20 men.
6. 108 miles.
7\. 2$\frac{2}{15}$ oz.
8. 30 yd.
9. \$37.50.
10. \$1.86$\frac{3}{4}$.
11. A \$1,383. B \$1,400. C \$770.
12. A \$504. B \$672. C \$784. D \$840.

13. $187\frac{1}{3}$ qt.
14. $105\frac{15}{17}$ T.
15. \$502.93 +.
16. 3 men.
17. 13 yr. $9\frac{3}{8}$ m.
18. $972\frac{4}{9}$ bu.
19. 4,275,000 ft. lumber.
20. $2,131\frac{9}{61}$ bu.
21. $12\frac{5}{7}$¢.
22. 11 hr. 6 m.
23. $145\frac{5}{7}$ ft.
24. \$242 and \$308.
25. \$$198.33\frac{1}{3}$.
26. \$3.121.51 +.
27. \$264,705.88 +.

Art. 332.

2. \$14,580.
3. \$176.
4. $34\frac{2}{13}$ days.
5. 225 men.
6. $66\frac{10}{27}$ rd.
7. $2\frac{2}{3}$ miles.
8. $4.35\frac{805}{17017}$ rd.
9. 5,315 lb.
10. \$4,536.
11. $78\frac{44}{135}$ Cd.
12. $12,270\frac{11}{39}$ lb.
13. 20,525 lb.
14. 1,808.1 lb.
15. $49\frac{13}{93}$ cu. ft.

Art. 333.

16. 149 m. 2 f. $39\frac{7}{72}$ rd.
17. \$3.94.
18. \$127.29.
19. \$2.06.
20. \$437.93.
21. \$83.125.
22. \$147.17.
23. \$76.56.
24. 439 m. 6 f. $38\frac{1}{3}$ rd.
25. 53 lb. 13 oz. $14\frac{7}{16}$ dr.
26. 13 A. 3 R. $13\frac{3}{4}$ P.
27. \$104.57.
28. \$19.375.
29. \$29,257.25.
30. \$30.43.
31. \$$367.81\frac{1}{4}$.

Art. 334.

1. $2\frac{26}{47}$ da.
A & B in $3\frac{3}{7}$ da.
B & C in $4\frac{12}{37}$ da.
A & C in $3\frac{3}{4}$ da.
2. A \$9, B \$8, C \$7.
3. 18 and 21.
4. Horse, \$100.
Harness, \$60.
5. 32; 24; 16.
6. \$4,800.
1st. \$1,400.
2d. \$1,300.
3d. \$1,100.
4th. \$1,000.
7. \$35,000.
\$40,000.
\$45,000.
8. \$1,800.
9. A \$16.
B \$32.
C \$48.
10. 2 hr. $10\frac{10}{11}$ m.
11. Corn 48¢, barley 70¢.
12. 7 hr.
13. $11\frac{9}{17}$ oz.
14. $7\frac{31}{77}$ da.
15. S. 60 yr. J. 26 yr.

Art. 336.

3. $14\frac{4}{5}$¢.
4. 70¢.
5. \$$1.39\frac{1}{4}$.
6. 10¢.
7. Entire gain $23\frac{1}{8}$¢.
8. 36 lb.
24 lb.
12 lb.
18 lb.
$17\frac{2}{13}$¢.
9. $92\frac{2}{19}$¢.
10. $6\frac{12}{25}$¢.

Art. 339.

3. 100 lb. @ 12¢,
100 lb. @ 15¢,
200 lb. @ 10¢,
100 lb. @ 8¢.
4. 30 gal.
5. 16 gal.
6. 15 oz.

Art. 342.

2. $2\frac{1}{2}$ lb. @ 7¢.
2 " 10
1 " 12
3. 1 lb. @ 50¢.
1 " 60
1 " 70
6 " 90
4. $1\frac{1}{3}$ bu. oats.
1 " corn.
1 " rye.
1 " wheat.
5. 1 lb. @ 60¢.
1 " 75
1 " 80
$2\frac{1}{3}$ " 100
6. 5 gal. @ \$1.20.
5 " \$1.40.
3 " water.
7. 3 parts @ 22.
1 " of each oth'r.
8. 8 gal. @ \$1.
4 " \$1.20.
8 " \$1.50.
16 " \$1.60.
9. 40 gal. water.
120 " brandy.
120 " rum.
10. 7 lb. @ \$.85.
13 " \$1.05.
4 " \$.95.
1 " \$1.10.
11. 10 lb. @ $14\frac{1}{3}$¢.
5 " $13\frac{1}{3}$
1 " $12\frac{2}{3}$
5 " $12\frac{1}{3}$
12. 1 lb. @ 16¢.
3 " $17\frac{1}{4}$
10 " $18\frac{3}{4}$
4 " $19\frac{1}{4}$

13. 9 gal. @ 80¢.
6 " 90
2 " 115
1 " 130
4 " 145

14. 60 lb. @ 14¢.
180 " $13\frac{1}{2}$
420 " 13
480 " $12\frac{1}{2}$
60 " 12

15. 1 turkey.
3 geese.
7 ducks.
2 chickens.

Art. 353.

1. 1,953,125.
2. 262,144.
3. 20.25.
4. 175,616.
5. $\frac{16}{49}$.
6. 81.
7. 9,801.
8. 997,001.
9. 243.
10. 970,299.
11. 984,003,999.
12. 1,000,000.
13. 1,785 [illegible].
14. 735,091 [illegible].
15. 1,418,519,112,-
256.
16. 2,357,947,691.
17. 1,833,82,804,125,-
988,210,868,224.
18. [illegible]
19. 28,722,900,390,-
625.
20. 30,517,578,125.
21. 918,330,048.
22. .00,003,656,158,-
440,581,376.
23. 2,565,784,513,-
950,347,900,-
390,625.
24. 50,805 [illegible].
25. [illegible]
26. 630 $\frac{7}{125}$.

Art. 356.

4. 1,664,100.
5. 614,125.
6. 474,552.
7. 103,823.
8. 1,521.
9. 31.36.
10. 9,604.
11. 1,368,900.

Art. 368.

3. 179.
4. 702.
5. 307.
6. 1.414+.
7. 15.3.
8. .25.
9. 240.
10. 20.615+.
11. 4.242+.
12. 1.808+.
13. 13.3.
14. [illegible]
15. [illegible]
16. $5\frac{1}{2}$.
17. $8\frac{1}{3}$.
18. 576.
19. 15.
20. 19,561.625+.
21. 6,889
22. $\frac{64}{125}$.

Art. 379.

1. 97.
2. 504.
3. 364.
4. 145.
5. 203.
6. 803.
7. 2.3.
8. 2.4836+.
9. $3\frac{4}{5}$.
10. .046.
11. $2\frac{1}{4}$.
12. $\frac{6}{35}$.
13. 2.42+.
14. .25.
15. $21\frac{1}{4}$.
16. 2.758017.
17. 81.32+.
18. 1.0196.
19. 1.5718.
20. 1.442241.

Art. 381.

1. 23.
2. 318.
3. 59.04.
4. 7.
5. 11.514+.
6. 2.1926+.
7. .0684.
8. 490.1853.

Art. 383.

2. 32.286.
3. 9.8902.
4. 2.4769.
5. 6.3095.

Art. 390.

1. \$3.97.
2. $1\frac{59}{81}$ lb.
72 lb.
401 days.
3. 78.
4. 49,500 ft.
5. \$667.95.
6. *a* $8\frac{3}{5}$, *s* $74\frac{2}{5}$.
7. *d* 4, *n* 10.
8. *s* 11,760, *a* 10.
9. *d* $5\frac{2}{5}$, *n* 17.
10. \$870, Dec. 1.
\$873.48 Cr.

Art. 397.

1. \$984.15 last.
\$1476.20 all.
2. 126.2477.
3. \$1,111,111.111.
4. $\frac{1}{3}$.
5. $\frac{7}{9}$.
7. 190 ft.
8. 1.979022+.
9. \$384 the 3d mo.
\$3,069 in all.
10. 104.04.
106.12058.
108.24325+.

Art. 398.

2. 362,880.

Art. 399.

2. 15,120.

Art. 414.

1. $281\frac{1}{4}$ mills.
   40,305 mills.
   $1,062\frac{1}{2}$ mills.
2. 1500¢.
   2165¢.
   $257\frac{1}{2}$¢.
3. 12,500 mills.
   7,050,000 mills.
   272,100 mills.
4. 90,000 mills.
   $7.05.
   $1 682$\frac{3}{4}$.
5. 2580¢.
6. $43 60.
   1.18\frac{1}{4}$.
   $ 23$\frac{1}{2}$.
7. 211.73\frac{1}{2}$.
8. 240.07\frac{1}{2}$.
9. $107 87$\frac{1}{2}$.
10. $ 50.
11. $124.35.
12. $261.
13. $12.
14. $176 08$\frac{1}{2}$.
16. 750 lb.
17. 40 bbl.
18. 5240.
19. $65.
20. $39,894.35.
21. $313 90.
22. $57 08.
23. 54.62\frac{1}{2}$.
24. 1924 bu.
25. $1 36$\frac{1}{4}$.
26. $2 12$\frac{1}{2}$.
27. $187.10.
28. 433.27\frac{1}{2}$.
29. 110 lb.
30. 18 turkeys.
31. $1.60.
32. $417 09.
33. 50.73\frac{1}{4}$.
34. $162.327.
35. $72.80.

Art. 415.

1. 6 bu. 1 pk.
   27 qr. doll.
2. $1\frac{11}{34}$ yd.
3. $8\frac{4}{5}$ tons.
4. $5\frac{1}{3}$.
5. 25%.
6. 25%.
7. $20\frac{1}{3}$¢.
8. $19.608.
9. $412\frac{1}{2}$ gr.
10. 1.07\frac{7}{64}$.
11. 10.65\frac{15}{16}$.
12. 50.04\frac{206}{261}$.
13. $4,442.79.
14. £2 $9\frac{3}{13}$s.
15. $\frac{3}{8}$%.
16. Designed ratio, 6 to 1.
    Actual ratio, 46 to 1.
17. 3.50\frac{10}{17}$.
18. $273.74 +.
19. $21\frac{9}{16}$ carats fine.
    1,161 gr.
20. 4,275 gr.

Art. 421.

3. $52.24.
4. $53.13.
5. $67.95.
6. 331.26\frac{1}{2}$.
7. $224.
8. $6,725.88.
9. 656.48\frac{1}{4}$.

Art. 423.

1. 171.48\frac{1}{4}$.
2. $45.
3. $21 76$\frac{3}{4}$.
4. 31.98\frac{1}{2}$.
5. $331.21.
6. $14,028.55.
7. $3,274.86.

Art. 429.

1. 766.38\frac{3}{5}$.
2. $973.39.
3. $2,434.65.
4. $45.02.
5. $8.
6. 5,210 fr.
7. $29.41.
8. 3,129.89 rix doll.
9. $48.91.
10. $1,520.83$\frac{1}{3}$.

Art. 437.

1. $5.20.
2. $6.30.
3. $6,757.50.
4. $12\frac{3}{5}$¢.
   14¢.
   $13\frac{3}{10}$¢.
5. $46\frac{2}{3}$¢.
   $61\frac{1}{3}$¢.
   $75\frac{1}{2}$¢.
6. $1.25.
7. $5,000.
8. $9.
9. $50,000.
10. $12\frac{1}{2}$%.
11. $23\frac{1}{13}$%.
12. 20%.
13. $8\frac{4}{7}$%.
15. Cost $122.
    Sold for $158.60.
16. $12,800.
17. $12,375.
19. $720.
20. 4% loss.
21. 0.
22. $23\frac{1}{3}$.
23. $6\frac{1}{4}$% loss.
    3.32\frac{1}{2}$.
24. $1,220 asked.
25. Sold for $65.
27. $2,725 profit.
28. $4.23.
29. 16¢.
30. $10.
31. $56\frac{1}{4}$%.
32. 40 lb.
    $1.65.

33. $10.75.
34. $1.15.
35. 25%.

Art. 444.

1. $414.40.
2. $$1.87\frac{1}{2}$.
3. $3\frac{1}{3}$%.
4. $600.
5. $54.03 paid.
6. $$1,271.87\frac{1}{2}$.
7. $750 expended.
8. $10,926.829.
9. $3,271.464.
10. $863.99 com's'n.
11. $10.596 com's'n.
12. $25.
13. $6,248.75.
14. $222.11.
15. $1\frac{1}{4}$%.

Art. 452a.

Ex. A to B $25 extra.
B to C $50 "
C to D $75 "
D $75 "
D $100 regular.

Art. 456.

2. $159.

Art. 475.

1. $112.50.
2. $39.
3. $\frac{7}{20}$% (last).
4. $738.
5. $58,000.
6. $950.
7. $\frac{2}{11}$%.
8. $424.
9. $44,880.375 loss.
10. $32,000.
11. $120,000.
12. $50,400.
13. $12,000.
14. $80,000.
15. 0.
16. $18.
17. 2. 44¢.
3. 50¢.
4. $3.75.

Art. 482.

1. $270,871.51 last.
2. A $203.40.
B 47.76.
C 6.76.
D 912.75.
E 1,790.70.
F 15,000.00.
4. $2\frac{4}{9}$ mills assessed.
5. $13\frac{9}{13}$¢ per day.
Mr. B. $30.12.

Art. 489.

1. $1,689.16.
2. $1,927.05 duty.
3. $3,286.44 duty.

Art. 493.

2. Loss 9%.
4. 6% of the risks.
5. Ship $5,254.17.
Cargo $3,041.88.
Freight $483.94.
A. $829.61.
B. $1,106.13.
C. $622.20.
D. $483.94.

Art. 497.

2. Pay 80%.
3. Pay 60%.

Art. 520.

1. $8.64.
2. $70.23.
3. $175.86.
4. $.78.
5. $20.30.
6. $116.10.
7. $803.33.
8. $80.
9. $87.25.
10. Nearly 10¢.
11. $698.85.
12. Gain $7.33.
13. $249.49.
18. $205.48.
19. $123 94 N. Y.
$124.00 Ohio.
20. $398.63.
22. $20,000.
23. 8 m. 18 d.
24. 20 yr.
25. 5%.
26. 3 y. 10 m. 3 d.
27. $145.926.
28. $559.89.
29. 7%.
30. 7 m. 20 d.
31. P. W. $917.431.
Disc't $82.569.
32. $$2,307.69\frac{3}{13}$.
33. $172.41.
$200.00.
$172.41.
34. Proc'ds $2,456.25.
P. W. $2,459.016.
35. 27 d.
30 d.
36. 6%.
37. 90 d. at 10%.
38. $20,000.
39. $7\frac{1}{2}$%.
40. £2 7s. 2d. 1 qr.

Art. 522.

2. $4.872.525.
3. $6,954.00.

Art. 531.

1. $833.22 int.
2. $2,651.67.
4. 25,937.
5. 3,200,000 bu.
6. 1 y. nearly 12.6%.
2 y. " 13.4%.
8 y. " 19.3%.
15 y. " 33%.
25 y. " 74%.

Art. 532.

1. $375.
2. $3,600.
3. $6,150.
4. $2,357.79.
5. $17,280.
6. $672.45.
$8,428.45.

Art. 533.

1. 4$\frac{1}{2}$%.
2. 7%.
3. 5%.
4. 9, 7, and 5%.
5. 7%.

Art. 534.

1. 5 yr.
2. 2 yr.
3. 33 yr.
4. 7 yr.; 4 yr.; 3$\frac{1}{2}$ yr.; 3 yr. nearly.
5. 3 yr. 6 mo. 12 d.

Art. 539.

3. $1,436.40, simple.
   $1,453.03, annual.
   $1,453.52, comp'd.
4. $1,107.375, simple.
   $1,126.35, annual.
   $1,127.24, comp'd.
5. $2,575.33$\frac{1}{3}$, simple.
   $2,641.41$\frac{1}{3}$, annu'l.
   $2,645.31$\frac{1}{3}$, comp.
   $2,095.33$\frac{1}{3}$ due.

Art. 541.

2. $551.347.
3. $170.16.

Art. 546.

3. U. S. $523.428.
   Vt. $503.32.
   Merc. $522.22.
4. U. S. $1,393.786.
   Vt. $1,090.125.
   Merc. $1,126.716.
5. U. S. $904.133.
   Vt. $794.414.
   Merc. $883.144.

Art. 596.

1. 53%.
   $78,845.625.
2. $120,000.
3. 177,646.60.
   $104,568,824.26.
   $195,160,175.74.

4. $202.04 in 1869.
   $220.59 in 1870.
   21 1% increase.
5. $298.34 N. Y.
   $276.41 N. Eng.
   7.934% increase.
6. $119,861,945.82.
   20.714%.
   19.2%.
   14.459%.
7. Nov. 19/22.
   $10.25 disc't.
   $739.75 proceeds.
8. July 10/13.
   $10.65 disc't.
   $1,196.35 proc'ds.
9. Oct. 16/19.
   $42 discount.
   $2,958 proceeds.
10. Dec. 23/26.
    $47.59 discount.
    $2,751.99 proc'ds.
12. $218.715.
13. $1,657.165.
14. $7,678.69 Note.
    $7,528.13 P. W.
16. 4 [illegible]%.
    6 [illegible]%.
    7 [illegible]%.
    10 [illegible]%.
18. 4 [illegible]%.
    5 [illegible]%.
    6 [illegible]%.
    7 [illegible]%.
19. $32.72 [illegible] gained.
20. $11,932.32.
    47 days.

Art. 607.

2. $862.555.
3. £506 15s. 1.6d.
4. $607.64.

Art. 622.

1. $1,268.75.
4. $6,718.74.
5. £640 1s. 11$\frac{1}{4}$d.
6. $2,270.
7. $7,498.33.

8. $195.75.
9. 1 m. k. = 35$\frac{1}{2}$¢ nearly.
10. .9518%.
11. m. k. @ 25% premium.
12. $3,792.50.
13. 1$\frac{1}{2}$%.
14. 1$\frac{1}{4}$%.
15. $28.20.
16. About 2% against Spain.
17. 111$\frac{1}{9}$%.
18. Would lose £500.
21. 1$\frac{1}{4}$%.
22. 1$\frac{1}{4}$%.
24. $5,534,203.
25. About 1 %.
26. $56,157,895.
    Actual balance of payments $56,675,123.
27. $7,675,123 against us.
28. $2,524,877 in our favor.

Art. 642.

1. $211.60.
2. $223.91.
3. $204.07.
4. $2,972.38.
5. $1,871.44.
6. $533.84.
7. $5.87.
8. $4.55 loss.
9. L. S. $20,425.
   O. & M. $14.300.
   W. U. T. $12,900.
   Bal. to Cr. $17,409.55.
10. Bal. to Cr. $8,753.11.
11. $525.08.
12. $1,283.94.
13. $46,353.03.
14. $9,090.01.
15. $5,937.50.
16. $431.08.

17. \$274.33.
18. \$1,849.26.
19. \$665.58.
20. \$116,552.74.

Art. 673.

1. \$2,200.
$8\frac{8}{10}\%$.
2. \$14,625.
3. \$1,380.
4. \$3,360.
$14\frac{14}{19}\%$.
5. 12.
\$232.
6. \$27,500.
\$137.50.
7. $7\frac{1}{2}\%$. $7\frac{32}{49}\%$.
8. \$3,000.
9. 50.
10. 201.
11. $\frac{2}{3}\%$ and use of dividend.
12. 15%.
13. 50%.
14. $33\frac{1}{3}\%$.
15. $187\frac{1}{2}\%$.
16. $83\frac{1}{3}\%$ of par stock.
$116\frac{2}{3}\%$ of "
17. £5,994,632 $15\frac{1}{5}$s.
18. $34\frac{58}{100}\%$.
19. Insurance stock.
20. \$400.
\$14,000.
21. $9\frac{1}{11}\%$.
\$30,000.
22. $90\frac{10}{11}\%$.
23. $\frac{72}{100}\%$.
24. \$700.
25. \$2,082.50.
\$134.40.
26. About $5\frac{24}{100}\%$.
27. About $4\frac{592}{1000}\%$.
28. \$1,629.375.
About $5\frac{492}{1000}\%$.
29. \$27,281.25.
30. 75%.
31. 125%.
32. $40\frac{615}{1000}\%$ better.
33. \$6,195.
34. \$252.26.
35. \$9,610.81.
36. \$2,567.77.
37. Seven 8% bonds.
\$207.21 cy.
38. $76\frac{6}{100}\%$.
39. $14\frac{886}{1501}\%$.
40. \$9,139.27.
41. \$6,001.
42. \$20,354.23.

Art. 682.

1. $7\frac{29}{39}$ months.
2. $5\frac{1}{2}$ months.
3. 61 days.
4. July 22.
5. Sept. 10.
6. May 19.

Art. 684.

1. Nov. 19.
2. Oct. 7, 1870.
3. \$364.04.
4. \$1,466.40.
5. Aug. 30.
\$925.65.
6. Purchase Jan. 4, 1868.
8. Aug. 14, 1868.
9. \$1,542.907.

Art. 686.

2. Apr. 26, 1872.
3. 8 months.
4. \$100.

Art. 693.

1. Feb. 25.
2. Jan. 13.
3. Nov. 6.
4. July 17.
5. 828 days previous to Feb. 15, 1871.
6. May 18.
7. 1 yr. from Dec. 17, 1868.
\$113.08.

Art. 697.

1. \$98.13.
2. \$307.492.
3. \$316.563.
4. \$2,106.98.
5. \$1,228.26.

Art. 708.

1. A, \$3,200.
B, \$1,800.
C, \$1,400.
2. A, \$3,000.
B, \$1,800.
3. Loss, \$724.12.
A Dr. to B, \$3,883.71.
4. Gain \$1,896.55.
C, \$1,087.75.
D, \$808.80.

Art. 711.

1. A, \$342.
B, \$504.
2. A, \$75.
B, \$65.
C, \$105.
3. A, \$5,750.
B, \$3,750.

Art. 714.

Ex. Interest in business.
Mr. J. \$31,656.67.
Mr. S. \$27,272.51.
Mr. W. \$17,310.28.
Mr. W. pays
to Mr. J. \$404.86 interest.
to Mr. S. \$555.98 interest.

Art. 715.

2. Mr. G. pays
to Mr. B. \$17.81.
to Mr. L. \$17.81.

Art. 716.

2. A paid B \$324.70.

Art. 717.

2. Present interest D, \$6,032.67.

E, $4,495.02.
F, $3,687.31.
E pays interest
to D. $49.67.
to F, $7.31.

Art. 718.

2. $2,620.

Art. 719.

2. C, $19,606.
D, $19,646.
E, $19,618.
3. F, $7,480.
G, $7,930.
H, $10,392.
I, $5,286.
4. Gain, $9,200.
J. $14,521.67.
K, $11,603.33.
L, $8,701.00.
M, $5,801.67.
N, $2,900.33.
5. Loss, $1,620.
O, $4,325.
P, $4,342.
Q, $4,505.
R, $4,371.

Art. 720.

2. Cap. $10,984.
C, $1,373.
D, $4,119.
E, $5,492.
3. Firm $8,000.
Each partner,
$2,000.

Art. 721.

2. Firm, $3,605.
Each partner,
$901.25.
Firm, $4,133.
C,
D,
E,
F,
3. First insolvency,
Firm, $3,540.
G, $354.
H, $531.
I, $708.
J, $885.
K, $1,062.

Art. 722.

1. B., $20,385.33.
G., $12,847.01.
F., $158.72.
Br., $182.47.
2. Gain, $1,985.58.
P., $7,409.43.
C., $5,00? 82.
R., $3,400.80.
3. Each gains,
$838 43.
1st Cap. of each,
$4,880.10.
H., at close,
$5,718.53.
E., at close,
$4,794.53.
S., at close,
$6,856.53.
4. Adverse.
S. Dr. to firm,
$454.13.
F. Dr. to firm,
$1,143.97.
Firm Dr. to C.,
$111 83.
Firm Dr. to W.,
$1,486.27.
5. Gain, $43 158.
C., $2,459.23.
R., $10 648.85.
G. owes the firm
$996.08.
R. receives
$11,548.85.
6. Gain, $3,242.64.
Gain of each,
$1,621.32.
A. owes B.,
$2,379.12.
7. Gain, $1,860.85.
C, $1,189.45.
D, $1,046.54.
8. 104 weeks.
9. C owes A $36.
10. A, $1,401.15.
B, $1,116.64.
C, $982.21.
12. A, $1,004.65.
B, $789.36.
C, $1,705.99.
13. A Cr. $444.40.
B Dr. $155.60.
C Dr. $155.60.
D Cr. $148.40.
E Dr. $281.60.
14. A, $1,206.15.
B, $?8.48.
15. G., $2,921.14.
W., $1,578.86.
16. Gain, $502.
R. Dr. to G.
$166.82.
R. Dr. to C.
$541 18.
17. Profit, $2 939.48.
R. & Co. Dr.
$4,651.48.
To A., $3,008.14.
To B., $1,233.34.
To hands, $410.
18. Cap., $19,335.60.
B., $7,566.62.
D., $5,903 71.
Y., $5,865.27.
19. 1st Cap., $25,342.
Closing Cap.,
$27,274.
Gain, $5,593.
A's interest,
$11,437 20.
B's interest,
$15,836.80.
20. 1st. yr. Firm,
$8,635.77.
1st yr. K,
$3,711.39.
1st yr. R.,
$4,924.38.
3d yr. Firm,
$7,039.37.

3d yr. K, $2,894.04.
3d. yr. Rice, $3,550.04.
3d yr. Ry., $595.29.
4th yr. Firm, $11,118.55.
4th yr. K, $3,749.33.
4th yr. Rice, $4,933.72.
4th yr. Ry., $2,435.50.

21. 1st. No.
2d. C loses $\frac{1}{3}$ the interest.
3d. A's interest, $29,422.71.
B's interest, $29,422.71.
C. Dr. to Firm, $3,892.42.
4th. Yes.

22. Gain, $205.85.
B. Dr. to A. $484.93.

23. Gain, $12,956.19.
A's " $6,251.87.
B's " $6,704.32.
A's interest, $3,692.01.
B's interest, $4,309.86.
100$\frac{6}{10}$% profit on sales.
170$\frac{9}{10}$% profit on investment.

24. Gain, $2,248.12.
A's interest, $43,441.70.
B's interest, $36,201.42.
C's interest, $28,961.14.

25. Gain, $81,947.50.
A's interest, $24,221.39.
B's interest, $26,401.90.
C's interest, $30,243.72.
D's interest, $31,724.86.
E's interest, $40,891.13.

26. At close of old books,
A's interest, $25,800.00.
B's interest, $10,533.33.
C's interest, $13,666.67.
On new books,
Cr. Cap. Stock, $50,000.
Dr. Property, $15,000.
Dr. Patent %, $35,000.
On Stock Ledger,
Cr. each Stockholder, $10,000.

27. Loss, $2,000.00.
A's interest, $10,687.31.
B's interest, $5,212.69.

28. Due A, $2,180.31.
Due B, $1,145.69.

29. A's net loss, $66.07.
B's net loss, $198 20.

Art. 727.

1. $2,253.87.
2. $4,655.19.
3. $3,566.33.
4. $75.024.
5. $72.12.

Art. 729.

2. $6,746.777.
3. $18,914.23.
4. $1,173.28.
5. $2,640.78.

Art. 730.

2. $60,000.
3. $60.900.
4. $26,666$\frac{2}{3}$.
5. $26,666$\frac{2}{3}$.
6. $26,272.58.
7. $27,060.84.

Art. 731.

2. $13,771.28$\frac{8}{9}$.

Art. 732.

2. Younger, $4,962.385.
3. $6,937.17.

Art. 733.

2. $1,200.
3. $500.
4. $100.
5. $407.22.
6. $1,359.83.

Art. 736.

4. $2,226.79.

Art. 737.

2. $1,955.06.
3. $6,496.48.

Art. 738.

2. $1,114.05.

Art. 745.

3. $.00319.

Art. 746.

5. $44.05.
6. $73.61.
7. $381.04.
8. $290.32.
9. $91.72.

Art. 750.

1. 600 sq. yd.
2. 58.7878 sq. ft.

3. Four times.
4. Base 150 yd.
Alt. 72 yd.
5. 75 rd.
6. 32 rd.
7. $9\frac{1}{3}$ ft.

Art. 754.

1. $11\frac{2}{3}$ sq. ft.
2. 14 sq. feet.
3. $1\frac{3}{4}$ sq. ft.
4. 850 sq. ft.
5. 769.420 sq. ft.
6. 688 191 sq. rd.
7. 60 yd.
8. 1 A. 3.9 sq. rd.
9. 3.51 rd.

Art. 757.

1. 68,093 mi.
2. 3.82 in.
3. 17.825 + in. sq.
4. 139.626 sq. yd.
5. 27.75 yd.

Art. 760.

1. 64 sq. in.
2. 14.13745 sq. in.
3. 106.3627 sq. in.
4. 33.8088 sq. ft.
5. 17.352 sq. ft.
6. 427.2576 sq. ft.
7. 7.794225 inches.
8. $22\frac{1}{2}$ in. sq.
9. .7089816 inches.
10. 3,562.5744 sq. ft.

Art. 762.

1. 69.1152 sq. ft.
2. 360,000 cu. ft.

Art. 764.

1. 3,769.92 sq. ft.
14,510.42 cu. ft.
2. Nearly $9\frac{1}{8}$ cu. ft.
3. 475 sq. ft.

Art. 766.

1. 196,663,355.7504 sq. mi.
2. 523.6 cu. in.
3. 14,832 sq. ft.
4. 5 ft.

Art. 771.

1. 4.
2. 7.8.

Art. 772.

1. 63.193 cu. in.
2. $80,631\frac{77}{128}$ lb.
3. 473.072 oz gold.

Art. 773.

1. 144 ft.
2. 57,600 ft.
1,904 ft.
3. 18 + sec.

Art. 774.

Ex. 192,000 m. per sec.

Art. 775.

Ex. 2,000.

Art. 781.

1. $82\frac{4}{5}$ lb.
$3\frac{3}{4}$ lb.
2. 4 ft.
3. 7.
4. 588 lb.
5. 320 lb.
6. 490 lb.
7. $\frac{1}{4}$.
8. $\frac{1}{16\pi}$.
9. 2,000 lb.

INDEX.

The numbers given refer to *Articles* in the book.

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